MOCK 2020-2021
MATH
COMPULSORY
PART

PAPER 1

Name	
Class	( )

St. Joseph's College

MATHEMATICS
Compulsory Part
PAPER 1
Question-Answer Book

($2\frac{1}{4}$ hours)

This paper must be answered in English

INSTRUCTIONS

1. Write your Name, Class and Class Number in the spaces provided on Page 1.

2. This paper consists of THREE sections, A(1), A(2) and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Graph paper and supplementary answer sheets will be supplied on request. Write your Name and mark the question number box on each sheet, and fasten them with string INSIDE this book.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

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	Marker's Use Only	**Examiner's Use Only**
	Marker No.	Examiner No.
Question No.	Marks	Marks
1−3		
4−6		
7−8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
Total		

SECTION A(1) (35 marks)

1. Simply $\frac{x^5y^{-8}}{(xy^{-2})^{-5}}$ and express the answer with positive indices. (3 marks)

$\frac{x^5 \cdot y^{-8}}{x^{-5} \cdot y^{10}}$ — (1M)

$= \frac{x^{5+5}}{y^{8+10}}$ — (1M)

$= \frac{x^{10}}{y^{18}}$ — (1A)

2. Change the subject as m of the formula $\frac{2m-5mn}{6m+7n} = \frac{5}{6}$. (3 marks)

$12m - 30mn = 30m + 35n$ — (1M)

$12m - 30mn - 30m = 35n$

$-18m - 30mn = 35n$ — (1M)

$m(-18 - 30n) = 35n$

$\therefore m = \frac{35n}{-18-30n}$ — (1A)

or $\left(= \frac{35n}{-(30n+18)}\right)$

3. a) Round up $\left(\frac{3\pi}{4}\right)^3$ to 2 decimal places.

b) Round down $\left(\frac{9\pi}{4}\right)^3$ to the nearest hundred.

c) Round off $\left(\frac{19\pi}{15}\right)^3$ to 3 significant figures. (3 marks)

a) 13.09 — (1A)

b) 300 — (1A)

c) 63.0 — (1A)

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4. Let x , y and z be non-zero numbers such that $x = y + 2z$ and $y : z = 2 : 5$. Find $x : y : z$.

(3 marks)

Let $y=2k$, $z=5k$ where k is a non-zero constant — (1M)

$\therefore x = 2k+2(5k)$

$x = 12k$ — (1M)

$\therefore x:y:z = 12:2:5$ // — (1A)

5. a) Factorize $4x^2 - 25y^2$.
   b) Using the result of (a), factorize $4x^2 - 25y^2 - 6x + 15y$.

(4 marks)

a) $(2x)^2-(5y)^2 = (2x-5y)(2x+5y)$ — (1A)

b) $(2x-5y)(2x+5y) - 3(2x-5y)$ — (1M) + (1M)
(Using (a)) (take out 3)

$= (2x-5y)(2x+5y-3)$ — (1A)

6. Mr. Chau bought a JPad at a price of \$$x$ as a gift of his birthday. He put it on sale in his shop with a marked price to be 80% above the price he had bought. The JPad was finally sold to a customer at a 10% discount. If the customer paid \$2916 for the JPad, find the value of x.

(4 marks)

(1M) (1M)

$2916 = \$x \times (1+80\%) \times (1-10\%)$ — (1M)

$\therefore x = 1800$ — (1A)

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7. Consider the compound inequality

$-9x > 72$ and $\frac{2x+5}{3} > 2(x+1)$ ……(*)

(a) Solve (*).

(b) Write down the greatest negative integer satisfying (*). (4 marks)

a) $x < -8$ and $2x+5 > 6x+6$ — (1M)

(1A)

$-4x > 1$

$x < -\frac{1}{4}$

$\therefore$ $x < -8$ — (1A)

b) -9 — (1A)

8. Consider G (2020, 2021).

(a) G is reflected in the x-axis to H. Find the coordinates of H.

(b) H is rotated about the origin through 180° to K. Find the coordinates of K.

(c) A student claims that ΔGHK is a right-angled triangle.
Is the claim correct? Explain your answer.

(5 marks)

a) $H(2020, -2021)$ — (1A)

b) $K(-2020, 2021)$ — (1A)

c) $\because$ x-coordinates of G & H are the same,
$\therefore$ GH is a vertical line (1M)

$\because$ x & y coordinates of K and G are the same
$\therefore$ KG is a horizontal line (1M)

$\therefore$ GH is perpendicular to KG

$\therefore$ ΔGHK is a right-angled Δ — (1A) p.t.

9. In **Figure 1**, *ABCD* is a parallelogram. *P* is a point lying on *CD* such that *BP* = *CP*.

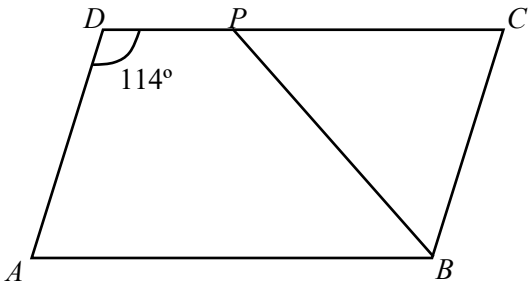

Figure 1

If $\angle ADC = 114^\circ$,

i. find $\angle ABP$;
ii. Are *D*, *P*, *B* and *A* concylic? Explain your answer.

(6 marks)

i) $\angle BCD + 114^\circ = 180^\circ$ (int. $\angle$s, $AD \parallel BC$)

$\angle BCD = 66^\circ$ — (1A)

$\angle PBC = 66^\circ$ (base $\angle$s, isos $\triangle PBC$) — (1A)

$\angle ABC = 114^\circ$ (opp. $\angle$s, $\parallel$gram) — (1A)

$\therefore \angle ABP = 114^\circ - 66^\circ$
$= 48^\circ$ — (1A)

ii) $\because \angle ADP + \angle ABP$
$= 114^\circ + 48^\circ$
$\neq 180^\circ$ } (1)

$\therefore$ D, P, B, A are not concyclic (1)

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SECTION A(2) (35 marks)

10. In **Figure 2**, AC is a diameter of circle $ABCD$. It is given that $BC = BD$ and $\angle ACD = 34°$.

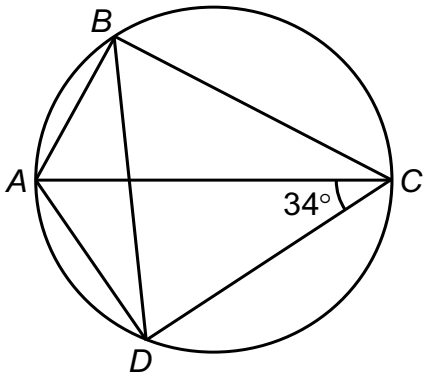

Figure 2

(a) Find $\angle ACB$.

(4 marks)

(b) A student claims that ΔABD is an isosceles triangle.
Do you agree? Explain your answer.

(3 marks)

a) $\angle ABC = 90°$ ($\angle$ in semi-circle) — 1A
$\angle ABD = 34°$ ($\angle$s in the same seg) — 1A
$\therefore \angle DBC = 90° - 34°$
$= 56°$
$\angle BCD = \angle BDC$ (base $\angle$s, isos Δ)
$\therefore \angle BCD = \frac{180° - 56°}{2}$
$\angle BCD = 62°$ — 1A
$\therefore \angle ACB = 62° - 34°$
$= 28°$ — 1A

b) $\angle BDA = \angle ACB$ ($\angle$s in the same seg) 1
$= 28°$
$\because \angle ABD \neq \angle BDA$ 1
$\therefore \Delta ABD$ is not an isos. Δ 1 f.t.
I disagree

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11. It is given that $f(x)$ is the sum of two parts, one part varies as x and the other part varies as $\sqrt{x}$. Suppose that $f(4) = 46$ and $f(16)=188$.

(a) Find $f(x)$.

(b) If the value of x is decreased from 16 to 9, find the percentage change of $f(x)$.

(6 marks)

11) $f(x) = k_1 x + k_2\sqrt{x}$ — (1M)

$\begin{cases} 46 = 4k_1 + 2k_2 \\ 188 = 16k_1 + 4k_2 \end{cases}$ (1M)

By solving, $k_1 = 12$, $k_2 = -1$

$\therefore f(x) = 12x - \sqrt{x}$ — (1A)

b) Original value $= 188$

New $f(9) = 12(9) - \sqrt{9}$

$= 105$ — (1A)

$\therefore$ % change $= \frac{105 - 188}{188} \times 100\%$ — (1M)

$= -44.14893617\%$ — (1A)

$\therefore$ It is decreased by 44.1% (corr. to 3 sig. fig.)

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12. **Figure 3(a)** shows a dartboard in the shape of a rectangle. Player throws a ball with a radius of 5 cm and hits the dartboard. If the whole ball lies inside the dartboard, the player wins.

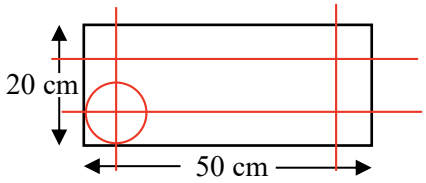


Figure 3(a)

a) Find the probability that the player wins.

(3 marks)

b) Now the shape of the dartboard is changed into a parallelogram with the same base length and height as shown in the **Figure 3(b)**.

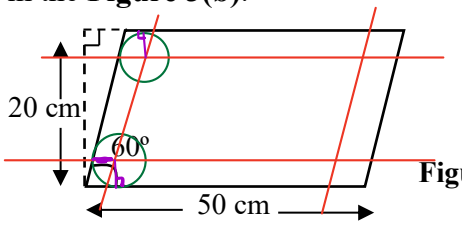


Figure 3(b)

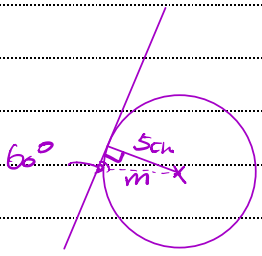
A student claims that the probability of the player wins will not be changed. Is the claim correct? Do you agree?

(3 marks)

12a) $P(\text{wins}) = \dfrac{(20-10)\times(50-10)}{20\times 50}$ — 1A — 1M

$= 0.4 \ / \ \frac{2}{5}$ — 1A

b)



$\frac{5}{m} = \sin 60°$ — 1M

$m = \frac{5}{\sin 60°} = \frac{10\sqrt{3}}{3}$

$P(\text{wins}) = \dfrac{(20-10)\times(50-2\times\frac{10\sqrt{3}}{3})}{20\times 50}$ — 1M for calculating the new area

≈ 0.384529946

$\neq 0.4$

$\therefore$ I disagree the claim. f.t. — 1

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13. Let P be a moving point in the rectangular coordinates plane such that the distance from P to a point $C(-5, 4)$ is 13. Denote the locus of P by Γ_p.

a) Find the equation of Γ_p.

(2 marks)

b) Γ_p cuts y-axis into two points A and B. There is an other moving point Q which has the same distance to CA and CB. Denote the locus of Q by Γ_q.

i. Describe the geometric meaning of Γ_q.

ii. Find the equation of Γ_q.

iii. Γ_p cuts the positive x-axis at a point E. Γ_q cuts the y-axis at K and cuts Γ_p at H at quadrant I.
Find the ratio of area ΔACK to area ΔKHE.

(5 marks)

a) Γ_p: $(x+5)^2+(y-4)^2=13^2$ — (2A)

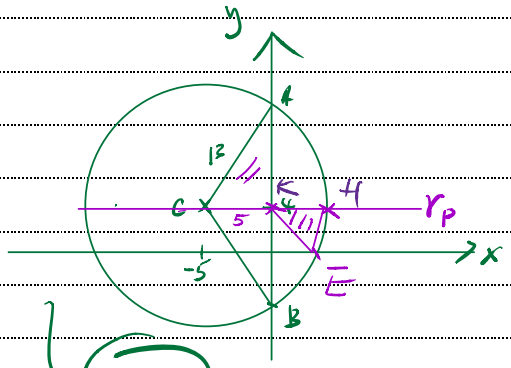
b) i) It is an angle bisector of $\angle ACB$ (1A)
(or perpendicular bisector of AB)

ii) $y=4$ — (1A)

iii) (Without solving A & B)
$AK^2+(13-5)^2=13^2$

$AK=12$

$\therefore$ Area of $\triangle ACK = \frac{1}{2}\times 5\times 12 = 30$ (1M)



$HK = 13-5 = 8$
$\therefore$ Area of $\triangle HKE = \frac{1}{2}\times 8\times 4 = 16$ (1M)

$\therefore$ Ratio of $\triangle ACK$ to $\triangle KHE = 30:16$
$= 15:8$ (1A)

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14. In the **Figure 4**, $\angle BAC = 30°$ and AB = 20 m and the area of ΔABC = 50 m^2.

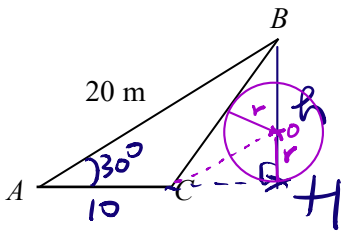

Figure 4

a) Let the height from B to the base AC be h m. Find the values of AC and h.

(3 marks)

b) A camping company is inventing some brand new camping equipment.

1. The designer makes an inflatable tent S_p by rotating the **Figure 4** along h, find the volume of the generated solid tent S_p.

2. In addition, the designer makes an inflatable sleeping bag which is a sphere. He wants this sphere can be placed inside the tent S_p completely.
Find the radius of the sphere such that it is maximum.

(6 marks)

a) $\frac{1}{2} AC \times 20 \times \sin 30° = 50$ — (1M)

$AC = 10$ — (1A)

$\frac{1}{2} \times AC \times h = 50$

$h = 10$ — (1A)

b) 1. $AH^2 = 20^2 - h^2$

$AH = \sqrt{300} = 10\sqrt{3}$ — (1A)

$\therefore CH = 10\sqrt{3} - 10$

$\therefore Vol\ S_p = \frac{1}{3}\pi(10\sqrt{3})^2 \times 10 - \frac{1}{3}\pi(10\sqrt{3}-10)^2 \cdot 10$ — (1M)

$\approx 2580\ m^3$ — (1A)

or $\left(\frac{2000\sqrt{3} - 1000}{3}\pi\right)$

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Method ①

(b) (ii) $\tan \angle BCH = \frac{10}{10\sqrt{3}-10}$ — 1M

$\angle BCH = 53.79597689°$

$\therefore \angle OCH = 26.89798844°$

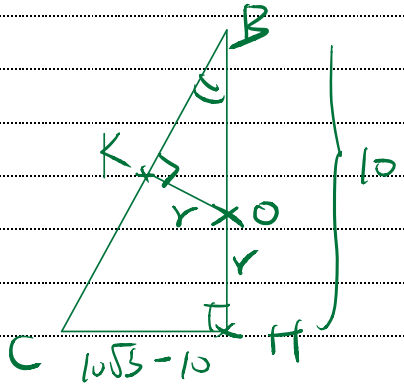
$\tan \angle OCH = \frac{r}{CH}$ — 1M

$\therefore r = 3.713421917$

$r \approx 3.71$ m — 1A

$\therefore$ radius $= 3.71$ m

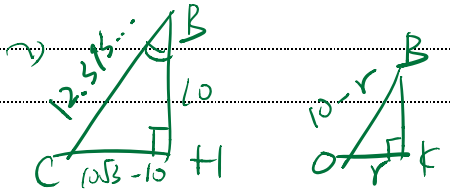
Method ②



$BC = 12.39313675$ — 1M

$\therefore \frac{r}{10\sqrt{3}-10} = \frac{10-r}{12.39313675\ldots}$ — 1M

$r = 3.71$ m — 1A



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SECTION B (35 marks)

15. 4 boys and 8 girls are randomly arranged to sit in a row.

(a) Find the probability that all the boys are sitting together. (2 marks)

(b) Find the probability that none of the boys are sitting next to each other. (2 marks)

a) $P(\text{Boys altogether}) = \dfrac{4! \times 9!}{12!}$ — (1M)

$= \dfrac{1}{55}$ — (1A)

b) $P(\text{Not sitting next to each other})$

$= \dfrac{8! \times P_4^9}{12!}$ — (1M) for P_4^9

$= \dfrac{14}{55}$ — (1A)

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16. In a English Speaking competition, John and Jaden score 68 marks and 86 marks respectively. The standard scores of John and Jayson are 0 and 1.5 respectively.

(a) Find the mean and the standard deviation of the scores in English Speaking competition. (3 marks)

(b) Later, John is found cheating in the competition and his mark is cancelled. Jaden claims that his standard score will increase after excluding John's score.

Is Jaden's claim correct? Explain your answer.

(2 marks)

a) the mean = 68 — (1A)

$\therefore 1.5 = \frac{86-68}{\sigma}$ — (1M)

$\therefore \sigma = 12$ — (1A)

b) The mean does not change.
S.D. increases as the data set spreads wider (1)

$\frac{86-68}{\sigma_{original}} > \frac{86-68}{\sigma_{new}}$

$\therefore$ the s. score of Jaden is decreased. f.t.
Jaden is incorrect. (1)

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17. For any positive integer n, let $a_n = 4n - 60$ and $b_n = \frac{10^8}{m^n}$, where $m > 1$. It is given that the sum to infinity of the series $b_1 + b_2 + b_3 + \ldots$ is 4×10^8.

(a) Find the value of m. (3 marks)

(b) For any positive integer k, $z_k = (a_1 + a_2 + a_3 + \ldots + a_k) + (b_1 + b_2 + b_3 + \ldots + b_k)i$,

where $i = \sqrt{-1}$. If z_j is purely imaginary where j is a positive integer, find z_j.

(4 marks)

a) $b_1 = \frac{10^8}{m}$ $b_2 = \frac{10^8}{m^2}$ $b_3 = \frac{10^8}{m^3}$ ..

$\therefore b_1, b_2, b_3 \ldots$ are g.s. with common ratio $= \frac{1}{m}$ (1M)

$\therefore 4 \times 10^8 = \frac{\frac{10^8}{m}}{1 - \frac{1}{m}}$ — (1M)

$\frac{4(m-1)}{m} = \frac{1}{m}$

$4m - 4 = 1$

$m = \frac{5}{4}$ — (1A)

b) $a_1 + \ldots + a_j = 0$ $a_1 = -56$

$\frac{j}{2}[-56 + 4j - 60] = 0$ — (1M)

$j = 29$ — (1A)

$b_1 + \ldots + b_j$

$= \frac{10^8}{1.25} + \ldots + \frac{10^8}{1.25^{29}}$

$= \frac{\frac{10^8}{1.25} \times \left[1 - \left(\frac{1}{1.25}\right)^{29}\right]}{1 - \left(\frac{1}{1.25}\right)}$ — (1M)

$= 399381030$

$\therefore z_j = 399381020\,i$ — (1A)

(or $\approx 3.99 \times 10^8\,i$)

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18. In **Figure 5(a)**, it is a paper card with DB =10 cm , $AD = DC$ = 15 cm, AB = BC = 22 cm.

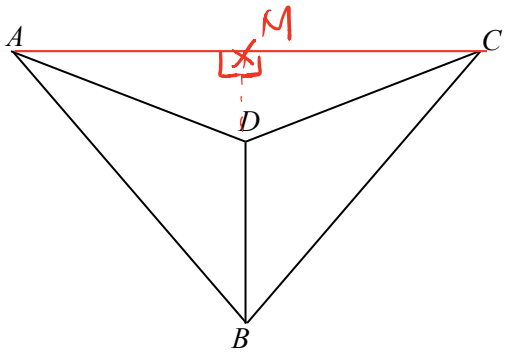

Figure 5(a)

a) Find the $\angle ABD$.

(2 marks)

b) An open-faced solid is formed when the above paper card is folded along DB. Then it is cut by a plane through the midpoint of AD, DC, AB and BC such that another solid with $APQC$ placed on the ground is formed(see **Figure 5(b)**). The four mid-points of the above line segments are M, N, P and Q respectively.

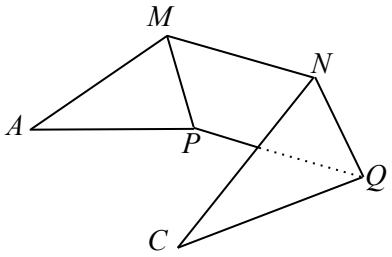

Figure 5(b)

Find the area of quadrilateral $MNPQ$ when the angle between the plane APM and plane CQN is 90º.

(5 marks)

a) $\cos \angle ABD = \frac{22^2+15^2-15^2}{2 \cdot 22 \cdot 15}$ — 1M

$\angle ABD \approx 35.322649849^\circ$

$\approx 35.3^\circ$ — 1A

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b) $\frac{AM}{22} = \sin \angle ABD$

$\therefore AM \approx 12.71996462$ — (1M)

$\therefore \overline{AC}$ on the ground

$= \sqrt{(AM)^2 + (AM)^2}$

≈ 17.98876688 — (1A)

$MN = 8.994573241$ (Mid-point theorem) (1A)

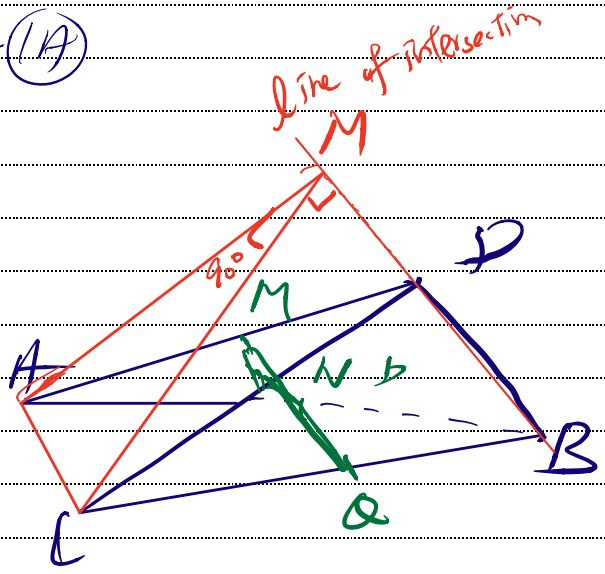
$\therefore$ Area of MNPQ

$= MN \times 5$

$= 44.97186621$

≈ 45.0 — (1A)

(1M) for Area of Rectangle of MNPQ



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19. A mask company introduces a new machine for manufacturing masks. At first, 20 masks are produced. The manager records the perimeter of the 20 masks in the following stem-and-leaf diagram.

Stem (10 cm)	Leaf (1 cm)
12	a 7 8 8 8 8 9
13	0 5 6 6 7 8
14	0 1 b 4
15	3 8 9

It is given that the range and the inter-quartile range of the perimeter of the 20 masks are 35 cm and 13 cm respectively.

(a) Find the values of a and b. (3 marks)

(b) Two extra masks are added to the above sample. It is found that the mean is decreased by 1 cm and the range is increased by 1 cm.

Find the perimeter of each of these extra masks. (4 marks)

(c) The boss of the company has to consider some constraints in order to decide which size of marks should be produced. He introduces rectangular coordinates system for handling the situation.

First, he sets the above each of the 22 data into (*stem*, *leaf*) format where *stem* as x-coordinate and *leaf* as y-coordinate.
For example, 127cm is (12,7) and 140 cm is (14,0).
The manager transforms his boss constraints as follow:

$$\mathbf{S} = \begin{cases} 0 \le y < 10 \\ 12 \le x \le 15 \\ x - 2y + 2 \ge 0 \\ x + 2y - 16 \ge 0 \\ x \text{ and } y \text{ are postive integers} \end{cases} .$$

i) If the perimeter of a mask is satisfied the above constraints, then that mask is qualified to be produced. Find the probability that the masks are qualified to be produced.

(2 marks)

ii) Now, his boss adds one more constraint to **S**, $x + y - 19 < 0$, in order to maximize the profit of making masks.
If the function of the profit of making each mask is \$P = 0.1x + 0.1y.
Find all possible qualified perimeter of masks which can maximize the profit of the company.

Hence, find the maximum profit of each of the masks that the company can make.

(3 marks)

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a) Range = 35

$159 - 120 - a = 35$

$a = 4$ — (1A)

$Q_3 = \frac{141 + 140+b}{2}$ $Q_1 = 128$ — (1M)

$\therefore \frac{281+b}{2} - 128 = 13$

$b = 1$ — (1A)

b) Let the two no. are x & y where $1 \le x \le y$ are integers

Total values of 20 data = 2740

Orignal mean = 137

New mean = 136 — (1M)

$\therefore \frac{2740 + x + y}{22} = 136$

$x + y = 252$ — (*) (1A)

the range is increased by 1,

$\therefore 159 - x = 36$

$x = 123$

or $y - 124 = 36$

$y = 160$

(1M) for considering two cases

Case (1) when $x = 123$, $y = 129$

(2) when $y = 160$, $x = 92$ (rejected)

$\therefore$ the two extra masks are 123 & 129 — (1A)

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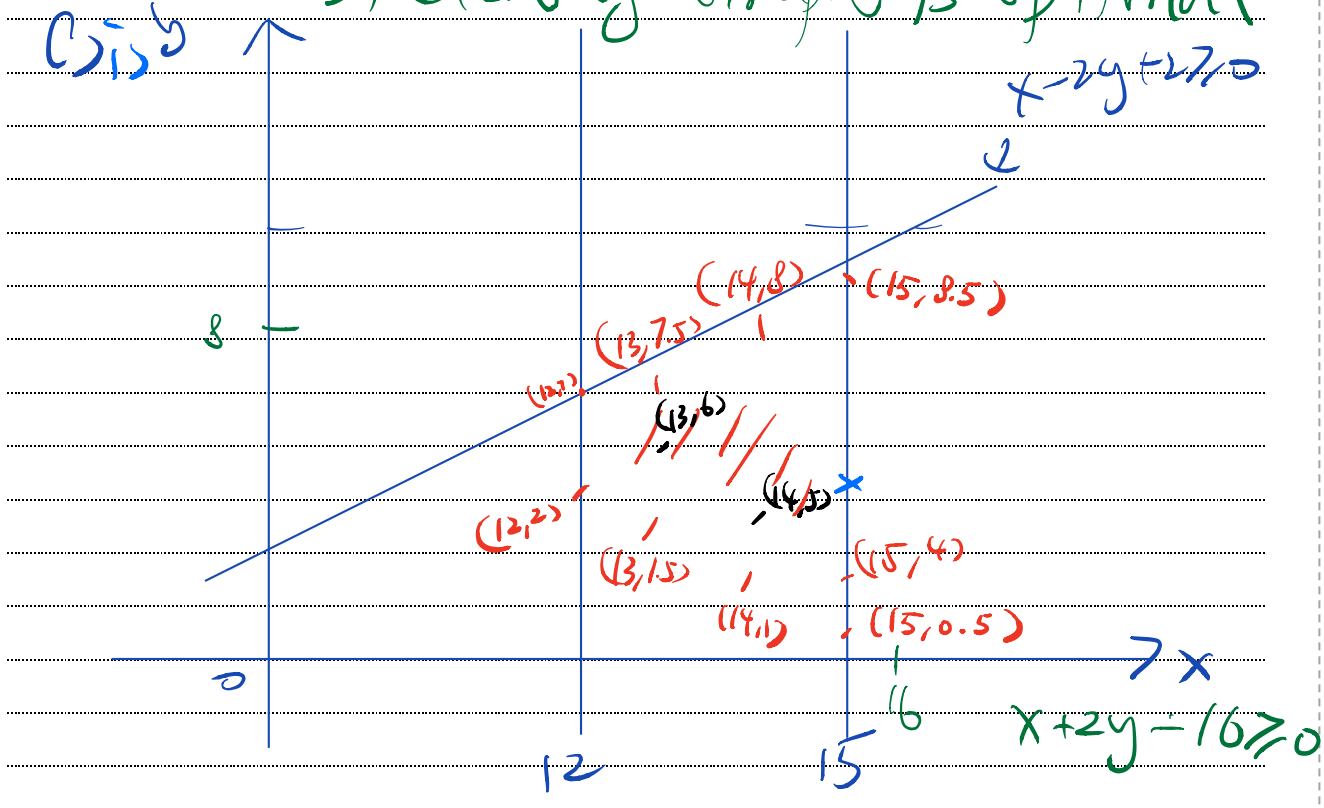
Answers written in the margins will not be marked

Sketching graph is optimal



∴ Satisfied no.

122 – 127
132 – 137
141 – 148
151 – 158

2 + 123 = 3
4
3
2

$\therefore P(\text{satisfied}) = \frac{12}{22}$ — (1A) for 12 correct choices

$= \frac{6}{11}$ — (1A)

ii) After setting an extra constraints,

$122 \le x < 127$ — 124 + 123
$132 \le x < 136$ — 135
$141 \le x < 145$ — 141 141 144
$151 \le x < 154$ — 153

$P(12,4) = 1.6$

$P(12,3) = 1.5$

$P(13,5) = 1.8$

$P(14,1) = 1.5$

$P(14,4) = 1.8$

$P(15,3) = 1.8$

(1M) for testing 6 choices

∴ There are three possible choice 135 cm, 144 cm and 153 cm and the maximum profit of each mask = $1.8 //

(1A) Only reward this mark by testing 6 choices.

— (1A)

Method (II)

If they use graph,

1M for sliding correct st. line or testing vertices

1A for 135 cm, 144 cm, 153 cm

1A for MAX profit = $1.8 each mask

~End of paper~

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