F. 6 Final Examination 2020 - 2021
Mathematics Compulsory Part Paper 1
Suggested Solutions

SECTION A1 (35 marks)

1.		$\frac{(mn^{-3})^{-4}}{m^2n^{-7}}$ $=\frac{m^{-4}n^{12}}{m^2n^{-7}}$ $=\frac{n^{19}}{m^6}$		
2.		$5-\frac{4a-7b}{b}=\frac{8a}{3}$ $15b-12a+21b=8ab$ $36b=8ab+12a$ $36b=4a(2b+3)$ $a=\frac{9b}{2b+3}$		
3.		$\frac{2}{4x+1}-\frac{6}{3-5x}$ $=\frac{2(3-5x)-6(4x+1)}{(4x+1)(3-5x)}$ $=\frac{6-10x-24x-6}{(4x+1)(3-5x)}$ $=\frac{-34x}{(4x+1)(3-5x)}$		
4.	(a)	$r^4-6r^3s+9r^2s^2$ $=r^2(r^2-6rs+9s^2)$ $=r^2(r-3s)^2$		
	(b)	$r^2-r^4+6r^3s-9r^2s^2$ $=r^2-r^2(r-3s)^2$ $=r^2[1-(r-3s)^2]$ $=r^2(1+r-3s)(1-r+3s)$		
5.	(a)	$\frac{9x+35}{3}\geq 4(x+4)$ $9x+35\geq 12x+48$ $-3x\geq 13$ $x\leq -\frac{13}{3}$		

		$-2x - 18 < 0$ $x > -9$		
	(b)	$\therefore$ The required range is $-9 < x \leq -\frac{13}{3}$. The required integers are $-8, -7, -6$ and -5.		
6.	(a)	Let $$y$ be the cost of the air purifier $y(1 + 50\%)(1 - 20\%) = 6600$ $y = 5500$ $\therefore$ The cost of the air purifier is $5500.		
		The cost price of the air purifier = $\$6600 \div (1 - 20\%) \div (1 + 50\%)$ = $5500		
	(b)	The percentage profit = $\frac{6600-5500}{5500} \times 100\%$ = 20%		
7.	(a)	$(2k)^2 - 4(1)(k + 6) = 0$ $4k^2 - 4k - 24 = 0$ $k = -2$ *or* $k = 3$ (*rejected*)		
	(b)	When $k = -2$, $x^2 + 2(-2)x + (-2) + 6 = 0$ $x^2 - 4x + 4 = 0$ $x = 2$ (repeated) $\therefore$ The required x-intercept is 2.		
8.	(a)	$x = 9$ $y = 11$		
	(b)	The mean = 51.75 cm The standard deviation = 13.9 cm		
	(c)	The required probability = $\frac{9+11+7}{40} = \frac{27}{40}$		
9.	(a)	Let x kg be the range of the actual weight of a *standard* bag of rice		

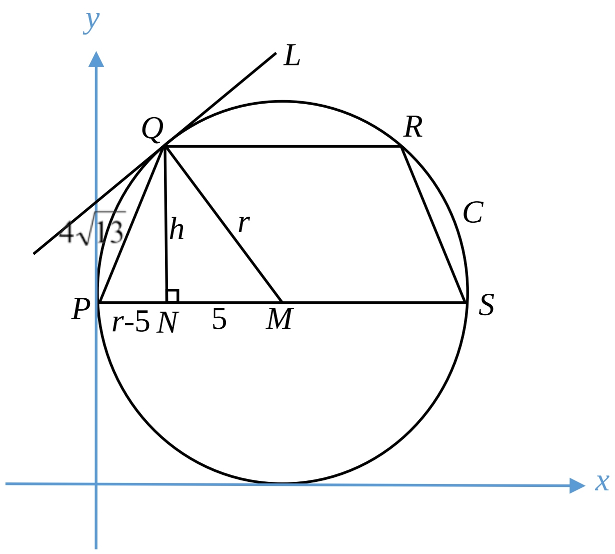
		$[8-\frac{1}{2}(\frac{10}{1000})] \leq x < [8+\frac{1}{2}(\frac{10}{1000})]$ $7.995 \leq x < 8.005$ $\therefore$ 7.995 kg ≤ The actual weight < 8.005 kg		
	(b)	$7.995(50) \leq 50x < 8.005(50)$ $399.75 \leq 50x < 400.25$ $399.8 \leq 50x < 400.3$ (cor. to the nearest 0.1kg) $\because$ 399.8 kg is in this range. $\therefore$ The claim is agreed.		

SECTION A2 (35 marks)

10.	(a)	Let $C = ax^2 + bx$, where a and b are non-zero constants. $\begin{cases} 42 = a(2)^2 + b(2) \\ 225 = a(5)^2 + b(5) \end{cases}$ Solving the equations, $a = 8$, $b = 5$ $\therefore C = 8x^2 + 5x$	
	(b)	Profit $= \$\{3600 - [8(20)^2 + 5(20)]\}$ $= \$300$ $> \$200$ $\therefore$ The claim is agreed.	
11.	(a)	Inter-quartile range $= \frac{78+70}{2} - 35$ $= 39$ $\frac{53+40+(a+b)}{2} = 49$ $a + b = 5$ $\because a \leq 1$ and $b \geq 5$ $\therefore a = 0$, $b = 5$	

		Range $=85-30$ $=55$	
	(b)	Mean = 53.9 Required probability $=\frac{11}{20}$	
12.	(a)	Let $f(x)=(x-1)(x+4)(ax+b)$, where a and b are constants. $\begin{cases} f(-1)=(-1-1)(-1+4)[a(-1)+b]=6 \\ f(2)=(2-1)(2+4)[a(2)+b]=30 \end{cases}$ Solving the equations, $a=2$, $b=1$ $\therefore f(x)=(x-1)(x+4)(2x+1)$	
	(b)	$\because g(-1)=2(-1)^3+11(-1)^2+13(-1)+4=0$ $\therefore x+1$ is a factor of $g(x)$ $\begin{array}{r} 2x^2+9x+4 \\ x+1\overline{)2x^3+11x^2+13x+4} \\ \underline{2x^3+2x^2} \\ 9x^2+13x \\ \underline{9x^2+9x} \\ 4x+4 \\ \underline{4x+4} \end{array}$ $\therefore g(x)=(x+1)(2x^2+9x+4)$ $=(x+1)(x+4)(2x+1)$ $\frac{f(x)}{g(x)}-\frac{g(x)}{f(x)}=\frac{(x-1)(x+4)(2x+1)}{(x+1)(x+4)(2x+1)}-\frac{(x+1)(x+4)(2x+1)}{(x-1)(x+4)(2x+1)}$ $=\frac{x-1}{x+1}-\frac{x+1}{x-1}$ $=\frac{(x-1)^2-(x+1)^2}{(x+1)(x-1)}$ $=\frac{4x}{1-x^2}$	

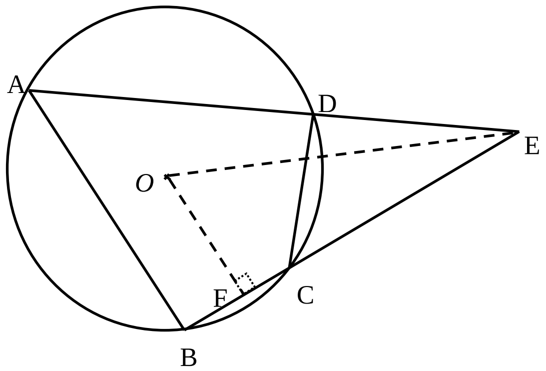
13.	(a)	$2\pi(2x)^2 + 2\pi(2x+5)^2 + \pi(2x+5)^2 - \pi(2x)^2 \le 2275\pi$ $3(2x+5)^2 + (2x)^2 \le 2275$ $16x^2 + 60x - 2200 \le 0$ $-13.75 \le x \le 10$ $\because -13.75 \le x \le 10$ and $x > 0$ $\therefore 0 < x \le 10$	
	(b)	Let h cm be the height of the cone. $\frac{1}{3}\pi(30)^2 h \times \frac{30^3 - 15^3}{30^3} = \frac{2}{3}\pi[(10)(2)+5]^3 - \frac{2}{3}\pi[(10)(2)]^3$ $h = \frac{1220}{63}$ The height of the frustum $= \frac{1220}{63} \times \frac{30-15}{30}$ cm $= \frac{610}{63}$ cm	
		Let h cm and H cm be the height of the frustum and bigger cone respectively. $\frac{H}{H-h} = \frac{30}{15}$ $H = 2(H-h)$ $H = 2h$ $\frac{1}{3}\pi(30)^2(2h) - \frac{1}{3}\pi(15)^2(2h-h) = \frac{2}{3}\pi[(10)(2)+5]^3 - \frac{2}{3}\pi[(10)(2)]^3$ $1575h = 15250$ $h = \frac{610}{63}$ $\therefore$ The height of the frustum is $\frac{610}{63}$ cm.	

14.	(a)	Consider the following diagram, where M is the centre of circle C and N is a point on PM such that $QN \perp PM$.  Let $h = QN$ and $r = QM$. $MN = \frac{10}{2} = 5$ $\begin{cases} h^2 + 5^2 = r^2 & \text{------(1)} \\ (r-5)^2 + h^2 = (4\sqrt{13})^2 & \text{------(2)} \end{cases}$ From (1), $h^2 = r^2 - 25$ ---------(3) Sub (3) into (2), $(r-5)^2 + r^2 - 25 = 208$ $r^2 - 5r - 104 = 0$ $(r+8)(r-13) = 0$ $r = -8$ (rejected) or $r = 13$ $M = (13, 13)$ Equation of C is $(x-13)^2 + (y-13)^2 = 13^2$ $x^2 + y^2 - 26x - 26y + 169 = 0$	
	(b) (i)	$Q = (13-5,\ 13+\sqrt{13^2 - 5^2})$ $= (8, 25)$	

	(b) (ii)	Slope of QM $=\frac{13-25}{13-8}$ $=-\frac{12}{5}$ Slope of L $=-1\div(-\frac{12}{5})$ $=\frac{5}{12}$	
	(b) (iii)	Equation of L is $\frac{y-25}{x-8}=\frac{5}{12}$ $12y-300=5x-40$ $5x-12y+260=0$	

SECTION B (35 marks)

15.	(a)	Required probability $=\frac{7!10!}{16!}$ $=\frac{1}{1144}$	
	(b)	Required probability $=\frac{9!P_7^{10}}{16!}$ $=\frac{3}{286}$	
	(c)	Required probability $=1-\frac{C_5^9}{C_5^{16}}-\frac{C_1^7C_4^9}{C_5^{16}}$ $=\frac{10}{13}$ Required probability $=\frac{C_2^7C_3^9}{C_5^{16}}+\frac{C_3^7C_2^9}{C_5^{16}}+\frac{C_4^7C_1^9}{C_5^{16}}+\frac{C_5^7}{C_5^{16}}$ $=\frac{10}{13}$	

16.	(a) (i)	$\angle CED = \angle AEB$ (common) $\angle CDE = \angle ABE$ (ext. $\angle$, cyclic. quad.) $\angle DCE = \angle BAE$ (ext. $\angle$, cyclic. quad.) $\therefore \Delta CDE \sim \Delta ABE$ (AAA) **Marking Scheme** Case 1 Any correct proof with correct reasons 2A Case 2 Any correct proof without correct reason 1A	
	(a) (ii)	$\because \Delta CDE \sim \Delta ABE$ $\therefore \frac{CD}{AB} = \frac{DE}{BE}$ (corr. sides, $\sim \Delta$ s) $\therefore \frac{CD}{93} = \frac{68}{39+85}$ $CD = 51\text{cm}$	
	(b) (i)	$CD^2 + DE^2 = 51^2 + 68^2 = 7225$ $CE^2 = 85^2 = 7225$ $\because CD^2 + DE^2 = CE^2$ $\therefore \angle CDE = 90°$ (converse of Pyth. Theorem) $\cos\angle CDE = \frac{51^2 + 68^2 - 85^2}{2(51)(68)}$ $\cos\angle CDE = 0$ $\angle CDE = 90°$	
	(b) (ii)	$\because \angle ABC = \angle CDE = 90°$ $\therefore$ AC is a diameter of the circle. (converse of $\angle$ in semi-circle) $AC = \sqrt{93^2 + 39^2} = \sqrt{10170}$ cm Let F be the mid-point of BC.  $OF = \sqrt{\left(\frac{\sqrt{10170}}{2}\right)^2 - \left(\frac{39}{2}\right)^2} = \sqrt{2162.25}$ cm (Pyth. Theorem)	

$FE = \frac{39}{2} + 85 = 104.5$ cm

$EO = \sqrt{\left(\sqrt{2162.25}\right)^2 + (104.5)^2} = \sqrt{13082.5} = 114.3787568$

$= 114$ cm (3 s.f.)

$\because \angle ABC = \angle CDE = 90°$

$\therefore$ AC is a diameter of the circle. (converse of $\angle$ in semi-circle)

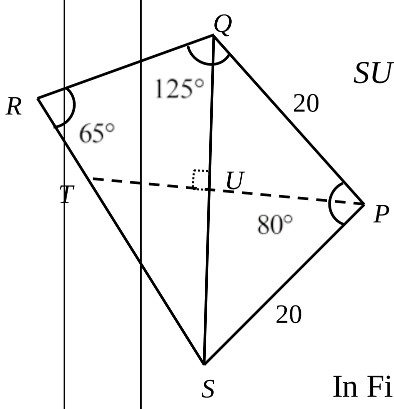
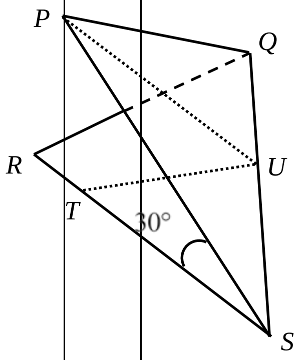
$AC = \sqrt{93^2 + 39^2} = \sqrt{10170}$ cm

$$\cos\angle OBC = \frac{\frac{39}{2}}{\frac{\sqrt{10170}}{2}}$$

$\angle OBC = 67.24902366°$

$$EO^2 = \left(\frac{\sqrt{10170}}{2}\right)^2 + (85+39)^2 - 2\left(\frac{\sqrt{10170}}{2}\right)(85+39)\cos 67.24902366°$$

$EO = 114$ cm (3 s.f.)

17.	(a) (i)	$QS^2 = 20^2 + 20^2 - 2(20)(20)\cos 80°$ $QS = 25.71150439$ cm $QS = 25.7$ cm (3 s.f.)	
	(a) (ii)	$\angle PQS = \angle PSQ$ (base. $\angle$s, isos. Δ) $\angle PQS = \dfrac{180° - 80°}{2} = 50°$ ($\angle$ sum of Δ) $\angle RQS = 125° - 50° = 75°$ $\dfrac{RS}{\sin 75°} = \dfrac{25.71150439}{\sin 65°}$ $RS = 27.40283872$ $RS = 27.4$ cm (3 s.f.)	
	(a) (iii)	$\angle RSQ = 180° - 65° - 75° = 40°$ $\dfrac{QR}{\sin 40°} = \dfrac{25.71150439}{\sin 65°}$ $QR = 18.23556708$ $QR = 18.2$ cm (3 s.f.)	
	(b)	Let T be a point on RS such that $PT \perp QS$ and let U be the point of intersection of PT and QS in Figure 5(a).  Figure 5(a) $SU = \dfrac{25.71150439}{2} = 12.85575219$ cm (property of isos. Δ) $TU = 12.85575219 \tan 40° = 10.78725692$ cm $TS = \dfrac{12.85575219}{\cos 40°} = 16.78199262$ cm In Figure 5(b), $PT^2 = (20)^2 + (16.78199262)^2 - 2(20)(16.78199262)\cos 30°$ $PT = 10.01448945$ cm  Figure 5(b) $PU = 20\sin 50°$ $\cos\angle PUT = \dfrac{(20\sin 50°)^2 + (10.78725692)^2 - (10.01448945)^2}{2(20\sin 50°)(10.78725692)}$ $\angle PUT = 40.64407403°$ $\angle PUT = 40.6°$ (3 s.f.) $\therefore$ The angle between the plane PQS and the plane RQS is $40.6°$.	

18.	(a)(i)(1)	Amount at the end of the first year $= \$8000(1+10\%)$ $= \$8800$	
	(a)(i)(2)	Amount at the end of the second year $= \$[8000(1.1)+8000(1+r\%)](1.1)$ $= \$[8000(1.1)^2+8000(1+r\%)(1.1)]$ $= \$[9680+8800(1+r\%)]$ $= \$(18480+88r)$	
	(a)(ii)	$[8000(1.1)^2+8000(1+r\%)(1.1)+8000(1+r\%)^2](1.1)=35244.88$ $8000(1+r\%)^2+8800(1+r\%)-22360.8=0$ $1+r\%=1.21$ or $1+r\%=-2.31$ (rejected) $r=21$ $[18480+88r+8000(1+r\%)^2](1.1)=35244.88$ $0.8r^2+248r-5560.8=0$ $r=21$ or $r=-331$ (rejected)	
	(b)(i)	Amount at the end of the nth year $= \$[8000(1.1)^n+8000(1.21)(1.1)^{n-1}+8000(1.21)^2(1.1)^{n-2}+...$ $..+8000(1.21)^{n-1}(1.1)]$ $= \$8000(1.1)^n[\frac{(\frac{1.21}{1.1})^n-1}{\frac{1.21}{1.1}-1}]$ $= \$80000(1.1)^n(1.1^n-1)$	
	(b)(ii)	$80000(1.1)^n(1.1^n-1)>1040000$ $(1.1^n)^2-1.1^n-13>0$ $1.1^n>4.140054945$ or $1.1^n<-3.14005$ (rejected) $\log(1.1^n)>\log(4.140054945)$ $n>\frac{\log(4.140054945)}{\log 1.1}$ $n>14.9$ $\therefore$ The amount will first exceed \$1040000 at the end of the 15th year.	