Name: ______________________________

Class: ___________ No.: ___________

SACRED HEART CANOSSIAN COLLEGE

S6 MOCK EXAMINATION
(2019-2020)

MATHEMATICS Compulsory Part

PAPER I

Question-Answer Book

Time Allowed: 2 hours 15 minutes

INSTRUCTIONS

1. Write your name, class and class number in the space provided on Page 1.
2. This paper consists of THREE sections, A(1), A(2) and B.
3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer book.
4. Supplementary answer sheets will be supplied on request. Write your name, class, class number and the question number on each sheet.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
7. The diagrams in this paper are not necessarily drawn to scale.

Section A Question No.	**Marks**
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
Section A Total	

Section B Question No.	**Marks**
15	
16	
17	
18	
19	
20	
Section B Total	

Paper 1 Total	

Section A(1) (35 marks)

1. Simplify $\dfrac{\left(a^0 b^{-4}\right)^{-2}}{\left(a^{-1} b^2\right)^5}$ and express your answer with positive indices. (3 marks)

2. Make n the subject of the formula $m = \dfrac{kn+m}{n-2}$. (3 marks)

3. Factorize
   (a) $121a^2 - 81$,
   (b) $121a^2 - 81 - 33ab - 27b$.

(4 marks)

4. The marked price of a vase is \$560. It is given that the marked price of the vase is 60% higher than the cost of the vase.
   (a) Find the cost of the vase.
   (b) If the vase is sold at a discount of 35% on its marked price, will there be a gain or a loss after selling the vase? Explain your answer.

(4 marks)

5. The side of a square is measured to be 10 m with a percentage error of 5%.
   (a) Find the range of the actual area of the square.
   (b) A student claims that the percentage error of the perimeter of the square is 20%. Is his claim correct? Explain your answer.

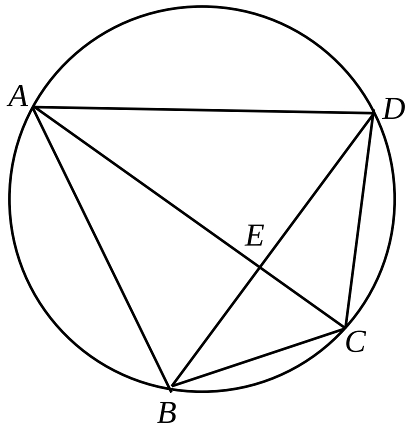
(4 marks)

6. In the figure, $ABCD$ is a circle. AC and BD intersect at the point E. $AB:BC:CD:DA = 2:1:1:x$.

(a) Show that $\angle BDC = \dfrac{180^\circ}{x+4}$.

(b) If $\angle CED = 90^\circ$, find the value of x.

(5 marks)



7. (a) Solve the inequality $\frac{3(y-27)-1}{5} \geq 10-y$.

(b) Find the range of the values of y which satisfy both $\frac{3(y-27)-1}{5} \geq 10-y$ and $32-4y > 0$.

(4 marks)

8. The number of candies owned by Fiona and Ginny is 80. If Fiona gives 4 candies to Ginny, the number of candies owned by Ginny will be 3 times that owned by Fiona. Find the number of candies owned by Fiona originally. (4 marks)

9. A ball is dropped at random into one of the six holes as shown in the figure below. The number under each hole indicates the score obtained when the ball drops into that hole. In a game, the ball is dropped twice.

○	○	○	○	○	○
4	3	2	2	3	1

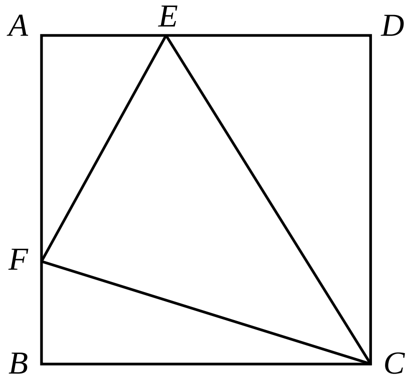
(a) Find the probability that a total score of 8 is obtained in the game.

(b) A prize is awarded for obtaining a total score of 7 or above in one game. If the game is played twice, find the probability that no prize is awarded.

(4 marks)

SECTION A(2) (35 marks)

10. In the figure, CEF is a triangle inscribed in a square $ABCD$ with $AB = x$ cm. It is given that $EF = 3$ cm, $FC = 4$ cm and $CE = 5$ cm.



(a) Show that $\Delta BCF \sim \Delta AFE$. (3 marks)

(b) Show that $FB = \dfrac{x}{4}$ cm. (2 marks)

(c) James claims that the area of $ABCD$ is greater than 16 cm^2. Do you agree? Explain your answer. (2 marks)

11. The data below show the numbers of lunch boxes ordered by a company for 12 days.

34	38	42	45	40	41
34	67	28	40	39	50

(a) Find the mean and median of the above set of data. (2 marks)

(b) The bill for the company is \$50 per lunch box plus \$100 delivery charge every day. Find the mean and the median of the bills for the company for the 12 days. (2 marks)

(c) The company ordered a and b lunch boxes for 2 other days respectively.

(i) Write down the least and greatest possible values of the median of the numbers of lunch boxes ordered for 14 days.

(ii) If the mean and median of the numbers of lunch boxes ordered for 14 days remain unchanged, find the least value of the difference between a and b.

(4 marks)

12. It is given that $g(x)$ is the sum of two parts, one part varies as x^3 and the other part varies as x^2. Suppose that $g(2)=8$ and $g(-1)=-4$.

(a) Find $g(x)$. (3 marks)

(b) Let $h(x)=g(x)+mx+8$, where m is a constant. It is given that $x-2$ is a factor of $h(x)$.

(i) Find the value of m.

(ii) How many rational roots does the equation $h(x)=8$ have ? Explain your answer.

(4 marks)

13. A solid metal right circular cylinder of base radius 4 cm and height 114 cm is melted and recast into two similar right circular cones. The ratio of the curved surface area of the smaller cone to that of the larger cone is 9 : 25.

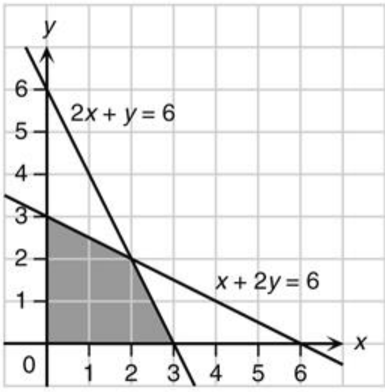
(a) (i) Find the volume of the cylinder in terms of π.

(ii) Find the volume of the smaller cone in terms of π.

(3 marks)

(b) A metal frustum is made by cutting off a part of the larger cone where the part being cut off is the same as the smaller cone. If the height of the frustum is 8 cm, find the curved surface area of the frustum in terms of π. (3 marks)

14. (a) In the figure, the equations of the two straight lines are $2x+y=6$ and $x+2y=6$. The shaded region represents the solution of a system of inequalities. Find the system of inequalities. (3 marks)



(b) Two kinds of drinks *A* and *B* contain lemon juice and tea. The amounts of lemon juice and tea in 1 L of drink *A* and 1 L of drink *B* are shown in the following table:

	Lemon juice	**Tea**
Drink *A*	6 units	4 units
Drink *B*	3 units	8 units

There are 18 units of lemon juice and 24 units of tea. Suppose x L of drink *A* and y L of drink *B* are produced. The profits of selling each litre of drink *A* and *B* are \$15 and \$10 respectively. The merchant claims that the maximum profit can be greater than \$50. Do you agree? Explain your answer. (4 marks)

Section B (35 marks)

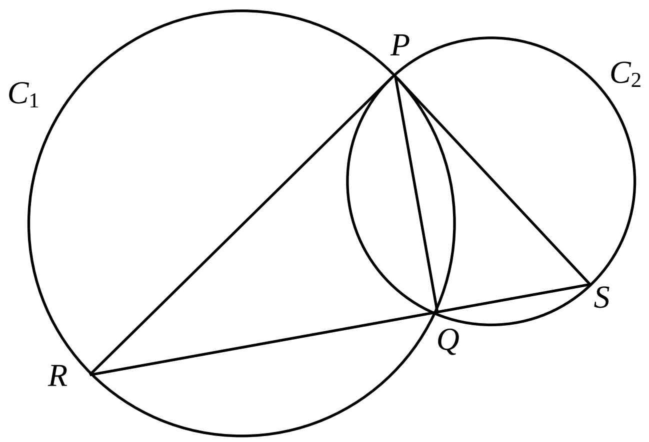
15. In the figure, the circles C_1 and C_2 intersect at the points P and Q. PR and PS are diameters of C_1 and C_2 respectively. PS is a tangent to C_1 at P. The radius of C_1 and C_2 are 9 cm and 3 cm respectively.

(a) Is PR a tangent to C_2? Explain your answer. (1 mark)

(b) Find the length of RS. (1 mark)
(Express your answer in surd form if necessary.)

(c) Let $QS = x$ cm. By considering ΔPQR and ΔPQS, or otherwise, find the value of x.
(Express your answer in surd form if necessary.) (2 marks)

(d) A coordinate system is introduced to the diagram such that Q is the origin and the positive y-axis is along PQ. Find the equation of C_2. (3 marks)



15.

16. Let f(x) = $kx^2 - 8kx - 6k^2 + 2$, where k is a real constant. It is given that the maximum value of f(x) is -4.

(a) (i) Using the method of completing the square, express, in terms of k, the coordinates of the vertex of the graph of y = f(x).

(ii) Find the value of k .

(3 marks)

(b) The graph of y = g(x) is obtained by reflecting the graph of y = f($x+6$) about the x-axis, and then translating upwards by 2 units. Let S and T be the vertices of the graph of y = f(x) and the graph of y = g(x) respectively. Denote the origin by O.

(i) Let Q be a moving point in the rectangular coordinate plane such that Q is equidistant from S and T. Find the equation of the locus of Q.

(ii) Mary claims that the circumcentre of ΔOST lies in the first quadrant. Do you agree? Explain your answer.

(4 marks)

16.

17. In Figure 17(a), PQR is a triangular cardboard. It is given that $PQ = 12\text{ cm}$, $\angle PRQ = 105°$ and $\angle QPR = 32°$.

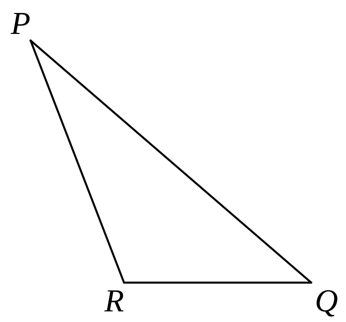


Figure 17(a)

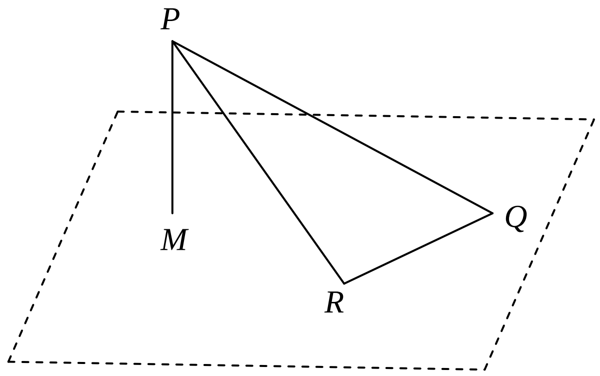


Figure 17(b)

(a) Find PR. (2 marks)

(b) In Figure 17(b), a rod of length 4 cm is standing vertically at the point M on the horizontal ground. The cardboard is supported by the rod such that P is at the top of the rod and QR touches the horizontal ground.

(i) Find the angle between PQR and the horizontal ground.

(ii) When the sun shines directly overhead, the shadow of the cardboard on the horizontal ground is MQR. Find the area of the shadow MQR.

(5 marks)

17.

18. Jimmy plans to save money for the down payment of a flat. He saves \$30000 at the beginning of each month for n years, where n is an integer. The interest is 6% per annum compounded monthly.

(a) Show that the total amount that Jimmy will get at the end of the nth year is given by $6030000\left[(1.005)^{12n}-1\right]$. (2 marks)

(b) Find the least value of n if the down payment is \$3,000,000. (3 marks)

18.

19. An auditor checked the invoices prepared by three chain stores *A*, *B* and *C*. A sample of 200 invoices, 70 from store *A*, 80 from store *B* and 50 from store *C*, was selected. The error rates of the invoices from stores *A*, *B* and *C* were found to be 5%, 3% and 4 % respectively. If one invoice was selected from the sample, find the probability that

(a) the invoice was from store *A*, (1 mark)

(b) the invoice contained an error, (2 marks)

(c) the invoice was from store *A*, given that it contained an error. (2 marks)

20. The graph in Figure 20 shows the linear relation between x and $\log_4 y$. The slope and the intercept on the vertical axis of the graph are -2 and $\frac{5}{2}$ respectively.

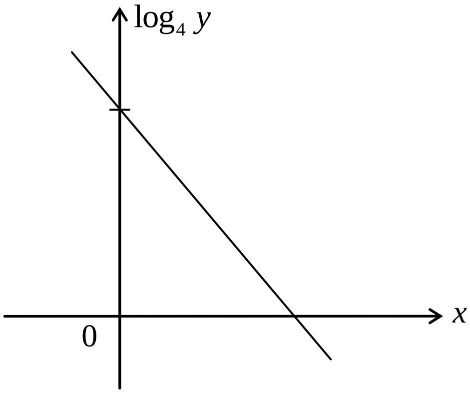


Figure 20

(a) Express the relation between x and y in the form $y = ka^{bx}$, where a, b and k are constants. (2 marks)

(b) Find the greatest value of x such that $y \geq 2^x$. (2 marks)

END OF PAPER