F. 6 2018/19

FINAL EXAMINATIONS

MATHEMATICS Compulsory Part

PAPER 1

Question-Answer Book

($2\frac{1}{4}$ hours)

This paper must be answered in English

INSTRUCTIONS

1. Write your Name, Class and Class Number in the spaces provided on Page 1.

2. This paper consists of THREE sections, A(1), A(2) and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Graph paper and supplementary answer sheets will be supplied on request. Write your Name, Class and Class Number, mark the question number box on each sheet and fasten them with string INSIDE this book.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

Name	
Class	( )

	Marker's Use Only	**Examiner's Use Only**
	Marker No.	Examiner No.
Question No.	Marks	Marks
1–2		
3–4		
5–6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
Total		

SECTION A(1) (35 marks)

1. Factorize

   (a) $16x^2 - y^2$;

   (b) $4x + y - 16x^2 + y^2$. (3 marks)

2. Simplify $\dfrac{(a^3b^{-4})^5}{\sqrt{a^{20}}b^{-13}}$ and leave your answer in positive indices. (3 marks)

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3. Make y the subject of the formula $\frac{y+5}{10x-2y}=\frac{3}{4}$. (3 marks)

4. When selling an article at a discount of 10%, a profit of \$40 is made. When selling it at a discount of 20%, a loss of 4% is made. Find the cost price of the article (4 marks)

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5. The length and breadth of a rectangular living room are 13 m and 10 m respectively, both correct to the nearest metre. Its floor will be covered by square tiles of side 25 cm, correct to the nearest cm.

(a) Find the maximum possible area of the living room.

(b) Someone claims that 2300 floor tiles must be enough to cover the floor of the living room. Do you agree? Explain your answer. (4 marks)

6. (a) Solve the compound inequality $\frac{3x+7}{5} \geq 6$ or $8-3x < 2(9+x)$.

(b) Write down the smallest integer satisfying the above compound inequality. (4 marks)

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7. There were 20 multiple-choice questions in an exam paper, which must be all attempted. For each correct answer, 3 marks will be given whereas for each wrong or blank answer, 1 mark will be deducted.

(a) Peter answered 14 questions correctly. How many marks did he get?

(b) Mary's mark was between 33 and 39. Peter said that Mary had the same mark as him. Do you agree? Explain your answer. (4 marks)

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8. It is given that z varies as x and inversely as $\sqrt{y}$. When $x = 8$ and $y = 225$, $z = \frac{32}{3}$.

(a) Find z in terms of x and y.

(b) Suppose y increases from 225 by 8 times and z decreases from $\frac{32}{3}$ to $\frac{16}{3}$. Find the change in x. (5 marks)

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9. Figure 1 shows the circumcircle of ΔABC. $\angle ABC = 70°$ and $\angle ACB = 47°$. D is a point on $\overset{\frown}{BC}$ such that $\overset{\frown}{BD} : \overset{\frown}{DC} = 2 : 1$.

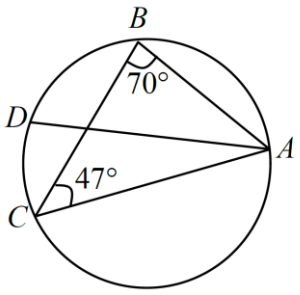

Figure 1

(a) Find $\angle BAD$.

(b) Is AD a diameter of the circle? Explain your answer. (5 marks)

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SECTION A(2) (35 marks)

10. The stem-and-leaf diagram below shows the distribution of the number of movies watched by members of a film appreciation club last year. It is given that the mean and the median of this distribution are the same.

Stem (tens)	Leaf (units)								
2	0	2	2	4	7	9			
3	0	1	1	a	5	7	7	7	9
4	0	2	3	5	7				

(a) Find the standard deviation of the above distribution. (4 marks)

(b) If 2 members are randomly selected from the club, find the probability that one member watched less than 30 movies last year while the other watched more than 30. (2 marks)

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11. The coordinates of points A, B, C and D are (1, 6), (7, 14), (−23, 13) and (16, k) respectively, where k is a constant. A, B and D are collinear.

(a) Find k. (2 marks)

(b) Show that $AC = AD$. (2 marks)

(c) Let P be a moving point in the rectangular coordinate plane. Denote the locus of P by Γ. If Γ bisects $\angle BAC$, find the equation of Γ. (3 marks)

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12. In Figure 2, AD intersects BG and CG at F and E respectively. A, B and C are collinear. $AF = CD$, $AB = CE$ and $\Delta ACD \sim \Delta GEF$.

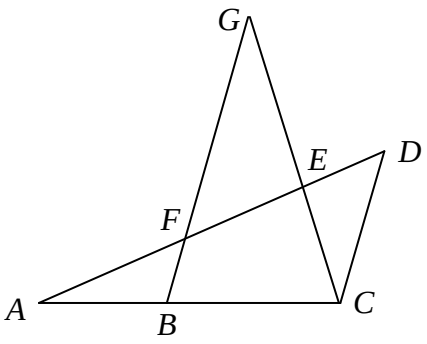

Figure 2

(a) Prove that BG is parallel to CD. (2 marks)

(b) Prove that $\Delta ABF \cong \Delta CED$. (2 marks)

(c) Suppose $AF = 13$ cm and $FB = 5$ cm. Find EF. (3 marks)

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13. Suppose a, p, q and r are constants. Let $f(x) = (px - 1)(18x^2 + qx + r)$. When $f(x)$ is divided by $12x^2 + ax - 9$, the quotient and remainder are $3x - 5$ and $-4x + 2$ respectively.

(a) Find p, q and r. (5 marks)

(b) Let $g(x)$ be a polynomial such that when $g(x)$ is divided by $x + 4$, the quotient and the remainder are $f(x)$ and 8 respectively. Someone claims that all the roots of the equation $g(x) = 8$ are rational. Do you agree? Explain your answer. (3 marks)

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14. A vessel in form of a frustum is made by cutting off the lower part of an inverted right circular cone. The radii of the upper base and the lower base are 16 cm and 4 cm respectively. The height of the frustum is 36 cm. The vessel contains 10000 cm^3 of water.

(a) Find the capacity of the vessel in terms of π. (2 marks)

(b) n solid metal spheres of radii 3 cm are put into the vessel and the spheres are totally immersed in the water.

(i) Find the maximum value of n such that the water in the vessel will not overflow.

(ii) If $n = 15$, find the depth of water in the vessel. (5 marks)

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SECTION B (35 marks)

15. In a bag, there are 2 red balls, 3 yellow balls and 4 blue balls. The red balls are numbered 1 and 2; the yellow balls are numbered 3, 4 and 5 while the blue balls are numbered 6, 7, 8 and 9. Four balls are drawn one by one without replacement from the bag at random. The numbers shown on the balls will form a 4-digit number with the first drawn number being the thousands digit, the second being the hundreds digit and so on. Find the probability that

(a) the number formed is larger than 4900; (2 marks)

(b) the balls drawn are of at most 2 colours and the numbers drawn are neither 1 nor 5. (2 marks)

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16. (a) In Figure 3(a), *ABCD* is a paper card in the shape of a rectangle with *AB* = 20 cm and *BC* = 16 cm. *E* is a point on *AB* such that $DE \perp AC$ and *DE* meets *AC* at *F*.

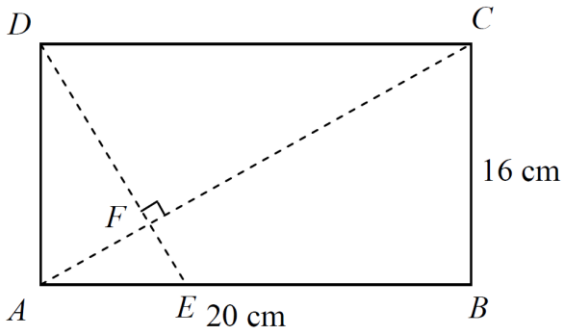

Figure 3(a)

Find the lengths of *AE*, *DF* and *FE*. (3 marks)

(b) The paper card in Figure 3(a) is folded along the diagonal *AC*. It is erected such that *A*, *B* and *D* lie on the same horizontal plane and plane *ABC* is vertical as shown in Figure 3(b).

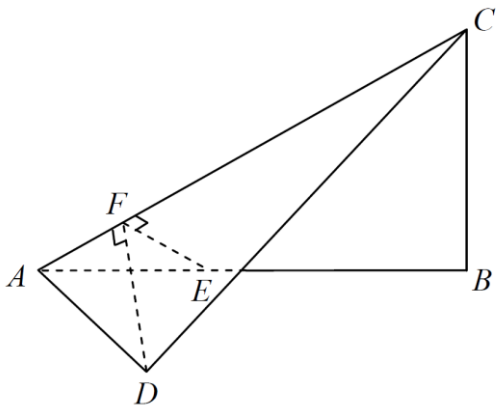

Figure 3(b)

(i) Find the angle between plane *ACD* and the horizontal plane.

(ii) Find $\cos\angle BAD$ by considering ΔABD and hence find *DE*.

(iii) Find the angle between plane *ABC* and plane *ACD*. (5 marks)

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17. ΔOPQ is a right-angled triangle with $\angle POQ = 90°$. Denote the in-centre of ΔOPQ by *I*. Denote the radii of the inscribed circle C_1 as *r*. Let *D*, *E*,and *F* be the points where C_1 meets *OP*, *OQ*, *PQ* respectively.

(a) By considering the areas of ΔOPQ, ΔOPI, ΔOQI and ΔIPQ, show that

$$r = \frac{OP \cdot OQ}{OP + OQ + PQ}.$$ (2 marks)

(b) A rectangular coordinate system is introduced so that the coordinates of *O* and *P* are (0, 0) and (0, 12) respectively, *Q* is on the positive *x*-axis and *PQ* = 20.

(i) Find the equation of C_1.

(ii) Let C_2 be the circumscribed circle of ΔOPQ. A tangent to C_2 from an external point *S* (0, 16) cuts the *x*-axis at *T*. Someone claims that the area of ΔSTQ exceeds 160. Is the claim correct? Explain your answer. (8 marks)

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18. On 31st December, 2018, Mr White retired and deposited \$4,000,000 into an investment fund at an interest rate of 4.8% per annum compounded quarterly. He will withdraw money from the fund for expenses at the end of every quarter according to one of the following plans A and B. In plan A, Mr White always withdraws \$60,000. In plan B, Mr White will withdraw \$60,000 at the end of March 2019. To cover inflation, he will increase the withdrawals by 0.4% each quarter.

(a) Suppose Mr White adopts plan A. Let $\$A_n$ be the balance of the fund after the nth withdrawal.

(i) Find A_1 and A_4, correct to the nearest integer.

(ii) Show that $A_n = (5 - 1.012^n) \cdot 10^6$.

(iii) Can the fund support Mr White's expenses for 30 years under plan A? Explain your answer. (7 marks)

(b) Suppose Mr White adopts plan B.

(i) How much, correct to the nearest dollar, will Mr White withdraw at the end of 2019?

(ii) Find the balance of the fund at the end of 2019 after withdrawal, correct to the nearest dollar.

(iii) For how many years can the fund support Mr White's expenses under plan B?

(6 marks)

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