Answers to Form 6 Mathematics Final Exam (2020–21) Compulsory Part Paper II

1.	C	11.	A	21.	D	31.	C	41.	C
2.	A	12.	C	22.	B	32.	C	42.	B
3.	D	13.	A	23.	A	33.	D	43.	C
4.	D	14.	D	24.	B	34.	D	44.	B
5.	C	15.	B	25.	B	35.	B	45.	D
6.	B	16.	A	26.	D	36.	A		
7.	B	17.	C	27.	B	37.	D		
8.	C	18.	D	28.	C	38.	A		
9.	A	19.	B	29.	A	39.	C		
10.	A	20.	D	30.	B	40.	A		

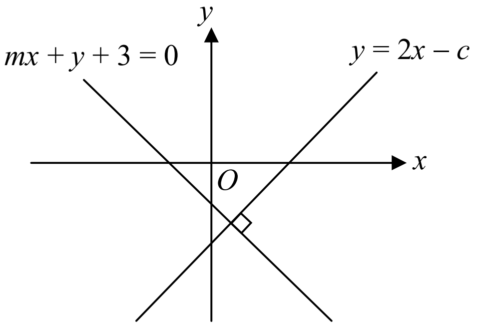
DISTRIBUTION:
A : 11
B : 12
C : 11
D : 11

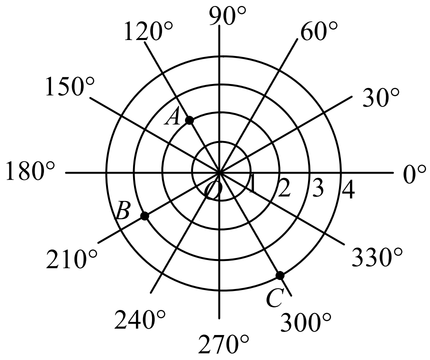
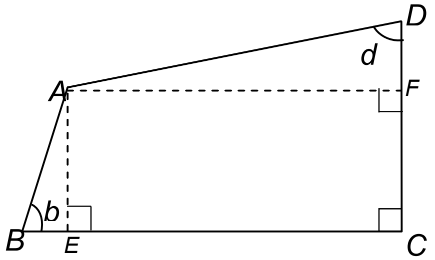
Topic	Junior F1-F3	Senior Syllabus		DSE Syllabus			
				Algebra	Geometry	Trigonometry	Statistics
No. of questions	**15**	**30**		**23**	**13**	**5**	**4**

Suggested Solutions

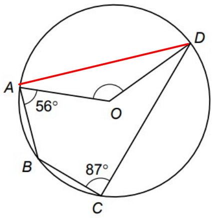
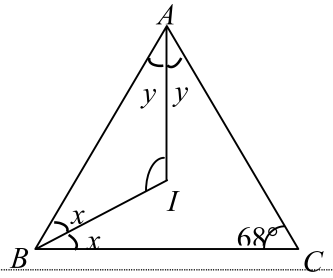
Section A

1.	**C**	$\frac{(-4)^{2020}}{-8^{2046}}=\frac{2^{4040}}{-2^{6138}}=\frac{-1}{2^{2098}}$
2.	**A**	x^2-4-9y^2-12y $=x^2-(9y^2+12y+4)$ $=x^2-(3y+2)^2$ $=(x+3y+2)(x-3y-2)$
3.	**D**	$ax^2+b(x-1)+c\equiv x^2-2x+3$ $ax^2+bx+(-b+c)\equiv x^2-2x+3$ $\therefore\ a=1,\quad b=-2\quad$ and $\quad -b+c=3$ i.e. $a-b+c=a+(-b+c)=1+3=4$
4.	**D**	$f(x)=(2x-1)(4x^2+1)+1$ The required remainder $=f\left(\frac{-1}{2}\right)=\left[2\left(\frac{-1}{2}\right)-1\right]\left[4\left(\frac{-1}{2}\right)^2+1\right]+1=-3$
5.	**C**	$\frac{1}{\pi^5}\approx 0.003\ 267\ 763$ $=0.003\ 27$ *(cor. to 5 d. p.)*
6.	**B**	$\frac{1}{2}-\frac{x}{3}<\frac{1}{4}\quad$ or $\quad 1+\frac{x}{2}>\frac{x}{3}$ $6-4x<3\quad$ or $\quad 6+3x>2x$ $-4x<-3\quad$ or $\quad x>-6$ $x>\frac{3}{4}\quad$ or $\quad x>-6$ $\therefore\quad >-6$
7.	**B**	Note that $y=\frac{5}{2}x$ and $z=\frac{4}{3}x$. $\frac{x+z}{y-x}=\frac{x+\frac{4}{3}x}{\frac{5}{2}x-x}=\frac{14}{9}$
8.	**C**	Let x and y be the price of the blue ball pen and the price of the correction pen respectively. Then we have $x(1+25\%)=y$ $1.25x=y$ $x=\frac{y}{1.25}$ $x=0.8y$

		$x = (1 - 20\%)y$ Thus, the price of the blue ball pen is 20% lower than the price of the correction pen.
9.	**A**	The coordinate of the vertex are $(h, -k)$. Thus, $h > 0$ and $k > 0$.
10.	**A**	Let $z = \dfrac{kx}{y}$, where k is a non-zero constant. Let x_0, y_0 and z_0 be the initial values of x, y and z respectively. If x is increased by 20% and y is decreased by 20%, the percentage change of z $= \dfrac{z - z_0}{z_0} \times 100\%$ $= \dfrac{\dfrac{kx_0(1+20\%)}{y_0(1-20\%)} - \dfrac{kx_0}{y_0}}{\dfrac{kx_0}{y_0}} \times 100\%$ $= \dfrac{1.5\dfrac{kx_0}{y_0} - \dfrac{kx_0}{y_0}}{\dfrac{kx_0}{y_0}} \times 100\% = 50\%$ Thus, z is increased by 50%.
11.	**A**	The amount with compound interest $= \$17000(1+3\%)^4$ $= 19133.64977$ The amount with simple interest $= \$17000 + \$17000 \times \dfrac{3}{100} \times 4$ $= \$19040$ The difference $= \$19134 - \$19040$ $= \$94$
12	**C**	$\because$ Slope of $mx + y + 3 = 0$ is less than 0. $\therefore$ $-m < 0$ i.e. $m > 0 \Rightarrow$ I is not true $\because$ y-intercept of $mx + y + 3 = 0$ is -3, and y-intercept of $y = 2x - c$ is $-c$. $\therefore$ $-c < -3$ i.e. $c > 3$ $\Rightarrow$ III is true $\because$ $2 \times (-m) = -1$ $\therefore$ $m = \dfrac{1}{2} < 3 = c$ $\Rightarrow$ II is true 
13	**A**	Let the original length and width of the square both be 1 unit respectively. The new length of the rectangle = 1.2 units

		The new width of the rectangle = 0.8 units Percentage change of the area $=\frac{1.2\times0.8-1^2}{1^2}\times100\%=-4\%$
14	**D**	By observing the patterns, we know that T(7) = 4 + 6 + 8 + 10 + 12 + 14 + 16 = (4 + 16) / 2 × 7 = 70 *<u>Alternative solution</u>*: $T(2) = T(1) + [2(1)+4] = 4 + 6 = 10$, $T(3) = T(2) + [2(2)+4] = 10 + 8 = 18$, $T(4) = T(3) + [2(3)+4] = 18 + 10 = 28$, $T(5) = T(4) + [2(4)+4] = 28 + 12 = 40$, $T(6) = T(5) + [2(5)+4] = 40 + 14 = 54$, $T(7) = T(6) + [2(6)+4] = 54 + 16 = \underline{\underline{70}}$
15	**B**	$OA = 2$, $OB = 3$ and $OC = 4$ $\because\ AC^2 = (2+4)^2 = 36$, and $AB^2 + BC^2 = (2^2 + 3^2) + (3^2 + 4^2) = 38$ $\therefore\ AC^2 \neq AB^2 + BC^2$ i.e. $\angle ABC \neq 90°$ ⇒ I is not true $AB=\sqrt{2^2+3^2}=\sqrt{13}\neq3\sqrt{2}$ ⇒ II is not true Area of $\Delta ABC=\frac{1}{2}\times(2+4)\times3=9$ ⇒ III is true 
16	**A**	Since AP = BP Let (x, y) be a point on the locus of P. $\therefore\ \sqrt{(x-1)^2+(y+2)^2}=\sqrt{(x+3)^2+(y-4)^2}$ i.e. $x^2 - 2x + 1 + y^2 + 4y + 4 = x^2 + 6x + 9 + y^2 - 8y + 16$ $\therefore\ 8x - 12y + 20 = 0$ i.e. $2x - 3y + 5 = 0$
17.	**C**	$\cos b=\frac{BE}{AB}$ $BE = AB\cos b$ $\sin d=\frac{AF}{AD}$ $AF = AD\sin d$ $\therefore\ BC = BE + AF$ $= AB\cos b + AD\sin d$ 

18.	**D**	$\frac{\cos(180^\circ+\theta)}{\sin(180^\circ-\theta)}\times\frac{1}{\tan(90^\circ-\theta)}$ $=\frac{-\cos\theta}{\sin\theta}\times\frac{1}{\frac{1}{\tan\theta}}$ $=-\frac{\cos\theta}{\sin\theta}\times\frac{\sin\theta}{\cos\theta}$ $=\underline{\underline{-1}}$
19.	**B**	Let $OC = r$ cm. $\pi(r+4)^2\times\frac{150}{360}-\pi r^2\times\frac{150}{360}=40\pi$ $(r+4)^2-r^2=96$ $r=10$ Thus, $OC = 10$ cm.
20.	**D**	$(n-2)\times 180^\circ = 3960^\circ$ $n = 24$ Size of each interior angle $=\frac{3960^\circ}{24}$ $= 165^\circ$ Number of diagonals $=\frac{24(24-3)}{2}$ $= 252$ Size of each exterior angle $=\frac{360^\circ}{24}$ $= 15^\circ$ ∴ D is true.
21.	**D**	$2x^2+2y^2-8x-6y-11=0$ i.e. $x^2+y^2-4x-3y-\frac{11}{2}=0$ Centre = $(2,\frac{3}{2})$ and radius $=\sqrt{2^2+\left(\frac{3}{2}\right)^2-\left(\frac{-11}{2}\right)}=\frac{\sqrt{47}}{2}$ ⇒ C is not true Distance between the origin and the centre $=\sqrt{2^2+\left(\frac{3}{2}\right)^2}=\frac{5}{2}<\frac{\sqrt{47}}{2}$ ∴ the origin lies inside C ⇒ A is not true When $x = 0$, $2y^2-6y-11=0$. Δ of "$2y^2-6y-11=0$" $=(-6)^2-4(2)(-11)=124>0$ ∴ C intersects the y-axis at two distinct points ⇒ B is not true Circumference of $C=2\pi\left(\frac{\sqrt{47}}{2}\right)\approx 21.53767413<30$ ⇒ D is true
22	**B**	Note that $AB : DE : EC = 2 : 4 : 3$.

		$\because \Delta ABF \sim \Delta CEF$ (AAA) $\therefore \dfrac{\text{Area of } \Delta ABF}{\text{Area of } \Delta CEF} = \left(\dfrac{AB}{CE}\right)^2$ $\therefore$ Area of ΔCEF $= 8\left(\dfrac{9}{4}\right) \text{cm}^2 = 18 \text{ cm}^2$ $\dfrac{\text{Area of } \Delta DEF}{\text{Area of } \Delta CEF} = \dfrac{DE}{EC}$ Area of $\Delta DEF = 18\left(\dfrac{4}{3}\right) \text{ cm}^2 = 24 \text{ cm}^2$
23.	**A**	Note that $\angle BAD + 87° = 180°$. So, we have $(\angle OAD + 56°) + 87° = 180°$. Thus, we have $\angle OAD = 37°$. Also note that $\angle ODA = \angle OAD = 37°$. Hence, we have $\angle AOD = 180° - 37° - 37° = 106°$ 
24.	**B**	$\because\ kx - 3y - 2k = 0$ and $(k-1)x + 2y - 18 = 0$ are perpendicular $\therefore\ \dfrac{k}{3} \times \dfrac{-(k-1)}{2} = -1$ $k^2 - k - 6 = 0$ $k = 3$ or $k = -2$ (rejected as $k > 0$)
25.	**B**	$2x + 2y + 68° = 180°$ ($\angle$ sum of Δ) $x + y = 56°$ $\angle AIB = 180° - x - y$ ($\angle$ sum of Δ) $= 180° - 56°$ $= 124°$ 
26.	**D**	$y = -3 \cos 2x$ Period $= \dfrac{360°}{2} = 180°$ Minimum value $= -3(1) = -3$
27.	**B**	$\because$ mode = 14 $\therefore$ at least two values of x, y and z are 14. Let $x = y = 14$. Arrange all the data exclude z in ascending order as follows: 12 14 14 14 20 21 26 26 28 30 30 $\because$ median = 23 $\therefore$ $21 < z < 26$ $\therefore$ 12 14 14 14 20 21 z 26 26 28 30 30 i.e. IQR $= \dfrac{26+28}{2} - \dfrac{14+14}{2} = 27 - 14 = 13$

<table>
<tr><td>28.</td><td>C</td><td>Required percentage $= \frac{22+7}{5+22+7+5+1} \times 100\% = 72.5\%$</td></tr>
<tr><td>29</td><td>A</td><td>α and β are the roots of $3x^2 - 3x + 4 = 0$
∴ $\alpha + \beta = \frac{-(-3)}{3} = 1 > 0$ ⇒ I is true and $\alpha\beta = \frac{4}{3} > 0$ ⇒ II is true
$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 1^2 - 2\left(\frac{4}{3}\right) = \frac{-5}{3} < 0$ ⇒ III is not true</td></tr>
<tr><td>30</td><td>B</td><td>The table below shows the sum of the numbers obtained.
<table>
<tr><td></td><td></td><td colspan="6">2nd dice</td></tr>
<tr><td></td><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td rowspan="6">1st dice</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td></tr>
<tr><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td></tr>
</table>
Number of favourable outcomes = 8
∴ $P(\text{a sum of 4 or 6}) = \frac{8}{36}$
$= \underline{\underline{\frac{2}{9}}}$</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td colspan="3"><i>Section B</i></td></tr>
<tr><td>31</td><td>C</td><td>$9x^2 - 6x + 1 = (3x-1)^2$
$18x^2 - 2 = 2(3x+1)(3x-1)$
$27x^3 - 1 = (3x-1)(9x^2 + 3x + 1)$
L.C.M. $= 2(3x+1)(3x-1)^2(9x^2 + 3x + 1)$</td></tr>
<tr><td>32.</td><td>C</td><td>$\frac{3}{2\log x - 1} + 4 = \frac{2}{\log x + 2}$
$3(\log x + 2) + 4(2\log x - 1)(\log x + 2) = 2(2\log x - 1)$
$3\log x + 6 + 4[2(\log x)^2 + 3\log x - 2] = 4\log x - 2$
$8(\log x)^2 + 15\log x - 2 = 4\log x - 2$
$8(\log x)^2 + 11\log x = 0$
∴ $\log x = 0$ or $\frac{-11}{8}$
i.e $\log\frac{1}{x} = 0$ or $\frac{11}{8}$ (as $\log\frac{1}{x} = -\log x$)</td></tr>
<tr><td>33.</td><td>D</td><td>$F6AB_{16} + CDE_{16}$
$= (15 \times 16^3 + 6 \times 16^2 + 10 \times 16^1 + 11 \times 16^0) + (12 \times 16^2 + 13 \times 16^1 + 14 \times 16^0)$
$= 15 \times 16^3 + 18 \times 16^2 + 23 \times 16^1 + 25$
$= 15 \times (2^4)^3 + 2(9) \times (2^4)^2 + 393$
$= 15 \times 2^{12} + 9 \times 2^9 + 393$</td></tr>
</table>

34. **D**

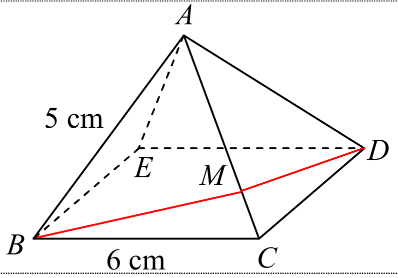
Let M be the point on AC such that $BM \perp AC$ and $DM \perp AC$.

Area of ΔABC = area of ΔACD = 12 cm^2.

$\therefore\ BM = DM = 4.8$ cm

By cosine formula, $\cos \angle BMD = \dfrac{4.8^2 + 4.8^2 - (6\sqrt{2})^2}{2(4.8)(4.8)}$.

$\therefore\ \angle BMD = 124°$ (cor. to the nearest degree)



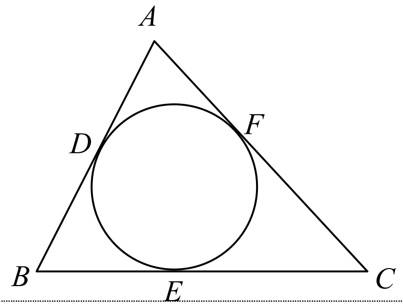
35. **B**

By Heron formula, area of $\Delta ABC = \sqrt{720}\ \text{cm}^2 = 12\sqrt{5}\ \text{cm}^2$

Let r cm be the radius of the circle.

$\therefore\quad \frac{1}{2}\times 7\times r + \frac{1}{2}\times 8\times r + \frac{1}{2}\times 9\times r = 12\sqrt{5}$

i.e. $\quad r = \dfrac{2\times 12\sqrt{5}}{7+8+9} = \sqrt{5}$



36 **A**

Let $P = 7x - 8y + 9$.

When $(x, y) = (-1, 2)$,

$P = 7(-1) - 8(2) + 9 = -14$.

When $(x, y) = (-5, -6)$,

$P = 7(-5) - 8(-6) + 9 = 22$.

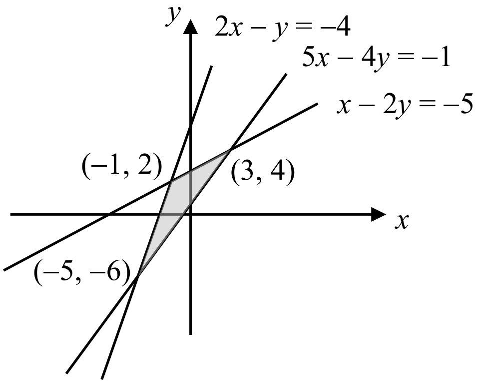
When $(x, y) = (3, 4)$,

$P = 7(3) - 8(4) + 9 = -2$.

Difference of the greatest and the least value

$= 22 - (-14)$

$= 36$



37. **D**

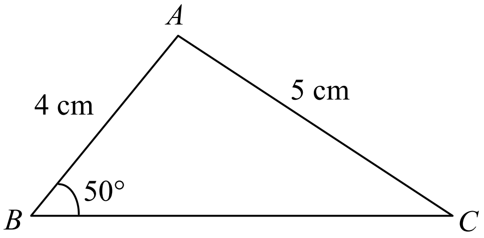
By sine formula, $\dfrac{4}{\sin \angle ACB} = \dfrac{5}{\sin 50°}$

$\therefore\ \angle ACB \approx 37.79481563°\quad \Rightarrow$ A is not true

$\angle BAC \approx 180° - 50° - 37.79481563°$

$\approx 92.20518437°$

$\therefore$ C is not true while D is true



By cosine formula, $BC = \sqrt{4^2 + 5^2 - 2(4)(5)\cos \angle BAC}\ \text{cm} \approx 6.522202777$ cm

$\Rightarrow$ B is not true

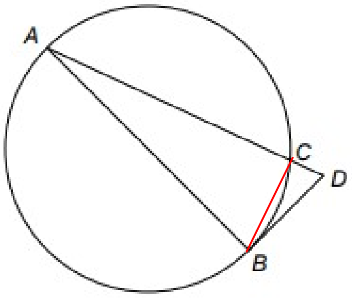
38. **A**

$$\frac{u}{v} = \frac{a+i}{3} \div \frac{3}{a-i} = \frac{a+i}{3} \times \frac{a-i}{3} = \frac{a^2+1}{9}$$

When $a = \sqrt[3]{3}$, $\dfrac{u}{v} = \dfrac{3^{\frac{2}{3}}+1}{9}$, which is not a rational number.

$\therefore$ I may not be true.

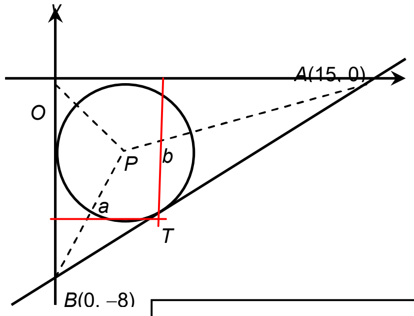
$$u = \frac{a+i}{3} = \frac{a}{3} + \frac{1}{3}i$$

		$\frac{1}{v}=\frac{a-i}{3}=\frac{a}{3}-\frac{1}{3}i$ $\therefore$ II is not true. $\frac{1}{u}=\frac{3}{a+i}=\frac{3}{a+i}\times\frac{a-i}{a-i}=\frac{3a}{a^2+1}-\frac{3}{a^2+1}i$ $v=\frac{3}{a-i}=\frac{3}{a-i}\times\frac{a+i}{a+i}=\frac{3a}{a^2+1}+\frac{3}{a^2+1}i$ $\therefore$ Only III must be true.
39	**C**	$5\sin^2\theta+3\sin\theta-2=0$ $(5\sin\theta-2)(\sin\theta+1)=0$ $\sin\theta=\frac{2}{5}$ or $\sin\theta=-1$ For $\sin\theta=\frac{2}{5}$, there are 2 roots. For $\sin\theta=-1$, the root is $\theta=270°$. $\therefore$ There are 3 roots.
40.	**A**	Note that $\angle ACB=\angle ABD=90°$. So, we have $\triangle ACB\sim\triangle ABD$. Thus, we have $\frac{AC}{AB}=\frac{AB}{AD}$. Hence, we have $AB=4\sqrt{3}$. 
41.	**C**	The required probability = 1 – P(3 balls are of different colour) $=1-\frac{C_1^3\times C_1^2\times C_1^1}{C_3^6}=\frac{7}{10}$
42.	**B**	The number of possible arrangements = 4 × 4! × 6! = 69 120
43.	**C**	Let m be the mean of the examination scores. $\therefore\ \frac{72-m}{5}=2$ i.e. $m=62$ Let x be the examination score of Chris. $\therefore\ \frac{x-62}{5}=-0.6$ i.e. $x=59$
44.	**B**	Let P be the centre of the circle. Join OP, AP and BP. Let r be the radius of the circle, then all the perpendicular distances from P to OA, OB and AB are r. *(tangent ⊥ radius)* $AB=\sqrt{(15-0)^2+[0-(-8)]^2}=17$ Area of ΔOAB = Area of ΔOAP + Area of ΔOBP + Area of ΔABP

$$\frac{1}{2}(15)(8)=\frac{1}{2}(15)r+\frac{1}{2}(8)r+\frac{1}{2}(17)r \qquad \therefore \quad r=3$$

$\therefore$ The coordinates of P are (3, –3).

$\therefore$ The equation of the circle is $(x-3)^2+(y+3)^2=9$.



The equation of the AB is $y=\frac{8}{15}x-8$.

$$\begin{cases}(x-3)^2+(y+3)^2=9 \quad \text{.........(1)} \\ y=\frac{8}{15}x-8 \text{...(2)}\end{cases}$$

Substituting (2) into (1),

$$(x-3)^2+(\frac{8}{15}x-5)^2=9$$

$$\frac{289}{225}x^2-\frac{34}{3}x+25=0$$

$$289x^2-2550x+5625=0$$

$$(17x-75)^2=0$$

$$x=\frac{75}{17}$$

Substituting $x=\frac{75}{17}$ into (2), $y=\frac{8}{15}\left(\frac{75}{17}\right)-8=-\frac{96}{17}$

$\therefore$ The coordinates of T are $\left(\frac{75}{17},-\frac{96}{17}\right)$.

$$\frac{a}{15}=\frac{5}{17}$$

$$a=\frac{75}{17}$$

$$\frac{b}{8}=\frac{12}{17}$$

$$b=\frac{96}{17}$$

$\therefore T$ are $\left(\frac{75}{17},-\frac{96}{17}\right)$.

45. **D**

New mean = (28 + 1) × 2 = 58

New IQR = 15 × 2 = 30

New variance = $13 \times 2^2 = 52$

END