SMCC

PREMOCK 19/20

MATHEMATICS Compulsory Part

PAPER 1

Question-Answer Book

(2¼ hours)

This paper must be answered in English

INSTRUCTIONS

1. Write your Name, Class and Class Number in the spaces provided on Page 1.

2. This paper consists of THREE sections, A(1), A(2) and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Graph paper and supplementary answer sheets will be supplied on request. Write your Name and mark the question number box on each sheet, and fasten them with string INSIDE this book.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

Name	
Class	( )

	Marker's Use Only	**Examiner's Use Only**
	Marker No.	Examiner No.
Question No.	Marks	Marks
1–2		
3–4		
5–6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
Total		

SECTION A(1) (35 marks)

1. Make a the subject of the formula $2(3a - 11) = 3a - 5b$. (3 marks)

2. Simplify $\dfrac{m^6 n^{-3}}{(m^5 n^{-4})^2}$ and express your answer with positive indices. (3 marks)

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3. Simplify $\frac{5}{3k+2} - \frac{4}{2k+7}$. (3 marks)

4. Factorize

(a) $25x^2 - 4$,

(b) $5x^2y - 17xy + 6y$,

(c) $5x^2y - 17xy + 6y - 25x^2 + 4$.

(4 marks)

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5. (a) Solve the inequality $-3(x-4) \geq \frac{5x+3}{6}$.

(b) How many integers satisfy both inequalities $-3(x-4) \geq \frac{5x+3}{6}$ and $6x + 24 > 0$?

(4 marks)

6. The marked price of a computer is \$7 000 which is 40% above its cost.

(a) Find the cost of the computer.

(b) If the computer is sold at a discount of 12% on its marked price, find the percentage profit.

(4 marks)

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7. Seven years ago, the ages of Peter and Irene were in the ratio 3 : 2. The ratio now becomes 4 : 3. Find the present age of Irene.

(4 marks)

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8. The stem-and-leaf diagram below shows the distribution of the weights of a group of students.

***Stem* (*10 kg*)**	*Leaf* (*1 kg*)
4	1 7 9
5	0 0 *a* 5 5
6	2
7	0 *a* *a*

It is given that the mean of the distribution is 57 kg.

(a) Find *a*.

(b) Find the range, the inter-quartile range and the standard deviation of the distribution.

(5 marks)

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9. In Figure 1, O is the centre of the circle $ABCD$. BAE and $CODE$ are straight lines. It is given that $\angle BDC = 48°$ and $AO = AE$.

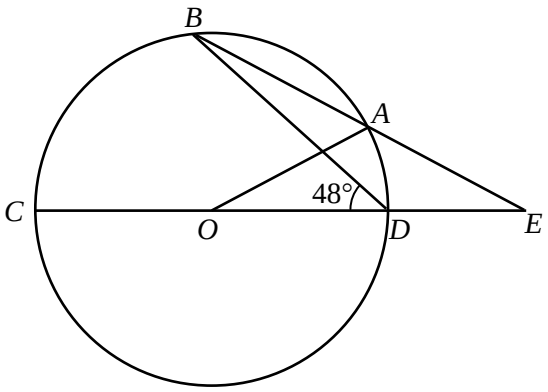

Figure 1

(a) Find $\angle AOE$.

(b) Someone claims that $\overset{\frown}{AB}$ is shorter than AE. Do you agree? Explain your answer.

(5 marks)

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SECTION A(2) (35 marks)

10. It is given that $f(x)$ is the sum of two parts, one part is a constant and the other part varies directly as x^2. Suppose that $f(-1) = 206$ and $f(3) = 254$.

(a) Find $f(x)$. (3 marks)

(b) Solve the equation $f(x) = 80x$. (2 marks)

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11. The following table shows the distribution of the numbers of films watched by a group of students last month, where $a > 5$, $b < 11$ and $c > 0$.

Number of films	0	1	2	3	4	5
Number of students	$c + 1$	4	a	8	$b - a$	c

The median of the distribution is 2.5.

(a) Find a and b. (3 marks)

(b) It is given that the mode of the distribution is greater than 2. Write down

(i) the least possible value of c,

(ii) the greatest possible value of c.

(2 marks)

(c) Suppose c is the value obtained in (b)(i). If a student is randomly selected from the group, find the probability that the selected student watched more than 3 films last month. (2 marks)

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12. Let $f(x) = -4x^3 + (a + 2)x^2 + 2x - 3b$, where a and b are constants. $x + 1$ is a factor of $f(x)$. When $f(x)$ is divided by $x - 2$, the remainder is 9.

(a) Find a and b. (3 marks)

(b) Let $g(x)$ be the quotient when $f(x)$ is divided by $-x^2 + 2x + 3$. Someone claims that the equation $kxg(x) = f(x)$ has more than one real root for all real values of k. Do you agree? Explain your answer. (4 marks)

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13. A right circular cylindrical container has base radius 16 cm and height 14 cm. The container is held vertically and some water is added into it. Then a solid metal sphere of surface area 144π cm^2 is put into the container. It is found that the metal sphere is totally immersed in the water and the water surface just reaches the top of the container.

(a) Find the volume of the solid metal sphere in terms of π. (2 marks)

(b) Find the original depth of water in the container. (3 marks)

(c) An inverted right circular conical vessel of curved surface area 720π cm^2 is formed by a paper sector of arc length 48π cm. Then the vessel is held vertically. The water in the circular cylindrical container in (b) is now poured into the vessel. Will the water overflow? Explain your answer. (3 marks)

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14. In Figure 2, *ABCD* is a square. *AB* is produced to *F*. *DF* cuts *AC* and *BC* at *E* and *G* respectively.

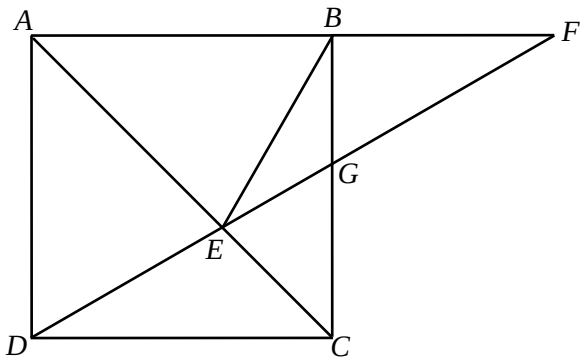

Figure 2

(a) Prove that

(i) $\triangle BCE \cong \triangle DCE$,

(ii) $\triangle BEG \sim \triangle FEB$.

(4 marks)

(b) Let $\angle AFD = \theta$.

(i) Show that $BE = FE \tan\theta$ and $EG = BE \tan\theta$.

(ii) Someone claims that when $0° < \theta < 30°$, $DE < \frac{\sqrt{3}}{2} FG$. Do you agree? Explain your answer. (4 marks)

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SECTION B (35 marks)

15. There are 8 boys and 5 girls in a dance class. 7 students are selected from the class to form a team.

(a) If exactly 5 boys are selected, how many different teams can be formed? (1 mark)

(b) If more girls are selected, how many different teams can be formed? (2 marks)

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16. Let $f(x) = -\frac{x^2}{16} + \frac{x}{2} + 11$.

(a) Using the method of completing the square, find the coordinates of the vertex of the graph of $y = f(x)$. (2 marks)

(b) The graph of $y = g(x)$ is obtained by translating the graph of $y = f(x)$ towards the left by 5 units and vertically by c units. If the graph of $y = g(x)$ cuts the straight line $y = 6$ at 2 distinct points, find the range of values of the y-intercept of the graph of $y = g(x)$. (4 marks)

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17. Let p and q be the sum of roots and the product of roots of the quadratic equation $(x + 2)(x - 2) = 8(x - 1)$ respectively.

(a) Write down the values of p and q. (2 marks)

(b) The 1st term and the 2nd term of a geometric sequence are $\log q$ and $\log p$ respectively. Find the greatest value of α such that the sum of the $(\alpha + 1)$th term and the $(2\alpha + 1)$th term of the sequence is less than $\log 2^{2\,020}$. (4 marks)

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18. Figure 3 shows a geometric model $ABCD$ in the shape of a tetrahedron. It is given that $AD = 15$ cm, $BC = 17$ cm, $CD = 27$ cm, $\angle ABD = 58°$, $\angle ADB = 65°$ and $\angle ABC = 116°$.

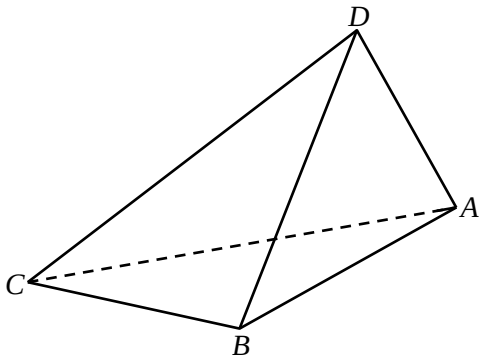

Figure 3

(a) Find AB and AC. (4 marks)

(b) Let K be a point on AD such that $BK \perp AD$. Someone claims that $\angle BKC$ is the angle between the face ABD and the face ACD. Do you agree? Explain your answer. (3 marks)

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19. Let P be a moving point. G is the circumcentre of $\triangle PQR$. The coordinates of Q, R and G are (6 , 9), (a , 11) and (h , 3) respectively, where $h > 0$.

(a) Express the coordinates of G in terms of a. (2 marks)

(b) It is given that the slope of RG is $\frac{4}{3}$. Denote the circumscribed circle of $\triangle PQR$ by C.

A straight line L: $y = kx$ cuts C at two distinct points S and T, where $k > 0$. M is the mid-point of ST.

(i) Find a.

(ii) Show that the x-coordinate of M is $\frac{14+3k}{1+k^2}$.

(iii) M is $2\sqrt{41}$ units from the origin O. Denote the location of P by a point A when P is farthest from M, and denote the location of P by a point B when P is nearest to the y-axis.

(I) Find the co-ordinates of B and M.

(II) If U is a point below the x-axis such that the area of the circle passing through A, B and U is the least, are A, M, B and U concyclic? Explain your answer.

(11 marks)

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