F.6 Final Examination (2018 – 2019)
Mathematics Compulsory Part Answers

1. (a) $16x^2 - y^2 = (4x+y)(4x-y)$

   (b) $4x + y - 16x^2 + y^2$

   $= 4x + y - (4x+y)(4x-y)$

   $= (4x+y)(1-4x+y)$

[3]

2. $\dfrac{(a^3b^{-4})^5}{\sqrt{a^{20}b^{-13}}} = \dfrac{a^{15}b^{-20}}{a^{10}b^{-13}}$

   $= a^{15-10}b^{-20+13}$

   $= \dfrac{a^5}{b^7}$

[3]

3. $\dfrac{y+5}{10x-2y} = \dfrac{3}{4}$

   $2(y+5) = 3(5x-y)$ (OR $4(y+5) = 3(10x-2y)$)

   $5y = 15x - 10$ (OR $10y = 30x - 20$)

   $y = 3x - 2$

[3]

4. Let the cost price be $C. Then

   $\dfrac{C+40}{0.9} = \dfrac{C(1-0.04)}{0.8}$

   $C + 40 = 1.08C$

   $C = 500$

   OR Let the marked price be $M.

   $0.9M - 40 = \dfrac{0.8M}{(1-0.04)}$

   $5.4M - 240 = 5M$

   $M = 600$

   $\therefore$ The cost price is $500.

[4]

5. (a) maximum possible area $= 13.5 \times 10.5\text{ m}^2 = 141.75\text{ m}^2$

   (b) smallest possible area of each tile $= 24.5 \times 24.5\text{ cm}^2 = 600.25\text{ cm}^2$

   maximum number of tiles required $= \dfrac{141.75 \times 10^4}{600.25} \approx 2362 > 2300$

   OR minimum total area of 2300 tiles $= \dfrac{2300 \times 600.25}{10^4}\text{ m}^2 \approx 138\text{ m}^2 < 141.75\text{ m}^2$

   $\therefore$ The claim is disagreed.

[4]

6. (a) $\frac{3x+7}{5} \geq 6$ or $8-3x < 2(9+x)$

$3x+7 \geq 30$ or $-5x < 10$

$x \geq \frac{23}{3}$ or $x > -2$

$x > -2$

(b) The smallest integer satisfying the compound inequality is -1.

[4]

7. (a) mark of Peter $= 14 \times 3 - (20-14) = 36$

(b) Suppose Mary answered x questions correctly and her mark was m. Then

$m = 3x - (20-x)$

$= 4x - 20 = 4(x-5)$

$\because$ x is an integer

$\therefore$ m is a multiple of 4.

$\because$ 36 is the only multiple of 4 between 33 and 39

$\therefore$ $m = 36$

> OR $33 \leq 4x - 20 \leq 39$ (OR $33 < 4x - 20 < 39$)
>
> $\frac{53}{4} \leq x \leq \frac{59}{4}$ (OR $\frac{53}{4} < x < \frac{59}{4}$)
>
> $\therefore$ $x = 14$

> OR When $x = 13$, $m = 32$; when $x = 14$, $m = 36$; when $x = 15$, $m = 40$.
>
> $\because$ Only 36 is between 33 and 39
>
> $\therefore$ $x = 14$

$\therefore$ The claim is agreed.

[4]

8. (a) $z = \frac{kx}{\sqrt{y}}$, where k is a non-zero constant

$\frac{32}{3} = \frac{k(8)}{\sqrt{225}}$

$k = 20$

$\therefore$ $z = \frac{20x}{\sqrt{y}}$

(b) $\frac{16}{3} = \frac{20x'}{\sqrt{225 \cdot 9}}$ where x' is the new value of x

$x' = 12$

$\therefore$ x increases by 4.

[5]

9. (a) Let $\angle CBD = \theta$. Then

$\angle BCD = 2\theta$ (arcs prop. to $\angle$s at circum.)

$\theta + 2\theta + 70° + 47° = 180°$ (opp. $\angle$s, cyclic quad.)

$\theta = 21°$

$\angle BAD = 2\theta = 42°$ ($\angle$s in the same segment)

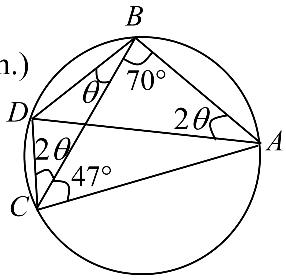


Figure 1

> OR Let $\angle CAD = \theta$. Then
> $\angle BAD = 2\theta$. (arcs prop. to $\angle$s at circum.)
> $\theta + 2\theta + 47° + 70° = 180°$ ($\angle$ sum of Δ)
> $\theta = 21°$
> $\therefore \angle BAD = 42°$

(b) $\because \angle ABD = 91° \neq 90°$ (OR $\angle ACD = 89° \neq 90°$)

$\therefore$ AD is not a diameter.

> OR $\angle CBD = 21°$ (or $\angle BCD = 42°$) ($\angle$s in the same segment)
> $\therefore \angle ABD = 91° \neq 90°$ (OR $\angle ACD = 89° \neq 90°$)
> $\therefore$ AD is not a diameter.

[5]

Section A(2)

10. (a)

$$\text{Median} = \frac{30 + a + 35}{2} = \frac{65 + a}{2}$$

$$\text{Mean} = \frac{668 + a}{20}$$

$$\frac{65 + a}{2} = \frac{668 + a}{20}$$
$$a = 2$$

$$\text{The standard deviation} = \frac{9\sqrt{3}}{2} \approx 7.79$$

(4)

(b)

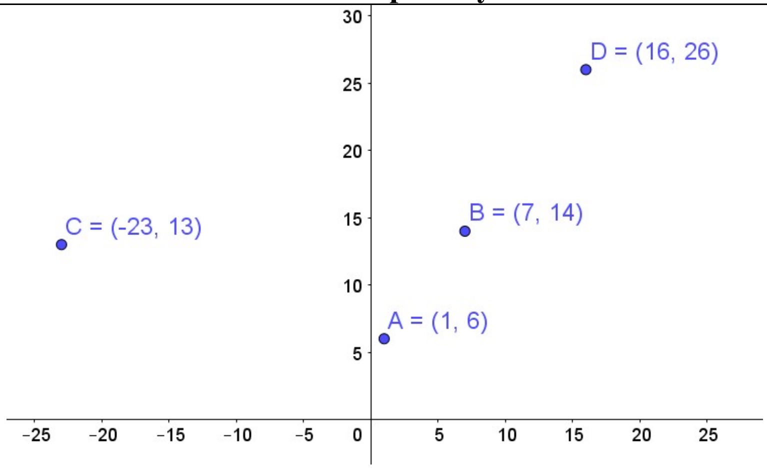
$$\text{The required probability} = 2\left(\frac{6}{20}\right)\left(\frac{13}{19}\right) = \frac{39}{95} \approx 0.411$$

Alternative method:

> $$\text{The required probability} = \frac{6 \times 13}{C_2^{20}} = \frac{39}{95} \approx 0.411$$

(2)

11. (a)



$\because A, B$ and D are collinear

$$\therefore \text{ slope of } AB = \text{slope of } AD$$
$$\frac{14-6}{7-1} = \frac{k-6}{16-1}$$
$$k = 26$$

Alternative method:

Equation of AB:
$$y - 6 = \frac{14-6}{7-1}(x-1)$$
$$y = \frac{4}{3}x + \frac{14}{3}$$
$$D \text{ is a point on } AB,\ k = \frac{4}{3}(16) + \frac{14}{3}$$
$$= 26$$

(2)

(b)
$$AC = \sqrt{(-23-1)^2 + (13-6)^2}$$
$$= 25$$
$$AD = \sqrt{(16-1)^2 + (26-6)^2} = 25 = AC$$

(2)

(c)
$$\because AC = AD$$
The angle bisector of $\angle BAC$ is the perpendicular bisector of CD
$$\therefore \Gamma \perp CD$$

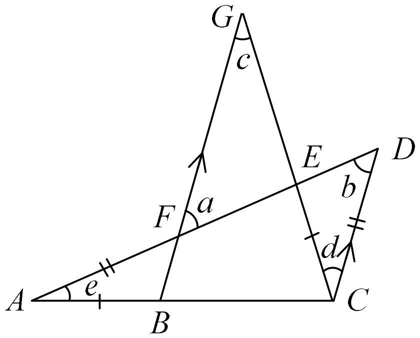
$$\text{slope of } CD = \frac{26-13}{16+23}$$
$$= \frac{1}{3}$$

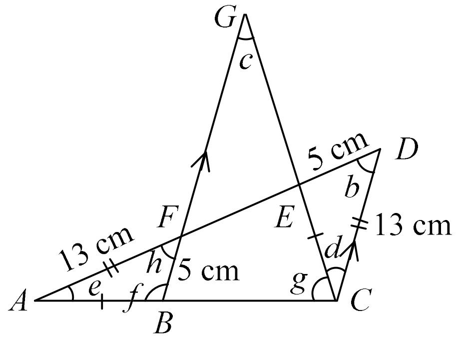
Equation of Γ:
$$\frac{y-6}{x-1} = \frac{-1}{1/3}$$
$$3x + y - 9 = 0$$

Alternative Method:
$$PC^2 = PD^2$$
$$(x+23)^2 + (y-13)^2 = (x-16)^2 + (y-26)^2$$
$$3x + y - 9 = 0$$

(3)

12. (a)





$$\because \Delta ACD \sim \Delta GEF \quad \text{(given)}$$
$$\therefore a = b \quad (\text{corr. } \angle\text{s}, \sim\Delta\text{s})$$
$$BG \text{ is parallel to } CD \quad (\text{alt. } \angle\text{s equal})$$

Marking Scheme:	
Case 1	Any correct proof with correct reasons.
Case 2	Any correct proof without reasons.

(2)

(b)

$$\because \Delta ACD \sim \Delta GEF \quad \text{(given)}$$
$$\therefore c = e \quad (\text{corr. } \angle\text{s}, \sim\Delta\text{s})$$
$$c = d \quad (\text{alt } \angle\text{s}, BG \,//\, CD)$$
$$\therefore e = d$$
$$\because AF = CD \quad \text{(given)}$$
$$AB = CE \quad \text{(given)}$$
$$\therefore \Delta ABF \cong \Delta CED \quad \text{(SAS)}$$

Marking Scheme:	
Case 1	Any correct proof with correct reasons.
Case 2	Any correct proof without reasons.

(2)

(c)

$$f = g + d \; (\text{corr. } \angle\text{s}, BG \,//\, CD)$$
$$h = b \; (\text{corr. } \angle\text{s}, BG \,//\, CD)$$
$$e \text{ is common}$$
$$\therefore \Delta ABF \sim \Delta ACD \quad \text{(AAA)}$$
$$ED = FB = 5 \text{ cm} \; (\text{corr. sides}, \cong\Delta\text{s})$$
$$CD = AF = 13 \text{ cm} \; (\text{corr. sides}, \cong\Delta\text{s})$$
$$\Delta ABF \sim \Delta ACD$$
$$\frac{AD}{DC} = \frac{AF}{FB} \quad (\text{corr. sides}, \sim\Delta\text{s})$$
$$\frac{EF + 18 \text{ cm}}{13 \text{ cm}} = \frac{13 \text{ cm}}{5 \text{ cm}}$$
$$\therefore EF = \frac{79}{5} \text{ cm} = 15.8 \text{ cm}$$

(3)

13. (a) $f(x) = (12x^2 + ax - 9)(3x - 5) - 4x + 2 = (px - 1)(18x^2 + qx + r)$

By comparing the coefficient of x^3,

$$36 = 18p$$
$$p = 2$$

By comparing the constant term,

$$45 + 2 = -r$$
$$r = -47$$

By comparing the coefficient of x^2,

$$3a - 60 = -18 + 2q$$
$$3a - 2q = 42 \quad \text{------------------ (1)}$$

By comparing the coefficient of x,

$$-5a - 27 - 4 = -94 - q$$
$$5a - q = 63 \quad \text{------------------ (2)}$$

Solving (1) and (2)

$$a = 12,\ q = -3$$

Alternative Method:

After getting $p = 2$ and $r = -47$, put $x = \frac{5}{3}$,

$$-4\left(\frac{5}{3}\right) + 2 = \left[2\left(\frac{5}{3}\right) - 1\right]\left[18\left(\frac{5}{3}\right)^2 + \frac{5q}{3} - 47\right]$$
$$q = -3$$

$$\therefore p = 2,\ q = -3 \text{ and } r = -47$$

(5)

(b)
$$g(x) = (x+4)f(x) + 8$$

If $g(x) = 8$, $(x+4)f(x) = 0$

$$(x+4)(2x-1)(18x^2 - 3x - 47) = 0$$

For $18x^2 - 3x - 47 = 0$,

$$\Delta = (-3)^2 - 4(18)(-47)$$
$$= 3393 \text{ which is not a perfect square}$$

Alternative Method:

For $18x^2 - 3x - 47 = 0$,

$$x = \frac{1 \pm \sqrt{377}}{12} \text{ which is irrational}$$

∴ The equation has irrational roots.

∴ The claim is disagreed.

(3)

14. (a) Let h cm = height of the removed circular cone.

$$\frac{h+36}{h} = \frac{16}{4}$$
$$h = 12$$

Capacity of the vessel $= \left[\frac{1}{3}\pi(16^2)(48) - \frac{1}{3}\pi(4^2)(12)\right]$ cm^3

or $\left[\frac{1}{3}\pi(4^2)(12)\left(\frac{4^3 - 1^3}{1^3}\right)\right]$ cm^3

$= 4032\pi$ cm^3

(2)

(b) (i) Volume of 1 metal sphere $= \frac{4}{3}\pi(3^3)$ cm^3

$= 36\pi$ cm^3

$$10000 + n(36\pi) \le 4032\pi$$
$$n \le \frac{4032\pi - 10000}{36\pi} \approx 23.58$$

∴ Maximum value of $n = 23$

(ii) Volume of water + Volume of spheres = $[10000 + 15(36\pi)]$ cm^3 = $(10000 + 540\pi)$ cm^3

Volume of the removed cone $= \frac{1}{3}\pi(4^2)(12)$ cm^3 $= 64\pi$ cm^3

Let d cm = depth of water

$$\left(\frac{d+12}{12}\right)^3 = \frac{64\pi + (10000 + 540\pi)}{64\pi}$$
$$d \approx 34.8$$

∴ Depth of water ≈ 34.8 cm

(5)

Section B

15. (a)

$$P(n > 4900) = \frac{1}{9}\cdot\frac{1}{8}+\frac{5}{9} = \frac{41}{72} \ (0.569)$$

$P(n>4900) = \frac{P_2^7 + 5\cdot P_3^8}{P_4^9} = \frac{41}{72}$

[2]

(b)

$$P(\text{at most 2 colours and no 1 and no 5}) = \frac{C_4^4 + C_3^4C_1^3 + C_2^4C_2^2}{C_4^9} \ \left(\text{or } \frac{C_4^7 - C_2^4C_1^2C_1^1 - C_1^4C_2^2C_1^1}{C_4^9}\right) = \frac{19}{126} \ (0.151)$$

[2]

16. (a) $\Delta ABC \sim \Delta DAE$

$$\frac{AE}{16\text{cm}} = \frac{16}{20}$$

$$AE = \frac{64}{5}\text{cm} = 12.8\text{ cm}$$

$$\tan\angle CAB = \frac{16}{20}$$

$\angle CAB = 38.7°$ (3 sig. fig.)

$\angle ADE = \angle CAB = 38.7°$ (3 sig. fig.)

$FE = 12.8\sin 38.7°\text{ cm} = 8.00\text{ cm}$ (3 sig. fig.) (or $\frac{64}{5}\cdot\frac{4}{\sqrt{41}}\text{cm} = \frac{256}{5\sqrt{41}}\text{cm}$)

$DF = 16\cos 38.7°\text{ cm} = 12.5\text{ cm}$ (3 sig. fig.) (or $16\cdot\frac{5}{\sqrt{41}}\text{cm} = \frac{80}{\sqrt{41}}\text{cm}$)

[3]

(b) (i) $\sin\angle CDB = \frac{16}{20}$

$\angle CDB = 53.1°$ (3 sig. fig.)

$\therefore$ The angle between plane ACD and the horizontal plane is $53.1°$.

(ii) $BD = \sqrt{20^2 - 16^2}\text{ cm} = 12\text{ cm}$

$\cos\angle BAD = \frac{16^2 + 20^2 - 12^2}{2(16)(20)} = 0.8$ (or $\frac{AD}{BD} = \frac{16}{20}$)

$DE^2 = [16^2 + 12.8^2 - 2(16)(12.8)(0.8)]\text{cm}^2$

$DE = 9.6\text{ cm}$

(iii) $\cos\angle DFE = \frac{12.5^2 + 8.00^2 - 9.6^2}{2(12.5)(8.00)}$

$\angle DFE = 50.2°$ (3 sig. fig.)

[5]

17. (a) Area of ΔOPQ = Area of ΔOPI + Area of ΔOQI + Area of ΔIPQ

$$\frac{1}{2}(OP)(OQ) = \frac{1}{2}(OP)(r)+\frac{1}{2}(OQ)(r)+\frac{1}{2}(PQ)(r)$$

$$(OP)(OQ) = r(OP+OQ+PQ)$$

$$r = \frac{OP\cdot OQ}{OP+OQ+PQ}$$

[2]

(b) (i) $OQ=\sqrt{20^2-12^2}=16$

$r = \dfrac{12\cdot 16}{12+16+20}=4$ (From (a))

$I=(4, 4)$

The equation of C_1 is $(x-4)^2+(y-4)^2=16$.

(ii) Centre of C_2 = $(\frac{16}{2},\frac{12}{2})$ = (8, 6) Radius of $C_2=\frac{1}{2}(20)=10$

The equation of C_2 is

$(x-8)^2+(y-6)^2=100$ or

$x^2+y^2-16x-12y=0$

Let the equation of the tangent be $y=mx+16$, where m is a constant.

Putting $y=mx+16$ into $x^2+y^2-16x-12y=0$, we have

$x^2+(mx+16)^2-16x-12(mx+16)=0$

$(1+m^2)x^2+(20m-16)x+64=0$

$\Delta=0$

$(20m-16)^2-4(1+m^2)(64)=0$

$9m^2-40m=0$

$m(9m-40)=0$

$m=\dfrac{40}{9}$ or $m=0$ (rejected)

Therefore, the equation of tangent is $y=\dfrac{40}{9}x+16$.

$T=(-16\div\dfrac{40}{9}, 0)=(-\dfrac{18}{5}, 0)$

Area of ΔSTQ $= \dfrac{1}{2}(16+\dfrac{18}{5})(16)$

$= 156.8$

< 160

Thus, the claim is incorrect.

[8]

18. (a) (i) $A_1 = 4\cdot10^6\cdot1.012-60000 = 3988000$

$A_2 = (4\cdot10^6\cdot1.012-60000)1.012-60000$

$= 4\cdot10^6\cdot1.012^2-60000\cdot1.012-60000$

$A_3 = 4\cdot10^6\cdot1.012^3-60000\cdot1.012^2-60000\cdot1.012-60000$

$A_4 = 4\cdot10^6\cdot1.012^4-60000\cdot1.012^3-60000\cdot1.012^2-60000\cdot1.012-60000$

$= 4\cdot10^6\cdot1.012^4-\dfrac{60000\cdot(1.012^4-1)}{1.012-1}$

≈ 3951129 (correct to the nearest integer)

(ii) $A_n = 4\cdot10^6\cdot1.012^n-60000(1.012^{n-1}+1.012^{n-2}+\cdots\cdots+1.012+1)$

$= 4\cdot10^6\cdot1.012^n-\dfrac{60000(1.012^n-1)}{1.012-1}$

$= 4\cdot10^6\cdot1.012^n-5\cdot10^6(1.012^n-1)$

$= (5-1.012^n)\cdot10^6$

(iii) $\because A_{120} = (5-1.012^{120})\cdot10^6 \approx 815327$

$\therefore$ The fund can support Mr White's expenses for 30 years under plan A.

[7]

(b) (i) amount withdrawn at end of 2019 = $\$60000\cdot1.004^3 \approx \$60723$ (nearest dollar)

(ii) balance at end of 2019 after withdrawal

$= \$(4\cdot10^6\cdot1.012^4-60000\cdot(1012^3+1.004\cdot1.012^2+1.004^2\cdot1.012+1.004^3)$

$= \$\{4\cdot10^6\cdot1.012^4-\dfrac{60000\cdot1.012^3\cdot[1-(\frac{1.004}{1.012})^4]}{1-\frac{1.004}{1.012}}\}$

$= \$[4\cdot10^6\cdot1.012^4-\dfrac{60000\cdot(1.012^4-1.004^4)}{1.012-1.004}] \approx \$3949674$ (nearest dollar)

(iii) Let $\$B_n$ be the balance of the fund after the nth withdrawal. Then

$B_n = 4\cdot10^6\cdot1.012^n-\dfrac{60000(1.012^n-1.004^n)}{0.008}$

$= (7.5\cdot1.004^n-3.5\cdot1.012^n)10^6 > 0$

$7.5\cdot1.004^n > 3.5\cdot1.012^n$

$(\dfrac{253}{251})^n < \dfrac{15}{7}$

$n < \dfrac{\log\frac{15}{7}}{\log\frac{253}{251}} \approx 96.03$

$\therefore$ The fund can support Mr White's expenses for 24 years under plan B.

[6]

The End