# F. 6 2020/21

# FINAL EXAMINATIONS

## MATHEMATICS Compulsory Part

## PAPER 1

## Question-Answer Book

($2\frac{1}{4}$ hours)

This paper must be answered in English

**INSTRUCTIONS**

1. Write your Name, Class and Class Number in the spaces provided on Page 1.

2. This paper consists of THREE sections, A(1), A(2) and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Graph paper and supplementary answer sheets will be supplied on request. Write your Name, Class and Class Number, mark the question number box on each sheet and staple them with this book.

5. Unless otherwise specified, all workings must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

| Name | |
|---|---|
| Class | ( ) |

| | **Marker's Use Only** | **Examiner's Use Only** |
|---|---|---|
| | Marker No. | Examiner No. |
| Question No. | Marks | Marks |
| 1–3 | | |
| 4–5 | | |
| 6–7 | | |
| 8 | | |
| 9 | | |
| 10 | | |
| 11 | | |
| 12 | | |
| 13 | | |
| 14 | | |
| 15 | | |
| 16 | | |
| 17 | | |
| 18 | | |
| **Total** | | |

**SECTION A(1)** (35 marks)

1. Simplify $\dfrac{(mn^{-3})^{-4}}{m^2n^{-7}}$ and express your answer with positive indices. (3 marks)

2. Make $a$ the subject of the formula $5-\dfrac{4a-7b}{b}=\dfrac{8a}{3}$. (3 marks)

3. Simplify $\dfrac{2}{4x+1}-\dfrac{6}{3-5x}$. (3 marks)



Answers written in the margins will not be marked.

4. Factorize

(a) $r^4 - 6r^3s + 9r^2s^2$;

(b) $r^2 - r^4 + 6r^3s - 9r^2s^2$.

(4 marks)

5. (a) Find the range of values of $x$ which satisfy both $\frac{9x+35}{3} \geq 4(x+4)$ and $-2x-18<0$.

(b) Find all the integers satisfying both the inequalities in (a).

(4 marks)



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6. The marked price of an air purifier is 50% above its cost. The air purifier is now sold at a discount of 20% on its marked price and the selling price is $6600.

(a) Find the cost of the air purifier.

(b) Find the percentage profit of the air purifier.

(4 marks)

7. The equation $x^2 + 2kx + k + 6 = 0$ has equal roots, where $k < 0$.

(a) Find $k$.

(b) Find the $x$-intercept of the graph of $y = x^2 + 2kx + k + 6$.

(4 marks)

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8. The frequency distribution table and the cumulative frequency distribution table below show the distribution of the lengths of 40 rods.

| Length (cm) | Frequency |
|---|---|
| 21 – 30 | 3 |
| 31 – 40 | 6 |
| 41 – 50 | $x$ |
| 51 – 60 | $y$ |
| 61 – 70 | 7 |
| 71 – 80 | 4 |

| Length less than (cm) | Cumulative frequency |
|---|---|
| 30.5 | 3 |
| 40.5 | 9 |
| 50.5 | 18 |
| 60.5 | 29 |
| 70.5 | 36 |
| 80.5 | 40 |

(a) Find $x$ and $y$.

(b) Find the mean and standard deviation of the above distribution.

(c) If a rod is randomly chosen, find the probability that the length of the rod is greater than or equal to 40.5 cm and less than 70.5 cm.

(5 marks)



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9. A bag of rice is termed *standard* if its weight is measured as 8 kg, correct to the nearest 10 g.

(a) Find the range of the actual weight of a *standard* bag of rice in kg.

(b) Someone measured the total weight of 50 *standard* bags of rice and finds that the total weight is 399.8 kg, correct to the nearest 0.1 kg. Is it possible? Explain your answer.

(5 marks)



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**SECTION A(2)** (35 marks)

10. The cost \$$C$ of decorating a wooden cube consists of two parts. The first part is the cost of painting, which varies directly as the square of its side length $x$ cm, and the second part is the cost of the drawing, which varies directly as $x$. The cost of decorating a cube of side length 2 cm is \$42. The cost of decorating a cube of side length 5 cm is \$225.

(a) Express $C$ in terms of $x$. (4 marks)

(b) Someone claims that the profit will exceed \$200 if the wooden cube of side length 20 cm is charged at \$3600. Is the claim correct? Explain your answer. (2 marks)



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11. The stem-and-leaf diagram below shows the distribution of the test marks of a class of students.

| Stem (10 marks) | Leaf (1 mark) | | | | | |
|---|---|---|---|---|---|---|
| 3 | $a$ | 1 | 3 | 3 | 5 | 5 |
| 4 | 0 | 2 | 2 | $a + b$ | | |
| 5 | 3 | 5 | 9 | | | |
| 7 | 0 | 0 | 8 | 8 | 9 | |
| 8 | 5 | $b$ | | | | |

It is given that the median of the distribution is 49 marks.

(a) Find the inter-quartile range and the range. (3 marks)

(b) If a datum is removed, find the probability that the mean will increase. (2 marks)



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12. Let $f(x)$ be a cubic polynomial. It is given that $f(1) = f(-4) = 0$, $f(-1) = 6$ and $f(2) = 30$.

(a) Factorize $f(x)$. (3 marks)

(b) Let $g(x) = 2x^3 + 11x^2 + 13x + 4$. Simplify $\dfrac{f(x)}{g(x)} - \dfrac{g(x)}{f(x)}$. (3 marks)

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13. Figure 1 shows a metallic hemispherical container of uniform thickness 5 cm. The radius of the internal hemisphere is $2x$ cm. The metallic container is then melted and recast into a right frustum as shown in Figure 2. The upper and lower base radius of the frustum are 15 cm and 30 cm respectively. The total surface area of the hemispherical container is at most $2275\pi$ cm$^2$.

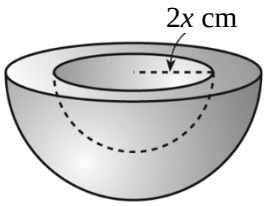


Figure 1

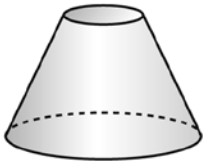

Figure 2

(a) Find the range of possible values of $x$. (5 marks)

(b) Suppose $x$ is the greatest possible value. Find the height of the frustum. (4 marks)



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14. In Figure 3, $PS$ a diameter of the circle $C$. $PQRS$ is a cyclic quadrilateral with $PQ = RS = 4\sqrt{13}$ and $QR = 10$. $C$ lies in quadrant I and touches the $x$-axis and the $y$-axis. $P$ is a point on the $y$-axis.

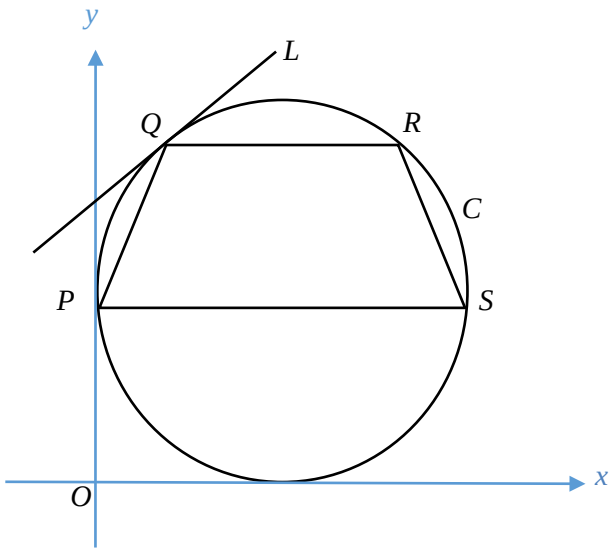


Figure 3

(a) Show that the equation of circle $C$ is $x^2 + y^2 - 26x - 26y + 169 = 0$. (4 marks)

(b) A straight line $L$ is the tangent to $C$ at $Q$.

(i) Find the coordinates of $Q$.

(ii) Find the slope of $L$.

(iii) Hence, find the equation of $L$.

(5 marks)



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**SECTION B** (35 marks)

15. 7 girls and 9 boys randomly stand in a row.

(a) Find the probability that 7 girls stand next to each other. (2 marks)

(b) Find the probability that no girls stand next to each other. (2 marks)

(c) 5 people are randomly selected from the row. Find the probability that at least 2 girls are selected. (2 marks)



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16. In Figure 4, *A*, *B*, *C* and *D* are four points lying on a circle with centre at *O*. *AD* and *BC* are produced to meet at *E*. $BC = 39\text{ cm}$, $CE = 85\text{ cm}$, $DE = 68\text{ cm}$ and $AB = 93\text{ cm}$ respectively.

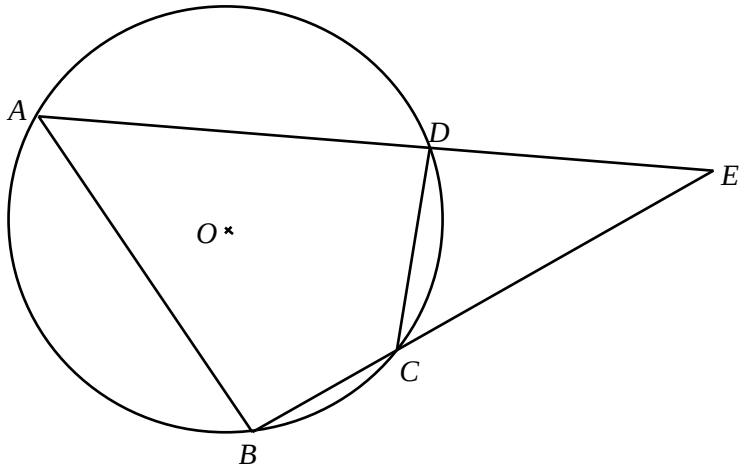


Figure 4

(a) (i) Prove that $\Delta CDE \sim \Delta ABE$.

(ii) Find *CD*.

(4 marks)

(b) (i) Prove that $\angle CDE = 90°$.

(ii) Hence or otherwise, find *EO*.

(4 marks)



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17. (a) In Figure 5(a), $PQRS$ is a paper card in the shape of a quadrilateral where $PQ = PS = 20\,\text{cm}$, $\angle QPS = 80°$, $\angle PQR = 125°$ and $\angle QRS = 65°$.

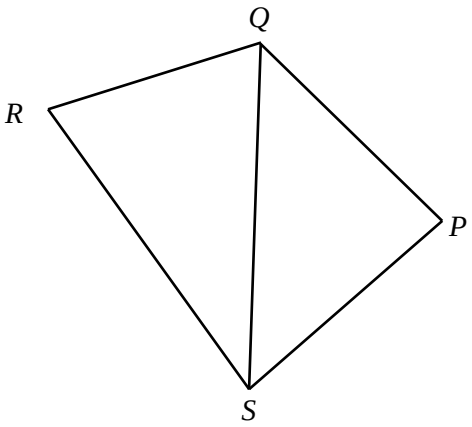


Figure 5(a)

(i) Find $QS$.

(ii) Find $RS$.

(iii) Find $QR$.

(6 marks)

(b) The paper card in Figure 5(a) is folded along $QS$ such that $\angle PSR = 30°$ (see Figure 5(b)). Find the angle between the plane $PQS$ and the plane $RQS$.

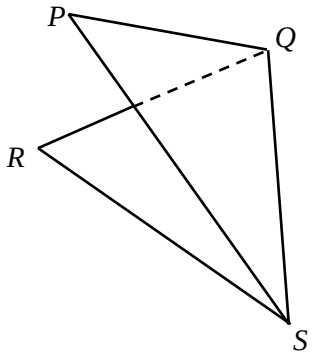


Figure 5(b)

(4 marks)

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18. Fanny deposited a principal of \$8000 into a bank account at the beginning of this year. Starting from the beginning of every subsequent year, she will deposit $r$% more than the previous year, where $r$ is a positive constant. The bank pays interest at a rate of 10% p.a., compounded yearly. The total amount in her account at the end of the 3rd year is \$35244.88 .

(a) (i) Find the total amount in her bank account at the end of
(1) the first year;
(2) the second year in terms of $r$.
(ii) Find $r$. (5 marks)

(b) (i) Express, in terms of $n$, the total amount in her account at the end of the $n$th year.
(ii) At the end of which year will the total amount in her account first exceed \$1040000? (6 marks)



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Page total

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