St. Joseph's College

2020-2021
S.6 Half-yearly Examination

MATHEMATICS Compulsory Part
PAPER 2

(1 1/4 hours)

INSTRUCTIONS

1. Read carefully the instructions on the Answer Sheet and insert the information required in the spaces provided.

2. When told to open this book, you should check that all the questions are there. Look for the words **'END OF PAPER'** after the last question.

3. All questions carry equal marks.

4. **ANSWER ALL QUESTIONS**. You are advised to use an HB pencil to mark all the answers on the Answer Sheet, so that wrong marks can be completely erased with a clean rubber.

5. You should mark only **ONE** answer for each question. If you mark more than one answer, you will receive **NO MARKS** for that question.

6. No marks will be deducted for wrong answers.

There are 30 questions in Section A and 15 questions in Section B.

The diagrams in this paper are not necessarily drawn to scale.

Choose the best answer for each question.

Section A

1. $3^{2020} \times \frac{(-9^{-1})^{-2021}}{27^{2022}} =$

   A. 27.

   B. $-\frac{1}{27}$.

   C. 81.

   D. $-\frac{1}{81}$.

2. $6a + 6b - a^2 + b^2 =$

   A. $(a + b)(6 - a + b)$.

   B. $(a + b)(6 - a - b)$.

   C. $(a - b)(6 - a + b)$.

   D. $(a - b)(6 - a - b)$.

3. If m and n are non-zero constants such that $m : n = 8 : 3$, then $(2m + 3n) : (4m + n) =$

   A. 3 : 2.

   B. 5 : 2.

   C. 5 : 7.

   D. 19 : 15.

4. If A and B are constants such that $Ax(x - 5) - x^2 \equiv Bx(x + 6) - 93x$, then $B =$

   A. 5.

   B. 6.

   C. 7.

   D. 8.

5. 0.828 456 9 =

A. 0.83 (correct to 3 decimal places).

B. 0.828 4 (correct to 4 significant figures).

C. 0.828 46 (correct to 5 significant figures).

D. 0.928 455 (correct to 6 significant figures).

6. A shop offers 'buy 2 get 1 free' promotion for the canned pineapples. John paid $84 for 10 cans of pineapples at the shop. What is percentage discount of John finally got from the shop?

A. 30 %

B. $33\frac{1}{3}\%$

C. $66\frac{2}{3}\%$

D. 70%

7. A sum of $25 000 is deposited at an interest rate of 4% per annum for 10 years, compounded half-yearly. Find the amount correct to the nearest dollar.

A. $30 474

B. $37 149

C. $53 973

D. $54 778

8. If $x \leq y$ and $h < 0$, which of the following must be true?

I. $x^2 \leq xy$.

II. $2x + h \leq 2y + h$.

III. $\frac{h^2}{x} \geq \frac{h^2}{y}$.

A. II only

B. I and II only

C. I and III only

D. II and III only

9. Solve the equation $x(x+\frac{n}{m})=\frac{m}{n}x+1$, where *m* and *n* are non-zero constants.

A. $x=\frac{m}{n}$ or $x=-\frac{n}{m}$.

B. $x=-\frac{m}{n}$ or $x=\frac{n}{m}$.

C. $x=mn$ or $x=-\frac{1}{mn}$.

D. $x=-mn$ or $x=\frac{1}{mn}$.

10. Let *k* be a constant. If the quadratic equation $x^2 + kx – k = 1$ has a repeated root, then *k* =

A. –2.

B. 1.

C. 0 or –4.

D. 0 or 2.

11. If an interior angle of a regular *n*-sided polygon is three times an exterior angle, which of the following is / are true?

I. Each interior angle of the polygon is 135°.

II. The number of axes of reflectional symmetry of the polygon is 4.

III. The number of folds of rotational symmetry of the polygon is 8.

A. I only

B. I and III only

C. II and III only

D. I, II and III

12. Let *p* and *q* be constants. If$(x-3)$is a factor of x^3+px^2+qx-3, find the remainder when $7x^3+(3p+q)x^2-x+2$ is divided by $(x+1)$.

A. −12

B. −8

C. 8

D. 12

13. It is given that c varies directly as a and inversely as the square of b where a, b and c are positive. $a:b:c = 3:1:5$ when $b = 3$. When $a:b:c = 3:4:5$, find the value of c.

A. $\frac{3}{2}$.

B. $\frac{10}{3}$.

C. $\frac{15}{4}$.

D. $\frac{21}{2}$.

14. In a liquid, the content of alcohol by volume is 80%. After 20 L of water is added into the liquid, the content of alcohol by volume becomes 55%. What is the original volume (in L) of the liquid?

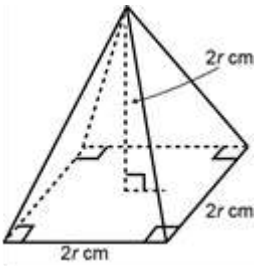
A. 42 L

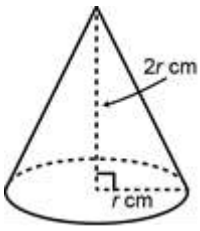
B. 44 L

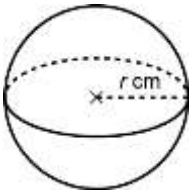
C. 52 L

D. 54 L

15.







The volumes of the right pyramid, right circular cone and sphere in the above figure are V_1, V_2 and V_3 respectively. Which of the following is true?

A. $V_2 < V_1 < V_3$.

B. $V_1 < V_3 < V_2$.

C. $V_2 < V_3 < V_1$.

D. $V_3 < V_2 < V_1$.

16. $A(8, 10)$, B and C are the vertices of $\triangle ABC$. $D(0, 6)$ and $E(8, 4)$ are the mid-points of AB and AC respectively. Which of the following is/are correct?

I. BC passes through origin O.

II. Area of ΔADE : Area of ΔBEC = 1:2 .

III. Area of ΔBDO : Area of quadrilateral $DECO$ = 1:2 .

A. II only

B. III only

C. II and III only

D. I, II and III

17. $\frac{cos(180°+\theta)}{sin(180°-\theta)} \times \frac{1}{tan(90°-\theta)} =$

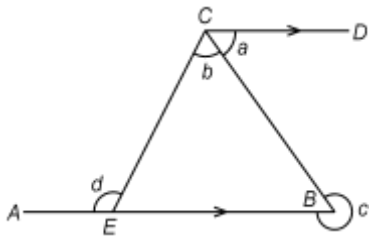
A. 1 .

B. –1 .

C. $\tan\theta$.

D. $\tan^2\theta$.

18. In the figure, AEB is a straight line.
Which of the following must be true?



I. $a + b = d$

II. $a + c = 360°$

III. $c + d = b + 180°$

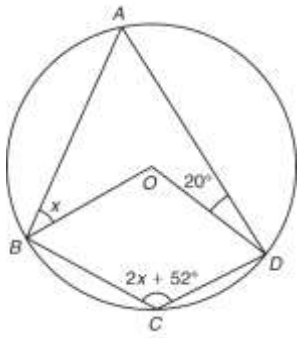
A. I and II only

B. I and III only

C. II and III only

D. I, II and III

19. In the figure, *O* is the centre of the circle ABCD. Find *x*.



A. 36º

B. 40º

C. 42º

D. 45º

20. Let a_n be the *n*th term of a sequence. If $a_1 = -2$, $a_2 = 1$ and $a_{n+2} = 4a_{n+1} - a_n$ for any positive integer *n*, then $a_5 =$

A. 26 .

B. 46 .

C. 66 .

D. 86 .

21. The side of a metal cube is measured as 8 cm, correct to the nearest cm. Now, the metal cube is melted and recast into a number of identical spheres of radius 2 cm, correct to the nearest cm. Find the maximum number of spheres that could be made.

A. 41

B. 42

C. 43

D. 44

22. Given two functions $f(x) = x^2 - 3$ and $g(x) = 1$. Which of the following is/are true?

I. Vertex of $y = f(x)$ is $(0,-3)$.

II. Maximum values of $f(x) = 3$.

III. Solution of $f(x) > g(x)$ is $x < -2$ or $x > 2$.

A. I only

B. II only

C. I and III only

D. II and III only

23. Find the equation of the locus of a point P which is equidistant from the origin and L: $y = 4$.

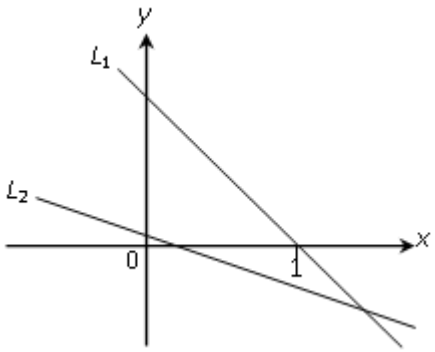
A. $y - 2 = 0$

B. $x + y + 2 = 0$

C. $x^2 + 8y - 16 = 0$

D. $x^2 + y^2 - 4 = 0$

24. The figure shows the straight lines L_1: $y + ax = b$ and L_2: $y + cx = d$. Which of the following is/are true?



I. $a > c > 0$

II. $c < d$

III. $b - d > a - c$

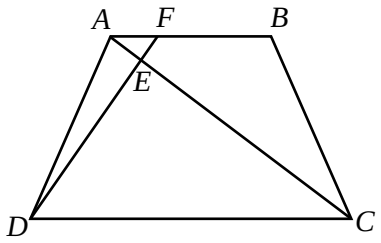
A. I only

B. II only

C. I and III only

D. II and III only

25. In the figure, $ABCD$ is a trapezium with AB // DC and $AB : DC = 1 : 2$. F is a point lying on AB such that $AF : FB = 1 : 3$. E is the point of intersection of DF and AC. If the area of $\triangle CED$ is 64 cm^2, then the area of $ABCD$ is



A. 96 cm^2.

B. 105 cm^2.

C. 108 cm^2.

D. 128 cm^2.

26. The rectangular coordinates of the point Q are $(2, -\sqrt{12})$. If Q is rotated anticlockwise about the origin through 180°, then the polar coordinates of its image are

A. (2, 120°).

B. (2, 150°).

C. (4, 120°).

D. (4, 150°).

27. If a diameter of the circle $x^2 + y^2 + kx - ky - 5 = 0$ cuts the y-axis at (0, 21) and the slope of the diameter is 5, then k =

A. 7.

B. 10.

C. −7.

D. −10.

28. There are 5 balls of different colours in a bag. Sammi draws a ball at random from the bag. Then she puts the ball drawn back into the bag and draws a ball at random from the bag again. Find the probability that the two balls drawn are of different colours.

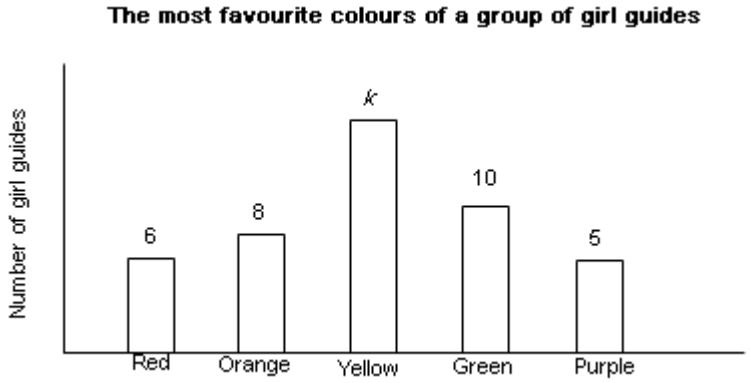
A. $\frac{4}{25}$

B. $\frac{1}{5}$

C. $\frac{1}{4}$

D. $\frac{4}{5}$

29. A group of girl guides are interviewed to find their favourite colours. Each girl guide can vote for one colour only. The results are shown in the bar chart below.



If a girl guide is selected at random from the group, then the probability that her favourite colour is green is $\frac{2}{9}$. Find the value of k.

A. 14

B. 16

C. 18

D. 20

30. Consider the following integers:

2 2 2 2 4 5 7 7 8 8 9 10 10 a

Let p, q and r be the mean, the median and the range of the above integers respectively.

If $5 \leq a \leq 7$, which of the following must be true?

I. $p > r$.

II. $r > q$.

III. $q > p$.

A. I only

B. II only

C. I and III only

D. II and III only

Section B

31. $15\times2^{10} + 255\times2^7 + 10\times2^6 =$

A. 101111100000000_2 .

B. 1011111000000000_2 .

C. 1101111000000000_2 .

D. 11011111000000000_2 .

32. It is given that $(1 + 5i)x = 6$. The imaginary part of *x* is

A. $\frac{3}{13}$.

B. $\frac{15}{13}$.

C. $-\frac{3}{13}$.

D. $-\frac{15}{13}$.

33. Given the equation $\log_5(5^{x+2} - 600) = x$. Find the value of *x*.

A. $\frac{1}{2}$

B. $\frac{1}{5}$

C. 2

D. 4

34. If α and β are the roots of the quadratic equation $x^2 + 5x - a = 0$, find the value of $\frac{1}{2\alpha^2} + \frac{1}{2\beta^2}$.

A. $\frac{5+2a}{a^2}$

B. $\frac{25+2a}{a^2}$

C. $\frac{25+2a}{2a^2}$

D. $\frac{25+2a}{4a^2}$

35. How many solutions of θ if $1 + \sin\theta\cos\theta = 3\sin^2\theta$ for $0° \leq \theta \leq 360°$.

A. 0

B. 2

C. 3

D. 4

36. There are 10 different balls, of which 4 are red, 3 are blue and the rest are yellow. Now 7 of the balls are selected to put in a row. How many permutations of 3 red balls are selected and any two of the red balls are not placed next to each other.

A. 8640

B. 51840

C. 86400

D. 518400

37. A coach of a basketball team is going to select 5 starters out of 11 team members for a match. They will be assigned to the point guard, shooting guard, small forward, power forward and centre. Andy and Ben are two of the members. Given that both Andy and Ben are selected, find the probability that Andy and Ben are assigned to the point guard and power forward respectively.

A. $\frac{1}{10}$

B. $\frac{1}{15}$

C. $\frac{1}{20}$

D. $\frac{2}{25}$

38. It is given that three positive numbers x, y and z are in geometric sequence. Which of the following must be true?

I. x^4, y^4, z^4 are in geometric sequence.

II. $4x$, $4y$, $4z$ are in geometric sequence.

III. $\log x^4$, $\log y^4$, $\log z^4$ are in arithmetic sequence.

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

39. The 3^{rd} term and the 5^{th} term of a geometric sequence are 72 and 648 respectively, which of the following must be true?

I. the first term of the sequence is 8.

II. The 4^{th} term of the sequence must be positive.

III. Sum of the first n term of the sequence is $4(3^n - 1)$ only.

A. I only

B. I and II only

C. II and III only

D. I, II and III

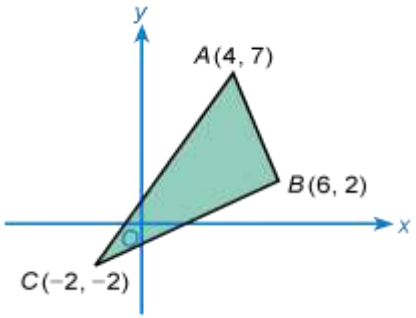
40. In the figure, the shaded region represents the feasible region. Given that (x, y) is any point in the feasible region and $P = ax + 2y$ (where a is a constant) attains its maximum value at point A, find the range of values of a.

A. $a \geq -3$

B. $a \leq -3$ or $a \geq 5$

C. $a \geq 5$

D. $-3 \leq a \leq 5$



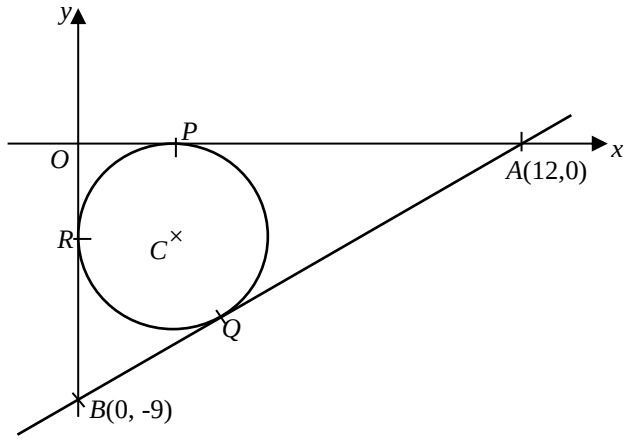
41. In the figure, a circle with its centre at C is the inscribed circle of ΔOAB, where P, Q and R are the points of contact. Find the coordinates of Q.

A. $\left(\frac{11}{3}, -\frac{23}{3}\right)$

B. $\left(\frac{17}{3}, -\frac{24}{5}\right)$

C. $\left(\frac{21}{5}, -\frac{18}{5}\right)$

D. $\left(\frac{24}{5}, -\frac{27}{5}\right)$



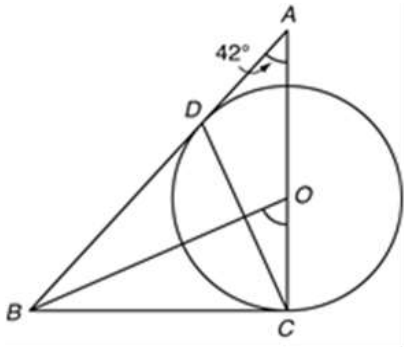
42. The figure shows a circle with centre O. BC and BA are the tangents to the circle at C and D respectively. If $\angle BAC = 42°$, find $\angle BOC$.

A. 66°

B. 72°

C. 84°

D. 90°



43. It is given that the circle S: $x^2 + y^2 - 4x = 0$ and the straight line L: $x - y = 0$ intersect at A and B. Find the length of AB.

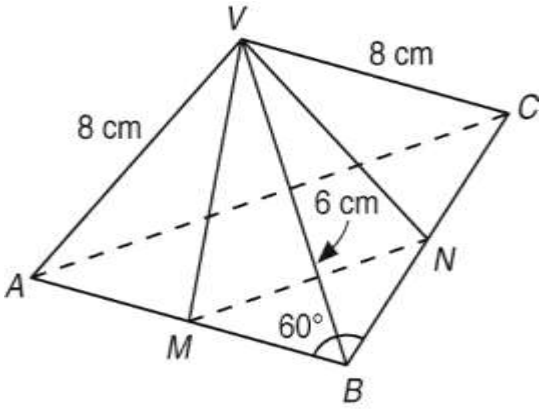
A. 2

B. $2\sqrt{2}$

C. 3

D. $3\sqrt{3}$

44. In the figure, BV is perpendicular to the plane VAC, $VA = VC = 8$ cm and $VB = 6$ cm. M and N are the mid-points of AB and CB respectively.
If $\angle ABC = 60°$, find the area of $\triangle VMN$.



A. $\frac{5\sqrt{19}}{2}$ cm^2

B. $\frac{5\sqrt{3}}{2}$ cm^2

C. $\frac{5\sqrt{19}}{4}$ cm^2

D. $\frac{25\sqrt{3}}{4}$ cm^2

45. The range and the variance of S:$\{a_1,..., a_{100}\}$ are m and n respectively. If every element of S are added 5 first and then multiplied by 10, what is the new range and the new variance?

A. New range = $10m$; new variance = $10n$.

B. New range = $10m+50$; new variance = $10n+50$.

C. New range = $10m$; new variance = $100n$.

D. New range = $10m + 5$; new variance = $10n + 5$.

~END OF PAPER~

Answers

1	**D**	**26**	**C**
2	**A**	**27**	**A**
3	**C**	**28**	**D**
4	**D**	**29**	**B**
5	**C**	**30**	**D**
6	**A**	**31**	**B**
7	**B**	**32**	**D**
8	**A**	**33**	**C**
9	**A**	**34**	**C**
10	**A**	**35**	**D**
11	**B**	**36**	**C**
12	**A**	**37**	**C**
13	**C**	**38**	**D**
14	**B**	**39**	**A**
15	**A**	**40**	**D**
16	**D**	**41**	**D**
17	**B**	**42**	**A**
18	**A**	**43**	**B**
19	**A**	**44**	**D**
20	**D**	**45**	**C**
21	**C**		
22	**C**		
23	**C**		
24	**C**		
25	**C**		