Compulsory Part Paper 1

	Solution	Marks
1.	$2(3a - 11) = 3a - 5b$	
	$6a - 22 = 3a - 5b$	1M
	$3a = 22 - 5b$	1M
	$a = \frac{22-5b}{3}$	1A
	$2(3a - 11) = 3a - 5b$	
	$3a - 11 = \frac{3a}{2} - \frac{5b}{2}$	1M
	$\frac{3a}{2} = \frac{22-5b}{2}$	1M
	$a = \frac{22-5b}{3}$	1A
		----------(3)
2.	$\frac{m^6 n^{-3}}{(m^5 n^{-4})^2}$	
	$= \frac{m^6 n^{-3}}{m^{10} n^{-8}}$	1M
	$= \frac{n^{-3-(-8)}}{m^{10-6}}$	1M
	$= \frac{n^5}{m^4}$	1A
		----------(3)
3.	$\frac{5}{3k+2} - \frac{4}{2k+7}$	
	$= \frac{5(2k+7)-4(3k+2)}{(3k+2)(2k+7)}$	1M
	$= \frac{10k+35-12k-8}{(3k+2)(2k+7)}$	1M
	$= \frac{-2k+27}{(3k+2)(2k+7)}$	1A
		----------(3)
4. (a)	$25x^2 - 4$	
	$= (5x + 2)(5x - 2)$	1A
(b)	$5x^2y - 17xy + 6y$	
	$= y(5x^2 - 17x + 6)$	
	$= y(5x - 2)(x - 3)$	1A

		Solution	Marks
	(c)	$5x^2y - 17xy + 6y - 25x^2 + 4$	
		$= 5x^2y - 17xy + 6y - (25x^2 - 4)$	
		$= y(5x - 2)(x - 3) - (5x + 2)(5x - 2)$	1M
		$= (5x - 2)[y(x - 3) - (5x + 2)]$	
		$= (5x - 2)(xy - 3y - 5x - 2)$	1A
			----------(4)
5.	(a)	$-3(x - 4) \geq \dfrac{5x+3}{6}$	
		$-18(x - 4) \geq 5x + 3$	
		$-18x + 72 \geq 5x + 3$	
		$-23x \geq -69$	1M
		$x \leq 3$	1A
	(b)	$6x + 24 > 0$	
		$6x > -24$	
		$x > -4$	1A
		$\therefore \quad -4 < x \leq 3$	
		The integers which satisfy both inequalities are –3, –2, –1, 0, 1, 2 and 3.	
		∴ 7 integers satisfy both inequalities.	1A
			----------(4)
6.	(a)	Let \$$x$ be the cost of the computer.	
		$x(1 + 40\%) = 7\ 000$	1A
		$1.4x = 7\ 000$	
		$x = 5\ 000$	
		∴ The cost of the computer is \$5 000.	1A
		The cost of the computer $= \$\dfrac{7\ 000}{1+40\%}$	1A
		$= \$5\ 000$	1A
	(b)	Selling price of the computer	
		$= \$7\ 000 \times (1 - 12\%)$	1M
		$= \$7\ 000 \times 0.88$	
		$= \$6\ 160$	
		∴ Percentage profit	
		$= \dfrac{6\ 160 - 5\ 000}{5\ 000} \times 100\%$	
		$= 23.2\%$	1A
			----------(4)

	Solution	Marks
7.	Let x and y be the present ages of Peter and Irene respectively.	
	$\begin{cases} \dfrac{x}{y} = \dfrac{4}{3} \text{ ....(1)} \\ \dfrac{x-7}{y-7} = \dfrac{3}{2} \text{ ....(2)} \end{cases}$	1A+1A
	From (1), $x = \dfrac{4}{3}y$ .... (3)	
	Substitute (3) into (2).	
	$\dfrac{\frac{4}{3}y - 7}{y-7} = \dfrac{3}{2}$	1M
	$\dfrac{8}{3}y - 14 = 3y - 21$	
	$-\dfrac{y}{3} = -7$	
	$y = 21$	
	∴ The present age of Irene is 21.	1A
	Let $4k$ and $3k$ be the present ages of Peter and Irene respectively, where k is a positive constant.	1A
	$\dfrac{4k-7}{3k-7} = \dfrac{3}{2}$	1M+1A
	$8k - 14 = 9k - 21$	
	$k = 7$	
	Present age of Irene = $3 \times 7 = 21$	1A
		----------(4)
8. (a)	$\dfrac{41+47+49+50\times 2+(50+a)+55\times 2+62+70+(70+a)\times 2}{12} = 57$	1M
	$669 + 3a = 684$	
	$3a = 15$	
	$a = 5$	1A
(b)	Range = (75 – 41) kg = 34 kg	1A
	$Q_1 = \dfrac{49+50}{2}$ kg = 49.5 kg	
	$Q_3 = \dfrac{62+70}{2}$ kg = 66 kg	
	Inter-quartile range = (66 – 49.5) kg = 16.5 kg	1A
	Standard deviation = 10.7 kg, *cor. to 3 sig. fig.*	1A
		----------(5)

Solution		Marks
9. (a) $\because AO = AE$		
$\therefore \angle AOE = \angle AEO$	(base $\angle$s, isos. Δ)	
$\angle ABD = \dfrac{\angle AOE}{2}$	($\angle$ at centre twice $\angle$ at circumference)	1M
In $\triangle BDE$,		
$\angle ABD + \angle AEO = \angle BDC$	(ext. $\angle$ of Δ)	
$\dfrac{\angle AOE}{2} + \angle AOE = 48°$		
$\dfrac{3\angle AOE}{2} = 48°$		
$\angle AOE = 32°$		1A
(b) Join OB.		
$\angle BOC = 2\angle BDC$	($\angle$ at centre twice $\angle$ at circumference)	
$= 2 \times 48°$		
$= 96°$		
$\angle BOC + \angle AOB + \angle AOE = 180°$	(adj. $\angle$s on st. line)	
$96° + \angle AOB + 32° = 180°$		1M
$\angle AOB = 52°$		
$\overset{\frown}{AB} = 2 \times \pi \times OA \times \dfrac{\angle AOB}{360°}$		
$= 2 \times \pi \times AE \times \dfrac{52°}{360°}$		1M
$\approx 0.907\ 571\ 211AE$		
$< AE$		
$\therefore$ The claim is agreed.		1A
Join OB.		
$\because OB = OD$		
$\therefore \angle OBD = \angle ODB = 48°$	(base $\angle$s, isos. Δ)	
In $\triangle OBD$,		
$\angle OBD + \angle BOD + \angle ODB = 180°$	($\angle$ sum of Δ)	
$48° + \angle AOB + 32° + 48° = 180°$		
$\angle AOB = 52°$		1M
$\overset{\frown}{AB} = 2 \times \pi \times OA \times \dfrac{\angle AOB}{360°}$		
$= 2 \times \pi \times AE \times \dfrac{52°}{360°}$		1M
$\approx 0.907\ 571\ 211AE$		
$< AE$		
$\therefore$ The claim is agreed.		1A
		----------(5)

	Solution	Marks
10. (a)	From the question, $f(x) = k_1 + k_2x^2$, where k_1 and k_2 are non-zero constants.	1A
	$f(-1) = 206$	1M for either substitution
	$k_1 + k_2(-1)^2 = 206$	
	$k_1 + k_2 = 206$ ....(1)	
	$f(3) = 254$	
	$k_1 + k_2(3)^2 = 254$	
	$k_1 + 9k_2 = 254$ ....(2)	
	(2) – (1): $8k_2 = 48$	
	$k_2 = 6$	
	Substitute $k_2 = 6$ into (1).	
	$k_1 + 6 = 206$	
	$k_1 = 200$	
	$\therefore \quad f(x) = 200 + 6x^2$	1A
		----------(3)
(b)	$f(x) = 80x$	
	$200 + 6x^2 = 80x$	1M
	$6x^2 - 80x + 200 = 0$	
	$3x^2 - 40x + 100 = 0$	
	$(x - 10)(3x - 10) = 0$	
	$x = 10 \text{ or } \dfrac{10}{3}$	1A
		----------(2)
11. (a)	$c + 1 + 4 + a = 8 + b - a + c$	1M
	$b = 2a - 3$	
	Note $b < 11$	
	$2a - 3 < 11$	
	$2a < 14$	
	$a < 7$	
	and $a > 5$	
	$\therefore \quad a = 6$ and $b = 2(6) - 3 = 9$	1A+1A
		----------(3)
(b)(i)	$\because \quad c > 0$ and the mode is greater than 2.	
	$\therefore$ The least possible value of c is 1.	1A
(ii)	$\because$ The mode is greater than 2.	
	$\therefore \quad c + 1 < 8$	
	$c < 7$	
	$\therefore$ The greatest possible value of c is 6.	1A
		----------(2)

Solution		Marks
(c)	$c = 1$	
	The required probability	
	$= \dfrac{(9-6)+1}{(1+1)+4+6+8+(9-6)+1}$	1M
	$= \dfrac{1}{6}$	1A
		----------(2)
12. (a)	$\because$ $x + 1$ is a factor of $f(x)$.	
	$\therefore$ $f(-1) = 0$	
	$-4(-1)^3 + (a + 2)(-1)^2 + 2(-1) - 3b = 0$	
	$4 + a + 2 - 2 - 3b = 0$	
	$a - 3b = -4$ .... (1)	1M for either one
	$\because$ When $f(x)$ is divided by $x - 2$, the remainder is 9.	
	$\therefore$ $f(2) = 9$	
	$-4(2)^3 + (a + 2)(2)^2 + 2(2) - 3b = 9$	
	$-32 + 4a + 8 + 4 - 3b = 9$	
	$4a - 3b = 29$ .... (2)	
	(2) – (1): $3a = 33$	
	$a = 11$	1A
	Substitute $a = 11$ into (1).	
	$11 - 3b = -4$	
	$-3b = -15$	
	$b = 5$	1A
		----------(3)
(b)	$f(x) = -4x^3 + (11 + 2)x^2 + 2x - 3(5)$	
	$= -4x^3 + 13x^2 + 2x - 15$	
	Using long division,	
	$\begin{array}{r} 4x \;-\; 5 \\ -x^2+2x+3 \overline{\big)\, -4x^3+13x^2+\;2x-15} \\ \underline{-4x^3+\;8x^2+12x\qquad} \\ 5x^2-10x-15 \\ \underline{5x^2-10x-15} \end{array}$	1M
	$\therefore$ $f(x) = (-x^2 + 2x + 3)(4x - 5)$	
	$g(x) = 4x - 5$	
	$kx\,g(x) = f(x)$	
	$kx(4x - 5) = (-x^2 + 2x + 3)(4x - 5)$	1M
	$(4x - 5)[kx - (-x^2 + 2x + 3)] = 0$	
	$(4x - 5)[x^2 + (k - 2)x - 3] = 0$	1M
	$x = \dfrac{5}{4}$ *or* $x^2 + (k - 2)x - 3 = 0$	

	Solution	Marks
	Consider the equation $x^2 + (k-2)x - 3 = 0$.	
	$\Delta = (k-2)^2 - 4(1)(-3)$	
	$= (k-2)^2 + 12$	
	> 0	
	$\therefore$ The equation $x^2 + (k-2)x - 3 = 0$ has two distinct real roots.	
	$\therefore$ The equation $kx\,g(x) = f(x)$ has more than one real root for all real values of k.	
	$\therefore$ The claim is agreed.	1A (f.t.)
		----------(4)
13. (a)	Let r cm be the radius of the metal sphere.	
	$4\pi r^2 = 144\pi$	1A
	$r^2 = 36$	
	$r = \sqrt{36}$	
	$= 6$	
	Volume of the metal sphere $= \frac{4}{3}\pi(6)^3$ cm^3	
	$= 288\pi$ cm^3	1A
		----------(2)
(b)	The original depth of water in the container	1M for numerator +1M for $\frac{\text{a}}{\pi(16)^2}$
	$= \dfrac{\pi(16)^2(14) - 288\pi}{\pi(16)^2}$ cm	
	$= \frac{103}{8}$ cm	1A
		----------(3)
(c)	Let R cm and ℓ cm be the base radius and the slant height of the circular conical vessel respectively.	
	$2\pi R = 48\pi$	
	$R = 24$	
	$\pi R\ell = 720\pi$	
	$\pi(24)\ell = 720\pi$	
	$\ell = 30$	
	Height of the vessel $= \sqrt{30^2 - 24^2}$ cm	1M
	$= 18$ cm	
	Capacity of the vessel $= \frac{1}{3}\pi(24)^2(18)$ cm^3	1M
	$= 3\,456\pi$ cm^3	

	Solution	Marks
	Volume of water in the circular cylindrical container	
	$= \pi(16)^2\left(\frac{103}{8}\right)\text{cm}^3$	
	$= 3\ 296\pi\ \text{cm}^3$	
	$\because\quad 3\ 456\pi\ \text{cm}^3 > 3\ 296\pi\ \text{cm}^3$	
	$\therefore$ The water will not overflow.	1A ----------(3)
14.	**Marking Schemes for (a)(i) and (a)(ii):**	
	Case 1 Any correct proof with correct reasons.	2
	Case 2 Any correct proof without all correct reasons.	1
(a)(i)	In $\triangle BCE$ and $\triangle DCE$,	
	$BC = DC$ (*by definition*)	
	$\angle BCE = \angle DCE = 45°$ (*property of square*)	
	$CE = CE$ (*common side*)	
	$\therefore\quad \triangle BCE \cong \triangle DCE$ (*SAS*)	
(ii)	In $\triangle BEG$ and $\triangle FEB$,	
	$\angle BEG = \angle FEB$ (*common angle*)	
	$\because\quad \triangle BCE \cong \triangle DCE$ (*proved in* (a)(i))	
	$\therefore\quad \angle EBC = \angle EDC$ (*corr.* $\angle$*s*, $\cong \triangle$*s*)	
	$\angle EDC = \angle EFB$ (*alt.* $\angle$*s*, *AF* // *DC*)	
	$\therefore\quad \angle EBC = \angle EFB$	
	i.e. $\angle EBG = \angle EFB$	
	$\angle BGE = 180° - \angle BEG - \angle EBG$ ($\angle$ *sum of* $\triangle$)	
	$= 180° - \angle FEB - \angle EFB$	
	$= \angle FBE$ ($\angle$ *sum of* $\triangle$)	
	$\therefore\quad \triangle BEG \sim \triangle FEB$ (*AAA*)	
		----------(4)
(b)(i)	$\because\quad \triangle BEG \sim \triangle FEB$ (*proved*)	
	$\therefore\quad \frac{BE}{FE} = \frac{EG}{BE} = \frac{BG}{FB}$ (*corr.* $\angle$*s of* $\sim \triangle$*s*)	1A
	$= \tan\theta$	
	$\therefore\quad BE = FE\tan\theta$ and $EG = BE\tan\theta$.	1A (f.t.)

	Solution	Marks
(b)(ii)	$\because$ $\triangle BCE \cong \triangle DCE$ *(proved)*	
	$\therefore$ $BE = DE$ (*corr.* $\angle$*s of* $\cong \triangle$*s*)	
	Note that when $0° < \theta < 30°$, $0 < \tan \angle AFD < \frac{\sqrt{3}}{3}$.	
	$DE = BE$	
	$= FE \tan \theta$	
	$= (EG + FG) \tan \theta$	
	$= (BE \tan \theta + FG) \tan \theta$	
	$= DE \tan^2 \theta + FG \tan \theta$	1M
	$< DE \left(\frac{\sqrt{3}}{3}\right)^2 + FG\left(\frac{\sqrt{3}}{3}\right)$	
	$DE < \frac{1}{3} DE + \frac{\sqrt{3}}{3} FG$	
	$\frac{2}{3} DE < \frac{\sqrt{3}}{3} FG$	
	$DE < \frac{\sqrt{3}}{2} FG$	
	$\therefore$ The claim is agreed.	1A (f.t)
		----------(4)
15. (a)	Number of teams formed = $C^8_5 \times C^5_2 = 560$	1A
		----------(2)
(b)	Number of teams formed = $C^8_3 \times C^5_4 + C^8_2 \times C^5_5$	1M
	$= 308$	1A
		----------(2)
16. (a)	$f(x) = -\frac{x^2}{16} + \frac{x}{2} + 11$	
	$= -\frac{1}{16}(x^2 - 8x) + 11$	
	$= -\frac{1}{16}\left[x^2 - 8x + \left(\frac{-8}{2}\right)^2 - \left(\frac{-8}{2}\right)^2\right] + 11$	1M
	$= -\frac{1}{16}(x^2 - 8x + 16) + 1 + 11$	
	$= -\frac{1}{16}(x - 4)^2 + 12$	
	$\therefore$ The coordinates of the vertex are (4 , 12).	1A
		----------(2)

Solution	Marks
(b) $g(x) = -\frac{1}{16}(x - 4 + 5)^2 + 12 + c$	1M
$= -\frac{1}{16}(x + 1)^2 + 12 + c$	
$12 + c > 6$ $c > -6$	1M
$g(0) = -\frac{1}{16}(0 + 1)^2 + 12 + c$	1M
$= \frac{191}{16} + c$	
$> \frac{191}{16} - 6$	
$= \frac{95}{16}$	
$\therefore$ y-intercept $> \frac{95}{16}$	1A ----------(2)
17. (a) $(x + 2)(x - 2) = 8(x - 1)$	
$x^2 - 4 = 8x - 8$	
$x^2 - 8x + 4 = 0$	
$\therefore$ $p = -\frac{-8}{1} = 8$	
$q = \frac{4}{1} = 4$	1M +1A (for p & q) ---------(2)
(b) Common ratio $= \frac{\log 8}{\log 4}$	1M
$= \frac{\log 2^3}{\log 2^2}$	
$= \frac{3\log 2}{2\log 2}$	
$= 1.5$	
$\therefore$ The general term $T_n = (\log 4)1.5^{n-1}$	
$T_{\alpha+1} + T_{2\alpha+1} < \log 2^{2\,020}$	
$(\log 4)1.5^{(\alpha+1)-1} + (\log 4)1.5^{(2\alpha+1)-1} < \log 2^{2\,020}$	1M
$(2\log 2)1.5^{\alpha} + (2\log 2)1.5^{2\alpha} < 2\,020\log 2$	
$(1.5^{\alpha})^2 + 1.5^{\alpha} - 1\,010 < 0$	
$\therefore$ $\frac{-1-\sqrt{1^2-4(1)(-1\,010)}}{2(1)} < 1.5^{\alpha} < \frac{-1+\sqrt{1^2-4(1)(-1\,010)}}{2(1)}$	

	Solution	Marks
	$\because\ 1.5^{\alpha} > 0$ for all α.	
	$\therefore\quad 1.5^{\alpha} < \dfrac{-1+\sqrt{1^2-4(1)(-1\,010)}}{2(1)}$	
	$\log 1.5^{\alpha} < \log\dfrac{-1+\sqrt{4\,041}}{2}$	1M
	$\alpha \log 1.5 < \log\dfrac{-1+\sqrt{4\,041}}{2}$	
	$\alpha < \dfrac{\log\dfrac{-1+\sqrt{4\,041}}{2}}{\log 1.5} = 8.49$, *cor. to 2 d.p.*	
	$\therefore$ The greatest value of α is 8.	1A
		----------(4)
18. (a)	In $\triangle ABD$, by the sine formula,	
	$\dfrac{AB}{\sin\angle ADB} = \dfrac{AD}{\sin\angle ABD}$	
	$\dfrac{AB}{\sin 65^\circ} = \dfrac{15\text{ cm}}{\sin 58^\circ}$	1A
	$AB \approx 16.030\,478\,54$ cm	
	$= 16.0$ cm, *cor. to 3 sig. fig.*	1A
	In $\triangle ABC$, by the cosine formula,	
	$AC^2 = AB^2 + BC^2 - 2 \times AB \times BC \times \cos\angle ABC$	
	$AC \approx \sqrt{16.03047854^2+17^2-2\times 16.03047854\times 17\times\cos 116^\circ}$ cm	1M
	$\approx 28.016\,145\,65$ cm	
	$= 28.0$ cm, *cor. to 3 sig. fig.*	1A
		----------(4)
(b)	In $\triangle ABD$,	
	$\angle BAD + \angle ABD + \angle ADB = 180^\circ$	
	$\angle BAD + 58^\circ + 65^\circ = 180^\circ$	
	$\angle BAD = 57^\circ$	
	In $\triangle ABK$,	
	$AK = AB\ \angle BAK \approx 16.0 \cos 57^\circ \text{ cm} \approx 8.730\,824\,363$ cm	
	In $\triangle ACD$, by the cosine formula,	
	$\cos\angle CAD = \dfrac{AC^2+AD^2-CD^2}{2\times AC\times AD} \approx \dfrac{28.01614565^2+15^2-27^2}{2\times 28.01614565\times 15}$	
	$\angle CAD \approx 70.475\,050\,57^\circ$	1M for any one
	In $\triangle ACK$, by the cosine formula,	
	$CK^2 = AK^2 + AC^2 - 2 \times AK \times AC \times \cos\angle CAD$	
	$CK \approx (8.730824363^2 + 28.01614565^2 - 2\times 8.730824363\times 28.01614565\times\cos 70.47505057^\circ)^{\frac{1}{2}}$ cm	
	$\approx 26.412\,684\,5$ cm	

Solution	Marks
$\cos \angle CKA = \dfrac{CK^2 + AK^2 - AC^2}{2 \times CK \times AK}$	
$\approx \dfrac{26.4126845^2 + 8.730824363^2 - 28.01614565^2}{2 \times 26.4126845 \times 8.730824363}$	1M
$\angle CKA \approx 91.372\ 522\ 28°$	
$\therefore$ $\angle CKA \neq 90°$	
$\therefore$ $\angle BKC$ is not the angle between the face *ABD* and the face *ACD*.	
$\therefore$ The claim is disagreed.	1A (f.t.)
In $\triangle ABD$,	
$\angle BAD + \angle ABD + \angle ADB = 180°$	
$\angle BAD + 58° + 65° = 180°$	
$\angle BAD = 57°$	
In $\triangle ABK$,	
$\cos \angle BAK = \dfrac{AK}{AB}$	
$\cos 57° \approx \dfrac{AK}{16.03047854\text{ cm}}$	
$AK \approx 8.730\ 824\ 363$ cm	
In $\triangle ACD$, by the cosine formula,	
$\cos \angle CAD = \dfrac{AC^2 + AD^2 - CD^2}{2 \times AC \times AD}$	
$\approx \dfrac{28.01614565^2 + 15^2 - 27^2}{2 \times 28.01614565 \times 15}$	1M for any one
$\angle CAD \approx 70.475\ 050\ 57°$	
In $\triangle ACK$, by the cosine formula,	
$CK^2 = AK^2 + AC^2 - 2 \times AK \times AC \times \cos \angle CAD$	
$\approx (8.730\ 824\ 363^2 + 28.016\ 145\ 65^2 -$ $2 \times 8.730\ 824\ 363 \times 28.016\ 145\ 65 \times \cos 70.475\ 050\ 57°)\text{ cm}^2$	
$\approx 697.629\ 902\ 6\text{ cm}^2$	
$CK^2 + AK^2 \approx (697.629\ 902\ 6 + 8.730\ 824\ 363^2)\text{ cm}^2 \approx 773.857\ 196\ 7\text{ cm}^2$	1M
$AC^2 \approx 28.016\ 145\ 65^2\text{ cm}^2 \approx 784.904\ 417\ 1\text{ cm}^2$	
$\because$ $AC^2 \neq CK^2 + AK^2$	
$\therefore$ $\angle CKA \neq 90°$	
$\therefore$ $\angle BKC$ is not the angle between the face *ABD* and the face *ACD*.	
$\therefore$ The claim is disagreed.	1A (f.t.)

	Solution	Marks
	In $\triangle ABD$,	
	$\angle BAD + \angle ABD + \angle ADB = 180°$	
	$\angle BAD + 58° + 65° = 180°$	
	$\angle BAD = 57°$	
	In $\triangle ABK$,	
	$\cos \angle BAK = \dfrac{AK}{AB}$	
	$\cos 57° \approx \dfrac{AK}{16.030\,478\,54\text{ cm}}$	
	$AK \approx 8.730\,824\,363$ cm	
	In $\triangle ACD$, by the cosine formula,	
	$\cos \angle CAD = \dfrac{AC^2 + AD^2 - CD^2}{2 \times AC \times AD}$	
	$\approx \dfrac{28.01614565^2 + 15^2 - 27^2}{2 \times 28.01614565 \times 15}$	1M for any one
	$\angle CAD \approx 70.475\,050\,57°$	
	Let N be a point on AD such that $CN \perp AD$.	
	In $\triangle ACN$,	
	$\cos \angle CAN = \dfrac{AN}{AC}$	
	$\cos 70.475\,050\,57° \approx \dfrac{AN}{28.01614565\text{ cm}}$	
	$AN \approx 9.363\,480\,57$ cm	
	$\because$ $AN \neq AK$	
	$\therefore$ CK is not perpendicular to AD.	1M
	i.e. $\angle BKC$ is not the angle between the face ABD and the face ACD.	
	$\therefore$ The claim is disagreed.	1A
		----------(3)
19. (a)	$\because$ G is the circumcentre of $\triangle PQR$.	
	$\therefore$ $GQ = GR$	
	$GQ^2 = GR^2$	1M
	$(6 - h)^2 + (9 - 3)^2 = (a - h)^2 + (11 - 3)^2$	
	$36 - 12h + h^2 + 36 = a^2 - 2ah + h^2 + 64$	
	$2ah - 12h = a^2 - 8$	
	$(2a - 12)h = a^2 - 8$	
	$h = \dfrac{a^2 - 8}{2a - 12}$	
	$\therefore$ The coordinates of G are $\left(\dfrac{a^2 - 8}{2a - 12}, 3\right)$.	1A
		----------(2)

Solution	Marks
(b)(i) Slope of $RG = \frac{4}{3}$	
$\frac{3-11}{h-a} = \frac{4}{3}$	
$\frac{2}{a-h} = \frac{1}{3}$	
$6 = a - h$	
$h = a - 6$	
Substitute $h = a - 6$ into $h = \frac{a^2-8}{2a-12}$.	
$a - 6 = \frac{a^2-8}{2a-12}$	1M
$(a-6)(2a-12) = a^2 - 8$	
$2a^2 - 24a + 72 = a^2 - 8$	
$a^2 - 24a + 80 = 0$	
$(a-4)(a-20) = 0$	
$a = 4$ *or* 20	
When $a = 4$, $h = \frac{4^2-8}{2(4)-12} = -2 < 0$	
When $a = 20$, $h = \frac{20^2-8}{2(20)-12} = 14 > 0$	
$\therefore \quad a = 20$	1A
(ii) Coordinates of $G = (14, 3)$	
Radius of $C = \sqrt{(6-14)^2+(9-3)^2}$	
$= 10$	
The equation of C is	
$(x-14)^2 + (y-3)^2 = 10^2$	1M
$(x-14)^2 + (y-3)^2 = 100$	
Substitute $y = kx$ into $(x-14)^2 + (y-3)^2 = 100$.	
$(x-14)^2 + (kx-3)^2 = 100$	1M
$(1+k^2)x^2 - (28+6k)x + 105 = 0$	
x-coordinate of $M = \frac{-\frac{-(28+6k)}{1+k^2}}{2}$	
$= \frac{14+3k}{1+k^2}$	1A (f.t.)

Solution	Marks
(iii) y-coordinate of $M = \dfrac{k(14+3k)}{1+k^2}$	
Note that $\angle OMG = 90°$.	
$OM = 2\sqrt{41}$	
$OM^2 = 164$	
$\left(\dfrac{14+3k}{1+k^2}\right)^2 + k^2\left(\dfrac{14+3k}{1+k^2}\right)^2 = 164$	1M
$\dfrac{(14+3k)^2}{1+k^2} = 164$	
$196 + 84k + 9k^2 = 164 + 164k^2$	
$155k^2 - 84k - 32 = 0$	
$(5k-4)(31k+8) = 0$	
$k = \dfrac{4}{5}$ *or* $-\dfrac{8}{31}$ (*rejected*)	1A
Coordinates of $M = \left(\dfrac{14+3\left(\frac{4}{5}\right)}{1+\left(\frac{4}{5}\right)^2}, \dfrac{\frac{4}{5}\left[14+3\left(\frac{4}{5}\right)\right]}{1+\left(\frac{4}{5}\right)^2}\right) = (10\ ,\ 8)$	1A
When P is farthest from M, MGA is a straight line.	
When P is nearest to the y-axis,	
coordinates of $B = (14 - 10\ ,\ 3)$	
$= (4\ ,\ 3)$	1A
Note that the area of the circle passing through A and B is the least when AB is a diameter of the circle. Hence, $\angle AUB = 90°$.	} 1A
Slope of AM = slope of GM	
$= \dfrac{8-3}{10-14}$	
$= -\dfrac{5}{4}$	
Slope of $BM = \dfrac{8-3}{10-4}$	
$= \dfrac{5}{6}$	
Slope of AM × slope of BM	
$= -\dfrac{5}{4} \times \dfrac{5}{6}$	
$= -\dfrac{25}{24}$	
$\neq -1$	
$\therefore\ \angle AMB \neq 90°$	
$\because\ \angle AUB + \angle AMB \neq 180°$	
$\therefore\ A$, M, B and U are not concyclic.	1A (f.t.)

	Solution	Marks
	y-coordinate of M = $\dfrac{k(14+3k)}{1+k^2}$ Note that $\angle OMG = 90°$. $OM = 2\sqrt{41}$ $OM^2 = 164$ $\left(\dfrac{14+3k}{1+k^2}\right)^2 + k^2\left(\dfrac{14+3k}{1+k^2}\right)^2 = 164$	1M
	$196 + 84k + 9k^2 = 164 + 164k^2$ $155k^2 - 84k - 3 = 0$ $(5k-4)(31k+8) = 0$ $k = -\dfrac{4}{5}$ *or* $-\dfrac{8}{31}$ (*rejected*)	1A
	Coordinates of M = $\left(\dfrac{14+3\left(\frac{4}{5}\right)}{1+\left(\frac{4}{5}\right)^2}, \dfrac{\frac{4}{5}\left[14+3\left(\frac{4}{5}\right)\right]}{1+\left(\frac{4}{5}\right)^2}\right) = (10\ ,\ 8)$	1A
	When P is farthest from M, MGA is a straight line. When P is nearest to the y-axis, coordinates of B = $(14 - 10\ ,\ 3)$ $= (4\ ,\ 3)$	1A
	Note that the area of the circle passing through A and B is the least when AB is a diameter of the circle. Hence, $\angle AUB = 90°$.	} 1A
	Substitute $x = 4$ and $y = 3$ into $y = \dfrac{4}{5}x$. L.H.S. = 3 R.H.S. = $\dfrac{4}{5}(4) = \dfrac{16}{5}$ $\because$ L.H.S. $\neq$ R.H.S. i.e. The ordered pair (4 , 3) does not satisfy the equation. $\therefore$ $B(4\ ,\ 3)$ does not lie on OM. $\therefore$ $\angle AMB \neq 90°$ $\because$ $\angle AUB + \angle AMB \neq 180°$ $\therefore$ A, M, B and U are not concyclic.	1A (f.t.) ----------(11)