

SACRED HEART CANOSSIAN COLLEGE  
2021-2022 MOCK EXAMINATION

**SECONDARY 6**  
**MATHEMATICS Compulsory Part**  
Paper 2 ( Solution )

Time allowed: 1 hour 15 minutes

**GENERAL INSTRUCTIONS**

- (1) The full mark of this paper is 45.
- (2) There are **TWO** sections, A and B, in this Paper. Section A and Section B consist of 30 and 15 multiple-choice questions respectively.
- (3) Read carefully the instructions on the Answer Sheet. After the announcement of the start of the examination, you should first insert the information required in the spaces provided. No extra time will be given for inserting information after the 'Time is up' announcement.
- (4) When told to open this paper, you should check that all the questions are there. Look for the words '**END OF PAPER**' after the last question.
- (5) All questions carry equal marks.
- (6) **ANSWER ALL QUESTIONS.** You are advised to use an HB pencil to mark all the answers on the Answer Sheet, so that wrong marks can be completely erased with a clean rubber. You must mark the answers clearly; otherwise you will lose marks if the answers cannot be captured.
- (7) You should mark only **ONE** answer for each question. If you mark more than one answer, you will receive **NO MARKS** for that question.
- (8) No marks will be deducted for wrong answers.

## Section A

1.  $3^x \cdot 9^{2022y} = 3^x \cdot 3^{4044y} = 3^{x+4044y}$

- A.  $3^{x+2022y}$  .  
 B.  $3^{x+4044y}$  .  
 C.  $27^{x+2022y}$  .  
 D.  $27^{2022xy}$  .

2. If  $\frac{3-a}{a} = \frac{3+b}{b}$ , then  $b =$

- A.  $a$  .  
 B.  $-a$  .  
 C.  $\frac{3-2a}{3a}$  .  
 D.  $\frac{3a}{3-2a}$  .

$$3b - ab = 3a + ab$$

$$3b - 2ab = 3a$$

$$b(3-2a) = 3a$$

$$b = \frac{3a}{3-2a}$$

3. Let  $a$  be a constant. Solve the equation  $(x-1)(x-4) = (a-1)(a-4)$  .

- A.  $x = a$   
 B.  $x = 1$  or  $x = 4$   
 C.  $x = a$  or  $x = 5$   
 D.  $x = a$  or  $x = 5 - a$

$$x^2 - 5x + 4 = a^2 - 5a + 4$$

$$x^2 - 5x - a^2 + 5a = 0$$

$$x^2 - 5x - a(a-5) = 0$$

$$(x-a)[x+(a-5)] = 0$$

$$x = a \text{ or } x = 5 - a$$

$$\begin{array}{r} x \quad -a \\ x \quad a-5 \\ \hline ax - 5x - ax \\ = -5x \end{array}$$

4. If  $p$  and  $q$  are constants such that  $x^2 + p(x-1) + q \equiv (x+6)(x-3)$ , then  $q =$

- A.  $-21$  .  
 B.  $-18$  .  
 C.  $-15$  .  
 D.  $3$  .

Put  $x = 1$ ,

$$1^2 + p(1-1) + q = (1+6)(1-3)$$

$$1 + q = -14$$

$$q = -15 \quad \checkmark$$

5. If  $0.06447 < x < 0.06454$ , which of the following must be true?

- A.  $x = 0.065$  (correct to ~~2~~ decimal places)  
 B.  $x = 0.065$  (correct to 2 significant figures)  
 C.  $x = 0.0645$  (correct to ~~3~~ decimal places)  
 D.  $x = 0.0645$  (correct to 3 significant figures)

6. Let  $a$  and  $b$  be non-zero constants. When the polynomial  $p(x)$  is divided by  $ax + b$ , the remainder is  $R$ . Find the remainder when  $p(x)$  is divided by  $3ax + 3b$ .

- A.  $R$   
 B.  $R - 3$   
 C.  $3R$   
 D.  $\frac{R}{3}$

$$p\left(-\frac{b}{a}\right) = R$$

When  $p(x)$  divided by  $3ax + 3b$ .

$$P\left(-\frac{3b}{3a}\right) = P\left(-\frac{b}{a}\right) = R.$$

7. In a school, 40% of the students are boys. If 80% of the boys and 60% of the girls go to school by bus, what percentage of the students does not go to school by bus?

- A. 32%  
 B. 36%  
 C. 68%  
 D. 72%

$$1 - 0.4 \times 0.8 - 0.6 \times 0.6 = 0.32$$

8. Let  $p$ ,  $q$  and  $r$  be non-zero numbers. If  $p:q = 3:4$  and  $q:r = 2:5$ , then  $(p+2q):(q+2r) =$

- A. 1:4.  
 B. 3:5.  
 C. 11:12.  
 D. 11:24.

$$p : q : r$$

$$\frac{3}{2} : 4 : 5$$

$$\frac{3}{3} : 4 : 10$$

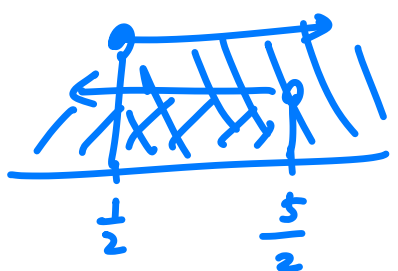
$$3 : 4 : 10$$

let  $p = 3k, q = 4k, r = 10k$ .

$$\frac{p+2q}{q+2r} = \frac{3k+2(4k)}{4k+2(10k)} = \frac{11k}{24k} = 11:24.$$

9. The solution of  $18x - 3 < 12x + 12 \leq 20x + 8$  is

$18x - 3 < 12x + 12$  and  $12x + 12 \leq 20x + 8$   
 $6x < 15$   $4 \leq 8x$   
 $x < \frac{5}{2}$   $x \geq \frac{1}{2}$

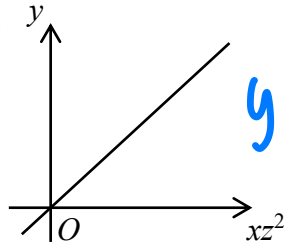
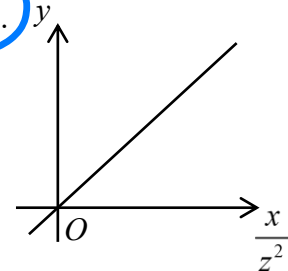
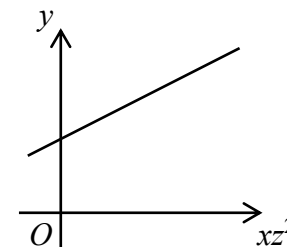
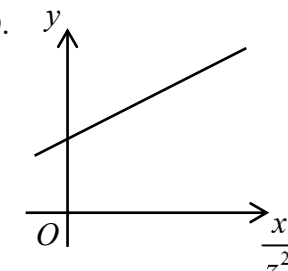


D.  $\frac{1}{2} \leq x < \frac{5}{2}$

$\therefore \frac{1}{2} \leq x < \frac{5}{2}$

10. It is given that  $y$  varies as  $x$  and inversely as the square of  $z$ . Which of the following must be true?

$y = \frac{kx}{z^2}$   
 $y = k \left( \frac{x}{z^2} \right)$

A. 
 B. 
 C. 
 D. 

11. Let  $a$  be a non-zero constant. Which of the following statements about the graph of  $y = (ax - a)^2 - a$  must be true?

$y = a^2x^2 - 2a^2x + a^2 - a$

A. The coordinates of the vertex of the graph are  $(a, -a)$ .  
 B. The graph opens upwards.  
 C. The  $y$ -intercept of the graph is  $a$ .  
 D. The graph does not cut the  $x$ -axis.

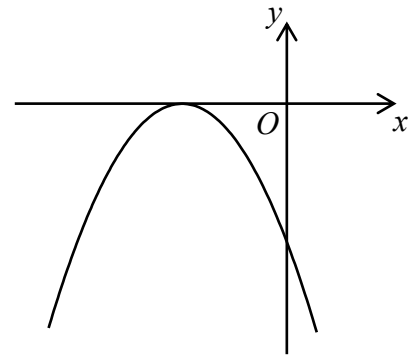
12. The figure shows the graph of  $y = ax^2 + bx + c$ , where  $a$ ,  $b$  and  $c$  are constants. Which of the following is/are true?  $a < 0, b < 0, c < 0$ .

I.  $\checkmark$   $ac > 0$

II.  $\checkmark$   $b < 0$

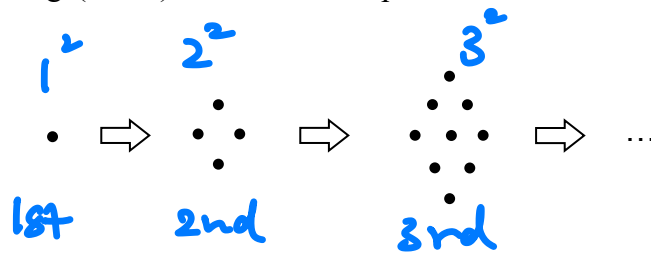
III.  $\checkmark$   $\frac{b^2}{4ac} = 1$

$b^2 - 4ac = 0$   
 $b^2 = 4ac$   
 $\frac{b^2}{4ac} = 1$



- A. I only
- B. I and II only
- C. II and III only
- D. I, II and III**

13. In the figure, the 1st pattern consists of 1 dot. For any positive integer  $n$ , the  $(n+1)$ th pattern is formed by adding  $(2n+1)$  dots to the  $n$ th pattern. Find the number of dots in the 8th pattern.



$8^2 = 64$

- A. 49
- B. 64**
- C. 72
- D. 81

14.  $ABCD$  is a rectangle and  $E$  is a point on  $AC$  such that  $AE : EC = 2 : 1$ . If  $AB = 15$  cm and  $BC = 36$  cm, then  $BE =$

- A.  $2\sqrt{61}$  cm .
- B.  $\sqrt{601}$  cm .**
- C.  $2\sqrt{313}$  cm .
- D.  $\sqrt{1105}$  cm .

$AC = \sqrt{15^2 + 36^2} = 39$

$AE = 39 \times \frac{2}{3} = 26$

$BE^2 = AB^2 + AE^2 - 2(AB)(AE)\cos\angle BAC$

$= 15^2 + 26^2 - 2(15)(26) \cdot \frac{15}{39}$

$\therefore BE = \sqrt{601}$

15. In the figure,  $ABC$  is a triangle.  $D$  is a point on  $AC$  such that  $AB = BD = DC$ . If  $\angle ABC = 90^\circ$ , then  $\angle BAD =$

A.  $30^\circ$ .  
 B.  $45^\circ$ .  
 C.  $60^\circ$ .  
 D.  $80^\circ$ .

$x + \frac{x}{2} + 90^\circ = 180^\circ$   
 $\frac{3x}{2} = 90^\circ$   
 $x = 60^\circ$

16. In the figure,  $ABCD$  is a parallelogram.  $E$  and  $F$  are points lying on  $AD$  and  $BC$  respectively.  $EF$  meets  $AC$  at  $G$ . If  $AE : ED = 3 : 2$ ,  $BF : FC = 1 : 4$  and the area of  $\triangle AEG$  is  $45 \text{ cm}^2$ , then the difference between the area of  $ABFG$  and the area of  $CDEG$  is

A.  $28 \text{ cm}^2$ .  
 B.  $30 \text{ cm}^2$ .  
 C.  $35 \text{ cm}^2$ .  
 D.  $38 \text{ cm}^2$ .

$\frac{\text{Area of } \triangle GFC}{45} = \left(\frac{4}{3}\right)^2$   
 Area of  $\triangle GFC = 80$   
 $80 + a = 45 + b$   
 $35 = b - a$   
 $\therefore \text{difference} = 35$

17. In the figure,  $ABCD$  is a rectangle.  $E$  and  $F$  are points lying on  $BC$  and  $CD$  respectively such that  $CF : FD = 1 : r$ . Find  $\frac{AE}{EF}$ .

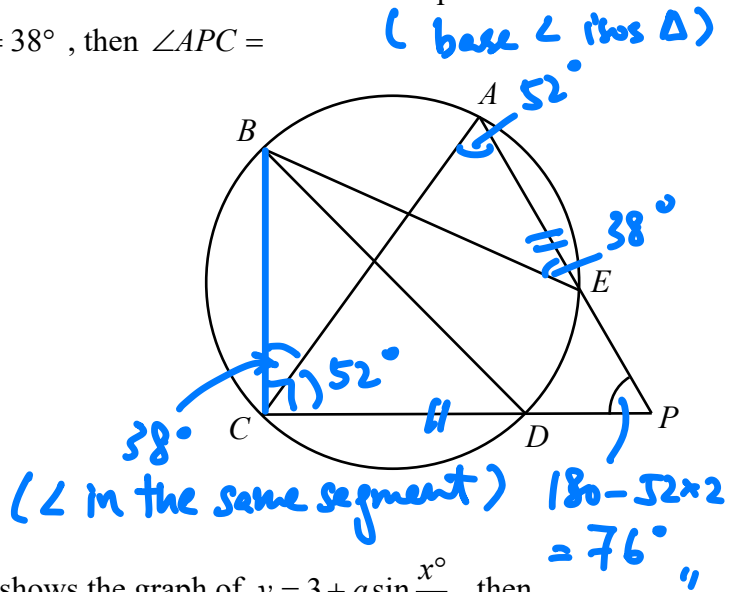
A.  $\frac{\sin \alpha}{(1+r) \sin \beta}$   
 B.  $\frac{\sin \beta}{(1+r) \sin \alpha}$   
 C.  $\frac{(1+r) \sin \alpha}{\sin \beta}$   
 D.  $\frac{(1+r) \sin \beta}{\sin \alpha}$

$\sin \alpha = \frac{1+r}{AE}$   
 $AE = \frac{1+r}{\sin \alpha}$   
 $\sin \beta = \frac{1}{EF}$   
 $EF = \frac{1}{\sin \beta}$

$\frac{AE}{EF} = \frac{\frac{1+r}{\sin \alpha}}{\frac{1}{\sin \beta}} = \frac{(1+r) \sin \beta}{\sin \alpha}$

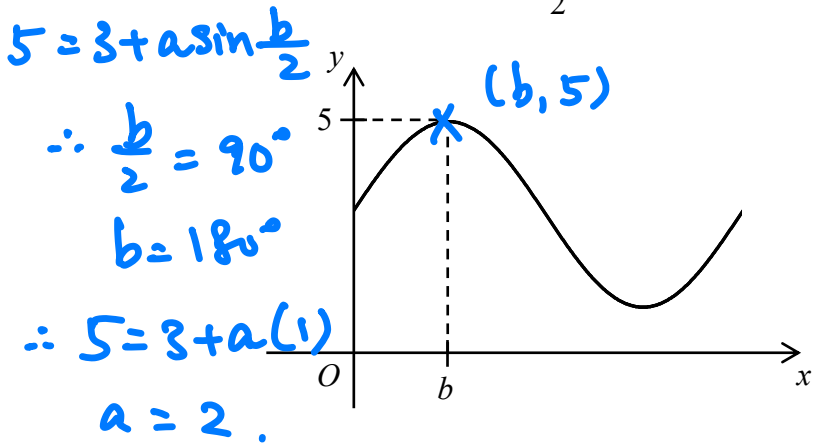
18. In the figure,  $BD$  is a diameter of the circle  $ABCDE$ .  $AE$  and  $CD$  are produced to meet at the point  $P$ . If  $PA = PC$  and  $\angle AEB = 38^\circ$ , then  $\angle APC =$

- A.  $68^\circ$ .
- B.  $70^\circ$ .
- C.  $76^\circ$ .**
- D.  $81^\circ$ .



19. Let  $a$  and  $b$  be constants. If the figure shows the graph of  $y = 3 + a \sin \frac{x^\circ}{2}$ , then

- A.  $a = -2$  and  $b = 90^\circ$ .
- B.  $a = -2$  and  $b = 180^\circ$ .
- C.  $a = 2$  and  $b = 90^\circ$ .
- D.  $a = 2$  and  $b = 180^\circ$ .**

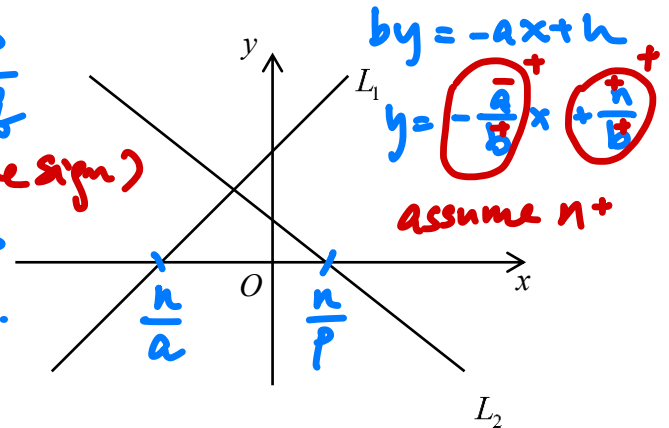


20. In the figure, the equations of the straight lines  $L_1$  and  $L_2$  are  $ax + by = n$  and  $px + qy = n$  respectively. Which of the following is/are true?

- I.  $aq - bp > 0$
- II.  $n(b - q) < 0$
- III.  $n(a - p) < 0$

I.  $-\frac{a}{b} > -\frac{p}{q}$   
 ( $\because b, q$  same sign)  
 $-aq > -bp$   
 $0 > aq - bp$

II.  $\frac{n}{b} > \frac{n}{q}$   
 ( $\because b, q$  same sign)  
 $nq > nb$   
 $0 > n(b - q)$



**D.** II and III only  
 III.  $\frac{n}{p} > \frac{n}{a}$   
 ( $\because a, p$  different sign)  
 $an < np$   
 $n(a - p) < 0$

$gy = -px + n$   
 $y = -\frac{p}{q}x + \frac{n}{q}$

21. The straight line  $4x + 3y - 24 = 0$  cuts the  $x$ -axis and the  $y$ -axis at the points  $P$  and  $Q$  respectively. If the straight line  $y = kx$  bisects  $PQ$ , then  $k =$

A.  $\frac{\sqrt{2}}{2}$  .  
 B.  $\frac{3}{4}$  .  
 C.  $\frac{4}{3}$  .  
 D.  $\sqrt{2}$  .

$P(6, 0) \quad Q(0, 8)$   
 $\therefore$  mid pt. of  $PQ = (3, 4)$   
 $4 = k(3)$   
 $k = \frac{4}{3}$

22. Find the area of the region bounded by the straight lines  $x - 2y + 6 = 0$ ,  $5x + 2y - 30 = 0$  and  $\frac{x}{6} + \frac{y}{3} = 1$ .

$\frac{x}{6} + \frac{y}{3} = 1$   
 $y = -\frac{x}{2} + 3$

A. 9.5  
 B. 12  
 C. 13.5  
 D. 15

$y = \frac{1}{2}x + 3 \quad y = -\frac{5}{2}x + 15$   
 area  
 $= \frac{(4+6)(5)}{2} - \frac{2 \times 4}{2} - \frac{3 \times 6}{2}$   
 $= 12$

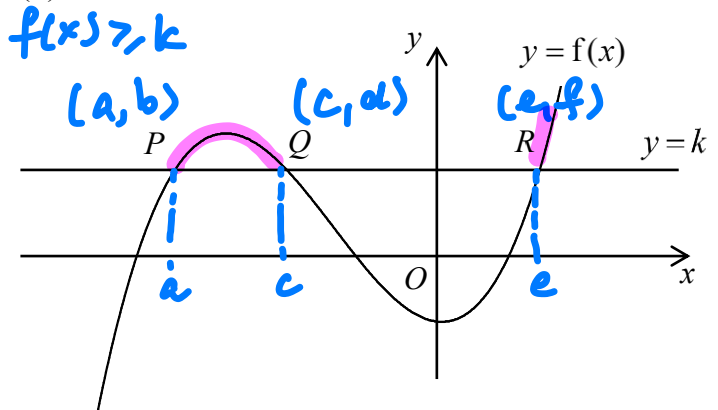
23. The equation of the circle  $C$  is  $(x - 5)^2 + (y + 3)^2 = 1$ . If  $C$  is rotated anticlockwise about the origin through  $90^\circ$  to the circle  $C'$ , then the equation of  $C'$  is

A.  $(x - 3)^2 + (y - 5)^2 = 1$  .  
 B.  $(x + 3)^2 + (y + 5)^2 = 1$  .  
 C.  $(x - 5)^2 + (y - 3)^2 = 1$  .  
 D.  $(x + 5)^2 + (y + 3)^2 = 1$  .



24. In the figure, the graph of  $y = f(x)$  and the straight line  $y = k$  intersect at the points  $P(a, b)$ ,  $Q(c, d)$  and  $R(e, f)$ . The solution of  $f(x) - k \geq 0$  is

- A.  $a \leq x \leq c$  or  $x \geq e$ .  
 B.  $b \leq x \leq d$  or  $x \geq f$ .  
 C.  $c \leq x \leq e$  or  $x \leq a$ .  
 D.  $d \leq x \leq f$  or  $x \leq b$ .



25. The equation of the circle  $C$  is  $4x^2 + 4y^2 - 40x + 56y + 27 = 0$ . Which of the following is/are true?

- I.  The centre of  $C$  lies in quadrant IV.  
 II.  The area of  $C$  is less than 210.  
 III.  The origin lies inside  $C$ .

$$x^2 + y^2 - 10x + 14y + \frac{27}{4} = 0$$

I. Centre =  $(5, -7)$

- A. I only  
 B. II only  
 C. I and III only  
 D. II and III only

III. Put  $(0, 0)$

II. Radius =  $\sqrt{5^2 + (-7)^2 - \frac{27}{4}}$

LHS

$$= 4(0)^2 + 4(0)^2 - 40(0) + 56(0) + 27$$

$$= 27 > 0$$

$\therefore$  outside.

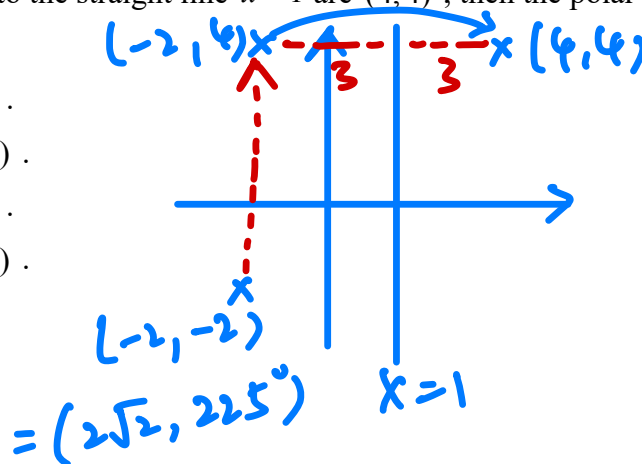
$$= \sqrt{\frac{269}{4}}$$

$$\text{Area} = \pi \left( \sqrt{\frac{269}{4}} \right)^2$$

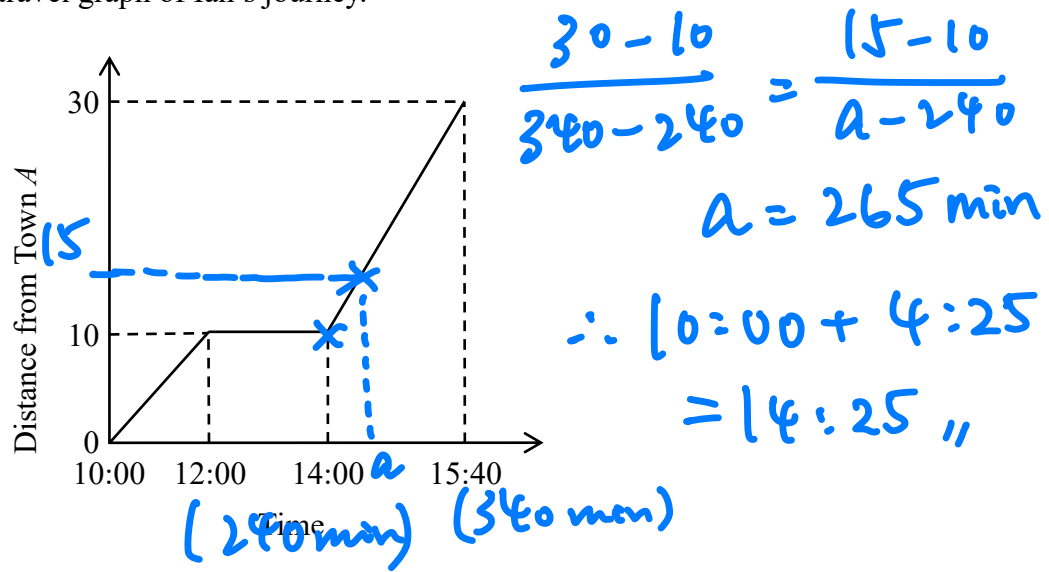
=

26. The point  $P$  is translated upwards by 6 units to  $Q$ . If the coordinates of the reflection image of  $Q$  with respect to the straight line  $x = 1$  are  $(4, 4)$ , then the polar coordinate of  $P$  are

- A.  $(2\sqrt{2}, 45^\circ)$ .  
 B.  $(2\sqrt{2}, 225^\circ)$ .  
 C.  $(4\sqrt{2}, 45^\circ)$ .  
 D.  $(4\sqrt{2}, 225^\circ)$ .



27. Town A and Town B are 30 km apart. Ian travelled from Town A to Town B on a straight road. The figure shows the travel graph of Ian's journey.



At what time does he reach the mid-point of Town A and Town B?

- A. 14:22
  - B. 14:25**
  - C. 14:35
  - D. 15:00
28. Two fair dice are thrown. Find the probability that the sum of the two numbers thrown is a multiple of 3 .

- A.  $\frac{1}{3}$**
- B.  $\frac{1}{4}$
- C.  $\frac{5}{7}$
- D.  $\frac{3}{4}$

Handwritten calculation:  $\frac{12}{36}$

	1	2	3	4	5	6
1		✓			✓	
2	✓			✓		
3			✓			✓
4		✓			✓	
5	✓			✓		
6			✓			✓

29. The following table shows the probability distribution of a loaded die.

The number thrown	1	2	3	4	5	6
Probability	0.2	0.1	0.2	0.1	0.3	$x = 0.1$

If the die is thrown twice, find the probability that the sum of the two numbers thrown is greater than 9 .

- A. 0.12
- B. 0.15
- C. 0.18
- D. 0.21

	1	2	3	4	5	6
1						
2						
3						
4						✓
5					✓	✓
6				✓	✓	✓

$$\begin{aligned}
 & 0.1 \times 0.1 \times 2 \\
 & + 0.3 \times 0.3 \\
 & + 0.3 \times 0.1 \times 2 \\
 & + 0.1 \times 0.1 \\
 & = 0.18 \text{ ,,}
 \end{aligned}$$

30. The stem-and-leaf diagram below shows the distribution of the numbers of books read by 11 students in a month.

Stem (tens)	Leaf (units)
1	2 4 5 8
2	0 m 5 7
3	0 1

If the mean and the inter-quartile range of the above distribution are 22 and 13 respectively, then the mode of the distribution is

- A. 20 .
- B. 25 .
- C. 27 .
- D. 30 .

$$\begin{aligned}
 20 + n - 16 &= 13 \\
 n &= 9 \text{ ,,} \\
 \frac{222 + 20 + m}{11} &= 22 \\
 m &= 0
 \end{aligned}$$

Section B

31.  $3 \times 2^9 + 2^7 - 6 \times 2^4 + 2^2 + 2 = (2+1) \times 2^9 + 2^3 \times 2^4 - 6 \times 2^4 + 2^2 + 2^1$   
 $= 2^{10} + 2^9 + 2 \times 2^4 + 2^2 + 2^1$   
 $= 2^{10} + 2^9 + 2^5 + 2^2 + 2^1$
- A. 11000100110<sub>2</sub> .
  - B. 11101010110<sub>2</sub> .
  - C. 1100010011<sub>2</sub> .
  - D. 1010010110<sub>2</sub> .

32. If  $\begin{cases} \log_3 y = 2 - x \\ 8(\log_9 y)^2 = x - 1 \end{cases}$ , then  $y =$
- by (2),  $8 \times \left(\frac{\log_3 y}{\log_3 9}\right)^2 = x - 1$

A. 1 or  $\frac{7}{2}$  .

B.  $\sqrt{3}$  or  $\frac{1}{3}$  .

C. 3 or  $\frac{\sqrt{3}}{9}$  .

D. 3 or  $\frac{3}{2}$  .

~~$8 \times \frac{(\log_3 y)^2}{4} = x - 1$~~

$2(2-x)^2 = x - 1$

$8 - 8x + 2x^2 = x - 1$

$2x^2 - 9x + 9 = 0$

$x = 3 \text{ or } \frac{3}{2}$

$\therefore \log_3 y = 2 - 3 \text{ or } 2 - \frac{3}{2}$

$3^{-1} = y \text{ or } 3^{\frac{1}{2}} = y$

$y = \frac{1}{3} \text{ or } y = \sqrt{3}$

33.  $1 + i + i^2 + i^3 + \dots + i^{2020} =$

A. -1 .

B. 1 .

C.  $i$  .

D.  $1 + i$  .

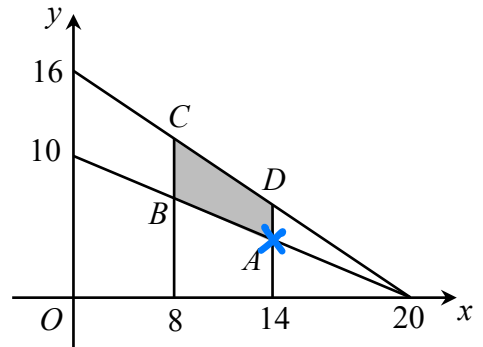
~~$1 + i + (-1) + (-i) + 1 + i + (-1) + \dots + (-1)$~~   
 $= 1$

34. In the figure,  $AD$  and  $BC$  are parallel to the  $y$ -axis. If  $(x, y)$  is a point lying in the shaded region  $ABCD$  (including the boundary), at which point does  $4 - 2y + 5x$  attain its greatest value?

- A. A
- B. B
- C. C
- D. D

$$4 - 2y + 5x$$

(Red arrow pointing down to  $-2y$ , red arrow pointing up to  $+5x$ )



35. For  $0^\circ \leq x < 360^\circ$ , how many roots does the equation  $5 \tan^2 x = \tan x$  have?

- A. 1
- B. 2
- C. 3
- D. 4

$$5 \tan^2 x - \tan x = 0$$

$$\tan x (5 \tan x - 1) = 0$$

$$\tan x = 0 \quad \text{or} \quad \tan x = \frac{1}{5}$$



36. Let  $a_n$  be the  $n$ th term of a geometric sequence. If  $a_3 + a_5 = 12$  and  $a_7 + a_9 = 48$ , which of the following must be true?

- I.  The first term of the sequence is 2.
- II.  All the terms of the sequence are positive numbers.
- III.  Some of the terms of the sequence are irrational numbers.

- A. I only
- B. II only
- C. I and III only
- D. II and III only

$r = \pm\sqrt{2}$

$$ar^2 + ar^4 = 12 \quad \wedge r = \sqrt{2}$$

$$ar^2(1 + r^2) = 12 \quad \text{--- (1)}$$

$$ar^6 + ar^8 = 48$$

$$ar^6(1 + r^2) = 48 \quad \text{--- (2)}$$

$$\frac{(2)}{(1)}, \quad \frac{ar^6(1+r^2)}{ar^2(1+r^2)} = \frac{48}{12}$$

$$r^4 = 4$$

$$r = \pm\sqrt{2}$$

I.

$$ar^2(1+r^2) = 12$$

$$a(2)(1+2) = 12$$

$$a = 2 \quad \text{,,}$$

37. In the figure, the inscribed circle of  $\triangle ABC$  touches  $AB$ ,  $BC$  and  $AC$  at the points  $D$ ,  $E$  and  $F$  respectively. If  $\widehat{DE} : \widehat{EF} : \widehat{FD} = 11 : 9 : 10$ , then  $\angle ABC =$

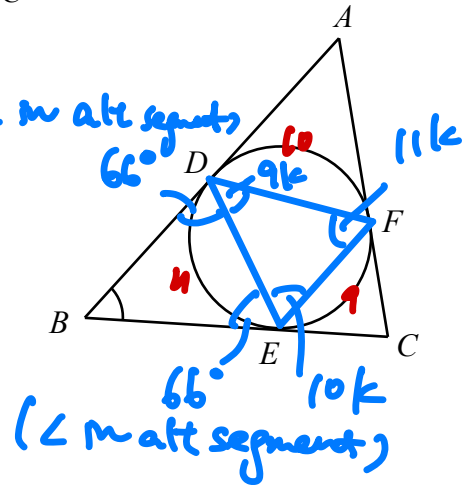
- A.  $48^\circ$ .
- B.  $60^\circ$ .
- C.  $66^\circ$ .
- D.  $72^\circ$ .

$$9k + 10k + 11k = 180$$

$$k = 6$$

$$\angle DFE = 66^\circ$$

$$\angle ABC = 180 - 66 \times 2 = 48^\circ$$



38. In the figure,  $ABCDEFGH$  is a cuboid.  $M$  is the mid-point of  $CH$ . Denote the angle between  $AE$  and  $ME$  by  $\theta$ . Find  $\tan \theta$ .

- A.  $\frac{1}{2}$
- B.  $\frac{3\sqrt{41}}{16}$
- C.  $\frac{\sqrt{61}}{5}$
- D.  $\frac{25}{16}$

$$AE = \sqrt{8^2 + 6^2} = 10$$

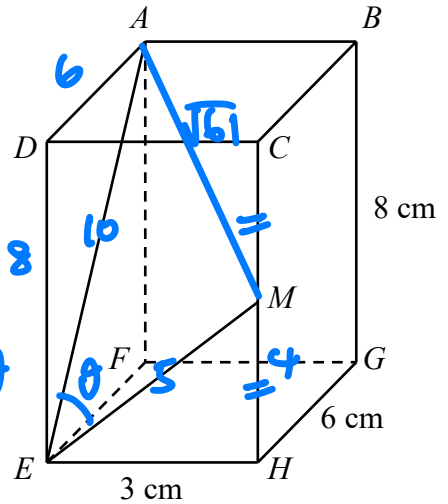
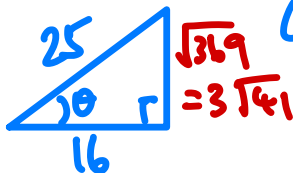
$$ME = \sqrt{3^2 + 4^2} = 5$$

$$AM = \sqrt{3^2 + 6^2 + 4^2} = \sqrt{61}$$

$$\sqrt{61}^2 = 10^2 + 5^2 - 2(10)(5)\cos\theta$$

$$\cos\theta = \frac{16}{25}$$

$$\therefore \tan\theta = \frac{3\sqrt{41}}{16}$$



39. Let  $k$  be a constant. The straight line  $y = kx$  and the circle  $x^2 + y^2 - 4x - 4 = 0$  intersect at the points  $P$  and  $Q$ . If the mid-point of  $PQ$  lies on the straight line  $2x + y - 1 = 0$ , then  $k =$

- A.  $-1$ .
- B.  $1$ .
- C.  $-1$  or  $3$ .
- D.  $-3$  or  $1$ .

$$x^2 + (kx)^2 - 4x - 4 = 0$$

$$(1+k^2)x^2 - 4x - 4 = 0$$

$$x\text{-corr. of mid pt. of } PQ = \frac{\frac{4}{1+k^2}}{2}$$

$$4 + 2k - (1+k^2) = 0$$

$$\therefore k = -1 \text{ or } 3$$

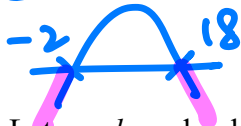
$$\therefore \text{mid pt. of } PQ = \left( \frac{2}{1+k^2}, \frac{2k}{1+k^2} \right)$$

$$2\left(\frac{2}{1+k^2}\right) + \left(\frac{2k}{1+k^2}\right) - 1 = 0$$

$$y = -2x - k$$

40. If the straight line  $2x + y + k = 0$  and the circle  $x^2 + y^2 + 12x - 8y + 32 = 0$  do not intersect with each other, find the range of the values of  $k$ .

- A.  $-18 < k < 2$
- B.  $-2 < k < 18$
- C.  $k < -18$  or  $k > 2$
- D.  $k < -2$  or  $k > 18$

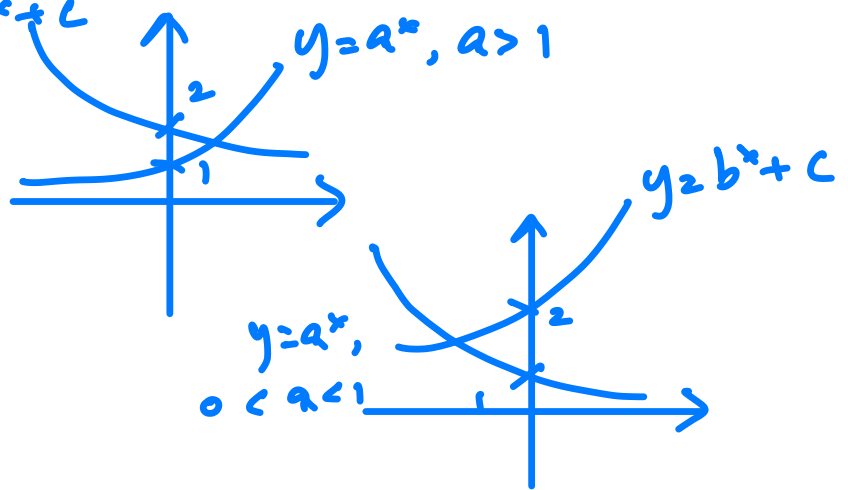


$$\begin{aligned}
 x^2 + (-2x - k)^2 + 12x - 8(-2x - k) + 32 &= 0 \\
 x^2 + 4x^2 + 4(-2x - k)^2 + 12x + 16x + 8k + 32 &= 0 \\
 5x^2 + (4k + 28)x + k^2 + 8k + 32 &= 0 \\
 \therefore \Delta < 0 \\
 (4k + 28)^2 - 4(5)(k^2 + 8k + 32) &< 0 \\
 -4k^2 + 64k + 144 &< 0
 \end{aligned}$$

41. Let  $a$ ,  $b$  and  $c$  be constants. The graph of  $y = b^x + c$  is obtained by reflecting the graph of  $y = a^x$  with respect to the  $y$ -axis and then translating the resulting graph upwards. If the  $y$ -intercept of the graph of  $y = b^x + c$  is 2, which of the following must be true?

- I.   $ab = 1$
- II.   $c = 1$
- III.   $a > 1$

- A. I only
- B. II only
- C. I and III only
- D. II and III only



42. A queue is formed by 7 boys and 4 girls. If no girls are next to each other, how many different queues can be formed?

$$7! \times P_4^8$$

- A. 241920
- B. 352800
- C. 8467200
- D. 39916800

43. A bag contains 10 red balls and 20 blue balls. Three balls are drawn randomly from the bag one by one without replacement. Given that both red balls and blue balls are drawn, find the probability that exactly 2 red balls are drawn.

A.  $\frac{45}{203}$   
 B.  $\frac{9}{28}$   
 C.  $\frac{19}{28}$   
 D.  $\frac{158}{203}$

$$P(2 \text{ red} \mid \text{have red \& blue}) = \frac{P(2 \text{ red \& have red \& blue})}{P(\text{have red \& blue})}$$

$$= \frac{\frac{C_2^{10} C_1^{20}}{C_3^{30}}}{\frac{C_2^1 C_1^{29} + C_1^1 C_2^{29}}{C_3^{30}}} = \frac{9}{28}$$

44. In a test, Edan gets 70 marks and his standard score is 2 while Marf gets 55 marks and her standard score is -1. If Anson gets 64 marks in the test, then his standard score is

A. -0.6  
 B. 0.4  
 C. 0.8  
 D. 1.2

$$\begin{cases} \frac{70 - \bar{x}}{\sigma} = 2 \text{ --- (1)} \\ \frac{55 - \bar{x}}{\sigma} = -1 \text{ --- (2)} \end{cases} \quad \therefore \bar{x} = 60, \sigma = 5$$

Anson standard score  
 $= \frac{64 - 60}{5} = 0.8$

45. If the mean and the standard deviation of the group of numbers  $\{a, b, c, d, m\}$  are  $m$  and  $\sigma$  respectively, then the standard deviation of the group of numbers  $\{a, b, c, d\}$  is

A.  $\frac{\sqrt{2}}{2} \sigma$   
 B.  $\frac{\sqrt{3}}{2} \sigma$   
 C.  $\frac{\sqrt{5}}{2} \sigma$   
 D.  $2\sigma$

$$\sigma = \sqrt{\frac{(a-m)^2 + (b-m)^2 + \dots + (d-m)^2 + (m-m)^2}{5}}$$

$$\sqrt{5}\sigma = \sqrt{(a-m)^2 + (b-m)^2 + \dots + (d-m)^2}$$

$$\therefore \text{the new } \sigma = \sqrt{\frac{(a-m)^2 + (b-m)^2 + \dots + (d-m)^2}{4}}$$

$$= \frac{\sqrt{5}\sigma}{\sqrt{4}} = \frac{\sqrt{5}\sigma}{2}$$

END OF PAPER