F.6 Final Examination (2021 – 2022)
Mathematics Compulsory Part Answers

1.
$$\frac{c^2\sqrt{b}}{\left(c^{-3}\sqrt[4]{b}\right)^{-2}} = \frac{c^2 b^{\frac{1}{2}}}{c^6 b^{-\frac{1}{2}}}$$
$$= \frac{b^{\frac{1}{2}+\frac{1}{2}}}{c^{6-2}}$$
$$= \frac{b}{c^4}$$

2.
$$b(2a+1) = \frac{2a}{3} - b$$
$$6ab + 3b = 2a - 3b$$
$$6ab - 2a = -6b$$
$$a(3b-1) = -3b$$
$$a = \frac{3b}{1-3b}$$

3. (a) $2m^2 + 7mn - 30n^2 = (2m - 5n)(m + 6n)$

(b)
$$2m^2 + 7mn - 30n^2 + 15n - 6m = (2m - 5n)(m + 6n) + 15n - 6m$$
$$= (2m - 5n)(m + 6n) - 3(2m - 5n)$$
$$= (2m - 5n)(m + 6n - 3)$$

4. (a)
$$\frac{2(x-3)}{5} > \frac{x}{2} - 2$$
$$4x - 12 > 5x - 20$$
$$4x - 5x > -20 + 12$$
$$x < 8$$

and

$$3 - 2x \le 8$$
$$-2x \le 5$$
$$x \ge -\frac{5}{2}$$

∴ The required range is $-\frac{5}{2} \le x < 8$

(b) Integers satisfying both inequalities in (a): –2, –1, 0, 1, 2, 3, 4, 5, 6, 7

∴ 10 integers satisfy both inequalities in (a).

5. Let \$$C$ be the cost.

marked price = \$$1.2C$

selling price = \$$1.2C(1 - 15\%) = C + 70$

$0.02C = 70$

$C = 3500$

Required cost = \$3500

6. Let x = number of 2-dollar coins

y = number of 5-dollar coins

$$\begin{cases} \dfrac{x}{y} = \dfrac{5}{3} \\ 2x + 5y = 100 \end{cases}$$

$$2x + 3x = 100$$
$$x = 20$$
$$y = 12$$

Total number of coins = 32

7. (a) $78° + 90° + a + b + 32° = 360°$

$b = 160° - a$

$$\text{mean} = \frac{1(78°) + 2(90°) + 3a + 4b + 5(32°)}{360°}$$

$$2.65 = \frac{1(78°) + 2(90°) + 3a + 4(160° - a) + 5(32°)}{360°}$$

$$1058° - a = 954°$$
$$a = 104°$$

(b) Note that $a + b = 160°$ and $b > 0°$

$$12° < a < 160°$$

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8. (a)

$$kx^2 - 20x + 25 = 0$$
$$\Delta = 0$$
$$\Delta = (-20)^2 - 4(k)(25) = 0$$
$$k = 4$$

(b)

$$f(x) = 4x^2 - 20x + 25$$

Consider

$$4x^2 - 20x + 25 = 36$$
$$(2x - 5)^2 = 36$$
$$2x - 5 = 6 \text{ or } -6$$
$$x = \frac{11}{2} \text{ or } -\frac{1}{2}$$

$$\text{Required product} = \left(\frac{11}{2}\right)\left(-\frac{1}{2}\right) = -\frac{11}{4}$$

Alternative Method:

$$4x^2 - 20x + 25 = 36$$
$$4x^2 - 20x - 11 = 0$$

$$\text{Required product} = -\frac{11}{4}$$

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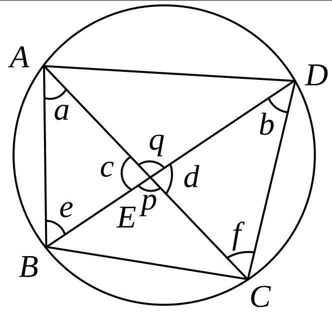
9. (a)



$a = b$ (∠s in the same segment)

$c = d$ (vert. opp. ∠s)

$e = f$ (∠ sum of Δ) / (∠s in the same segment)

$\Delta ABE \sim \Delta DCE$ (AAA)

Marking Scheme

Case 1:	Any correct proof with correct reasons.
Case 2:	Any correct proof without reasons.

(b) (i)

∵ $AE = BE = DE$

∴ E is the centre of the circle

∴ $\angle ABC = 90°$ (converse of ∠ in semi-circle)

∴ ΔABC is a rt ∠d triangle.

(ii)

$BE = ED$ (radii)

$CE = AE$ (radii)

∴ $p = q$ (vert. opp. ∠s)

∴ $\Delta AED \cong \Delta CEB$ (SAS)

Marking Scheme

Case 1:	Any correct proof with correct reasons.

Case 2: Any correct proof without reasons.

10. (a) Let $N = a + bt$, where a and b are non-zero constants.

We have $a + 5b = 598$ and $a + 12b = 1032$.
Solving, we have $a = 288$ and $b = 62$.
$\therefore\ N = 288 + 62t$

(b) Let $M = ct^2$, where c is a non-zero constant.

When $t = 16$, $2M = N$.

$$2c(16^2) = 288 + 62(16)$$
$$c = \frac{5}{2}$$
$$\therefore\ M = \frac{5}{2}t^2$$

11. (a)

$$\text{Inter-quartile range} = \frac{35+37}{2} - \frac{24+20+a}{2}$$
$$11 = 14 - \frac{a}{2}$$
$$a = 6$$

(b)

$$\frac{22(3)+24(2)+26+27+(20+b)(2)+(30+c)(2)+31(2)+35(2)+37+40(2)+42+44}{20} = 31$$
$$b + c = 9$$
$$\because 7 \le b \le 9 \text{ and } 0 \le c \le 1$$
$$\text{we have } \begin{cases} b = 8 \\ c = 1 \end{cases} \text{ or } \begin{cases} b = 9 \\ c = 0 \end{cases}$$

(c) Case 1: $b = 8$ and $c = 1$
The standard deviation ≈ 6.782329983

Case 2: $b = 9$ and $c = 0$
The standard deviation ≈ 6.752777206

∴ The least possible standard deviation is 6.75 minutes.

Alternative Method:

Note that $2(28-31)^2 + 2(31-31)^2 > 2(29-31)^2 + 2(30-31)^2$ When $b = 9$ and $c = 0$, the standard deviation is the least. The least possible standard deviation is 6.75 minutes.

12. (a) Let R cm be the radius of the sphere.

$$4\pi R^2 = 1764\pi$$
$$R = 21$$

$$\text{Volume of sphere} = \frac{4}{3}\pi(21)^3 \text{ cm}^3 = 12348\pi \text{ cm}^3$$

(b) Let V cm^3 be the volume of the larger cone.

$$\text{Volume of the smaller cone} = \left(\frac{16}{25}\right)^{\frac{3}{2}} V \text{ cm}^3 = \frac{64}{125}V \text{ cm}^3$$
$$\frac{64}{125}V + V = 12348\pi$$
$$V = \frac{24500}{3}\pi$$

Let r cm be the radius of the larger cone.

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$$V = \frac{1}{3}\pi r^2\left(\frac{4}{7}r\right) = \frac{4}{21}\pi r^3$$

$$\frac{4}{21}\pi r^3 = \frac{24500}{3}\pi$$

$$r = 35$$

Slant height of the larger cone = $\sqrt{35^2 + \left(\frac{4}{7}\times 35\right)^2}$ cm $= 5\sqrt{65}$ cm

Curved surface area of larger cone = $\pi(35)(5\sqrt{65})$ cm^2

= $175\sqrt{65}\pi$ cm^2 ≈ 4430 cm^2 (3 s.f)

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13. (a) Let $ax + b$ be the quotient when $f(x) \div (2x^2 - 7x - 4)$ where a and b are constants.

$$f(x) = (ax + b)(2x^2 - 7x - 4) + 2x - k - 3$$

$$f\left(-\frac{1}{2}\right) = 0$$

$$\left(-\frac{1}{2}a + b\right)\left[2\left(-\frac{1}{2}\right)^2 - 7\left(-\frac{1}{2}\right) - 4\right] + 2\left(-\frac{1}{2}\right) - k - 3 = 0$$

$$0 - 1 - k - 3 = 0$$

$$k = -4$$

(b)

$$f(x) = (ax + b)(2x^2 - 7x - 4) + 2x + 1$$

$$f\left(\frac{7}{2}\right) = 4$$

$$\left(\frac{7}{2}a + b\right)\left[2\left(\frac{7}{2}\right)^2 - 7\left(\frac{7}{2}\right) - 4\right] + 2\left(\frac{7}{2}\right) + 1 = 4$$

$$-14a - 4b = -4$$

Constant term of $f(x) = -4b + 1 = 25$

$$b = -6$$

∴ $a = 2$

∴ The required quotient is $2x - 6$.

(c)

$$f(x) = (2x - 6)(2x^2 - 7x - 4) + 2x + 1$$

$$= 2(x - 3)(2x + 1)(x - 4) + 2x + 1$$

$$= (2x + 1)(2x^2 - 14x + 25)$$

$$f(x) = 0$$

$x = -\frac{1}{2}$ or $2x^2 - 14x + 25 = 0$

$$\Delta = (-14)^2 - 4(2)(25)$$

$$= -4 < 0$$

∴ The quadratic equation $2x^2 - 14x + 25 = 0$ has no real roots

$-\frac{1}{2}$ is a rational root

∴ The equation $f(x) = 0$ has no irrational root.

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14. (a) Slope of $L_1 = -\frac{3}{4}$

$$G = (-2, 7)$$

Equation of L_2:

$$y - 7 = -\frac{3}{4}(x + 2)$$

$$3x + 4y - 22 = 0$$

Alternative Method:

Let the equation of L_2 be $3x + 4y + k = 0$, where k is a constant.

$$3(-2) + 4(7) + k = 0$$
$$k = -22$$

Equation of L_2 is $3x + 4y - 22 = 0$.

(b) (i) Γ is the perpendicular bisector of AB.
$\therefore \Gamma \perp L_1$ and passes through the centre G.

Equation of Γ:

$$y - 7 = \frac{-1}{\left(-\frac{3}{4}\right)}(x + 2)$$
$$4x - 3y + 29 = 0$$

(ii) Let M = mid-point of AB.

Consider $\begin{cases} 4x - 3y + 29 = 0 \\ 3x + 4y - 42 = 0 \end{cases}$

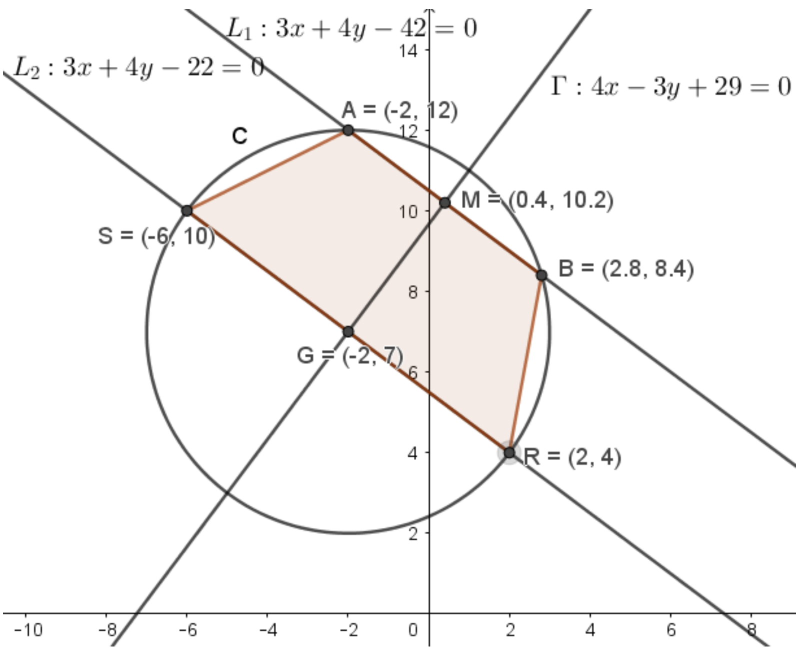
$$x = \frac{2}{5},\ y = \frac{51}{5}$$
$$M = \left(\frac{2}{5}, \frac{51}{5}\right)$$

$$GM = \sqrt{\left(\frac{2}{5} + 2\right)^2 + \left(\frac{51}{5} - 7\right)^2} = 4$$
$$AB = 2\sqrt{\text{radius}^2 - GM^2}$$
$$= 2\sqrt{5^2 - 4^2} = 6$$

$$RS = 2(\text{radius}) = 10$$

Area of $ABRS = \frac{(6 + 10)(4)}{2} = 32 > 30$

The claim is agreed.



15. (a) The required probability

$$=\frac{C_3^5+C_3^7}{C_3^{14}}$$

$$=\frac{45}{364}$$

Alternatively method

The required probability

$$=\frac{5}{14}\bullet\frac{4}{13}\bullet\frac{3}{12}+\frac{7}{14}\bullet\frac{6}{13}\bullet\frac{5}{12}$$

$$=\frac{45}{364}$$

(b) The required probability

$$=\frac{C_1^2\times C_1^5\times C_1^7}{C_3^{14}}$$

$$=\frac{5}{26}$$

Alternatively method

The required probability

$$=\frac{2}{14}\bullet\frac{5}{13}\bullet\frac{7}{12}\times 6$$

$$=\frac{5}{26}$$

16. Common ratio $= \dfrac{\log 9}{\log 3} = 2$

$$\frac{1}{2}(\log 3)(2)^{k+1} + \frac{1}{2}(\log 3)(2)^{2k+1} < \log 3^{2022}$$

$$2^{2k} + 2^k - 2022 < 0$$

$$\therefore \quad \frac{-1-\sqrt{1^2-4(1)(-2022)}}{2(1)} < 2^k < \frac{-1+\sqrt{1^2-4(1)(-2022)}}{2(1)}$$

$$\frac{-1-\sqrt{8089}}{2} < 2^k < \frac{-1+\sqrt{8089}}{2}$$

$\because \quad 2^k > 0$

$$\therefore \quad 0 < 2^k < \frac{-1+\sqrt{8089}}{2}$$

$$\log 2^k < \log \frac{-1+\sqrt{8089}}{2}$$

$$k \log 2 < \log \frac{-1+\sqrt{8089}}{2}$$

$$k < \frac{\log \frac{-1+\sqrt{8089}}{2}}{\log 2}$$

$k < 5.47$ *cor. to 2 d.p.*

$\therefore$ The greatest value of k is 5.

17.	(a)	$\angle BCD = \angle CAD$ ($\angle$in alt. segment) $\angle CDB = \angle ADC$ (common $\angle$) $\angle DBC = \angle DCA$ ($\angle$sum of Δ) $\Delta DBC \sim \Delta DCA$ (AAA) $\frac{DB}{DC} = \frac{DC}{DA}$ (corr. sides, ~Δs) $\frac{27}{45} = \frac{45\ cm}{DA}$ DA = 75 cm $AB = (75 - 27)$ cm $= 48$ cm	
	(b) (i)	$\because$ $BC \perp AC$ $\therefore$ AB is the diameter of the circle. (converse of $\angle$ in semi-circle) Area of the circle $=\left(\frac{48}{2}\right)^2 \pi$ cm^2 $=576\pi$ cm^2	
	(b) (ii)	$AF = CF$ (tangent properties) $AB \perp AF$ (tangent $\perp$ radius) Let $AF = k$ cm $AF^2 + AD^2 = DF^2$ (Pyth. theorem) $k^2 + 75^2 = (45 + k)^2$ $k^2 + 5625 = 2025 + 90k + k^2$ $k = 40$ Area of ΔAFD $= \frac{1}{2}(40)(75)$ cm^2 $= 1500$ cm^2 Difference of area of circle and the area of ΔAFD $=(576\pi - 1500)$ cm^2 ≈ 309.5573685 cm^2 > 0.03 m^2 $\therefore$ The claim is agreed.	

18. (a)

$\angle ABC = 180° - 60° = 120°$

$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos\angle ABC$

$AC^2 = 18^2 + 30^2 - 2(18)(30)\cos 120°$

$AC = 42$ cm

(b) (i)

$AC^2 = PQ^2 + (AP - CQ)^2$ (Pyth. theorem)

$42^2 = PQ^2 + (8 - 5)^2$

$PQ \approx 41.89272013$

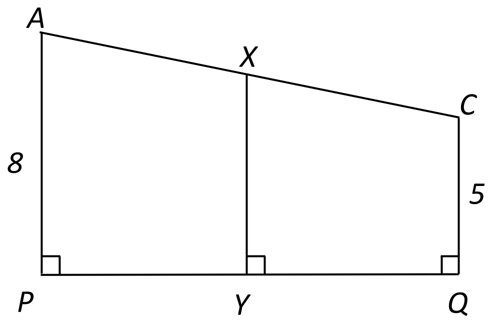
$PQ \approx 41.9$ cm

(b) (ii)

Let *F* be the projection of *B* on the horizontal ground.

Suppose *AC* intersects *BD* at *X*.

Y is the projection of *X* on the horizontal ground.



$XY = \dfrac{AP + CQ}{2} = \dfrac{8 + 5}{2} = \dfrac{13}{2}$ cm

$BF = 2XY = 13$ cm

Alternatively method

Let *F* be the projection of *B* on the horizontal ground

Note that *PDQF* is a parallelogram.

$PF = DQ$

$= \sqrt{DC^2 - CQ^2}$ (Pyth. theorem)

$= \sqrt{18^2 - 5^2}$

$= \sqrt{299}$ cm

Let *G* be a point on *BF* where *GF* = *AP*.

$AB^2 = AG^2 + BG^2$ (Pyth. theorem)

$18^2 = (\sqrt{299})^2 + (BF - 8)^2$

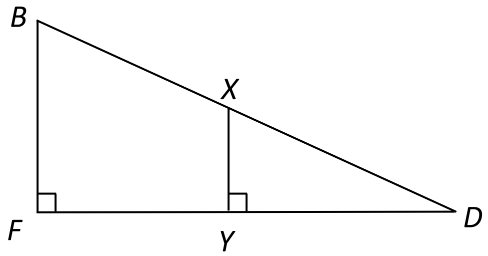
$BF = 13$ cm

$BD^2 = AB^2 + AD^2 - 2(AB)(AD)\cos\angle BAD$

$BD^2 = 18^2 + 30^2 - 2(18)(30)\cos 60°$

$BD \approx 26.15339366$

$BD \approx 26.2$ cm



Required angle $= \angle BDF$

$= sin^{-1}(\frac{BF}{BD})$

$\approx 29.80617121°$

$\approx 29.8°$

(b) (iii)

Since $\Delta RAP \sim \Delta RBF$, *we have*

$$\frac{RA}{RA+AB} = \frac{AP}{BF}$$

$$\frac{RA}{RA+18} = \frac{8}{13}$$

$RA = 28.8$ cm

Since $\Delta SCQ \sim \Delta SBF$, *we have*

$$\frac{CS}{CS+BC} = \frac{CQ}{BF}$$

$$\frac{CS}{CS+30} = \frac{5}{13}$$

$CS = 18.75$ cm

Area of ΔBRS

$= \frac{1}{2}(BR)(BS)\, sin\angle RBS$

$= \frac{1}{2}(18+28.8)(30+18.75)\, sin\, 120°$

$= \frac{1}{2}(46.8)(48.75)\, sin\, 120°$

≈ 987.91847937 cm^2

In ΔBRS,

$RS^2 = BR^2 + BS^2 - 2(BR)(BS)\, cos\angle RBS$

$RS = \sqrt{46.8^2 + 48.75^2 - 2(46.8)(48.75)\, cos\, 120°}$

≈ 82.75447118 cm

(b) (iii) Let *U* be the foot of the perpendicular from *B* to *RS*.

$$Area\ of\ \Delta BRS = \frac{1}{2}(RS)(BU)$$

$$987.91847937 = \frac{1}{2}(82.75447118)(BU)$$

$$BU \approx 23.87589372 \text{ cm}$$

Required angle $= \angle BUF$

$$\sin\angle BUF = \frac{BF}{BU}$$

$$\sin\angle BUF \approx \frac{13}{23.87589372}$$

$$\angle BUF \approx 32.98928856° > 32°$$

∴ The claim is agreed.

Alternatively method

Since $\Delta RAP \sim \Delta RBF$, *we have*

$$\frac{RA}{RA+AB} = \frac{AP}{BF}$$

$$\frac{RA}{RA+18} = \frac{8}{13}$$

$$RA = 28.8 \text{ cm}$$

In ΔABD,

$$\cos\angle ABD = \frac{AB^2 + BD^2 - AD^2}{2(AB)(BD)}$$

$$= \frac{18^2 + 26.15339366^2 - 30^2}{2(18)(26.15339366)}$$

$$\angle ABD \approx 83.41322445°$$

Area of ΔBDR

$$= \frac{1}{2}(BR)(BD)\ sin\angle ABD$$

$$= \frac{1}{2}(18+28.8)(26.15339366)\ sin\, 83.41322445°$$

$$\approx 607.9498334 \text{ cm}^2$$

In ΔBDR,

$$DR^2 = BD^2 + BR^2 - 2(BD)(BR)\ cos\angle ABD$$

$$DR \approx 50.92582842 \text{ cm}$$

Let U be the foot of the perpendicular from B to RD.

$$Area\ of\ \Delta BDR = \frac{1}{2}(BU)(DR)$$

$$607.9498334 = \frac{1}{2}(BU)(50.92582842)$$

$$BU \approx 23.87589372 \text{ cm}$$

Required angle $= \angle BUF$

$$\sin\angle BUF = \frac{BF}{BU}$$

$$\sin\angle BUF = \frac{13}{23.87589372}$$

$$\angle BUF \approx 32.98928856° > 32°$$

∴ The claim is agreed.

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19. (a) $f(x) = 4x^2 - 16kx - 8x + 16k^2 + 15k + 13$

$= 4(x^2 - 4kx - 2x) + 16k^2 + 15k + 13$

$= 4[x^2 - 2(2k+1)x + (2k+1)^2 - (2k+1)^2] + 16k^2 + 15k + 13$

$= 4(x - 2k - 1)^2 - 16k^2 - 16k - 4 + 16k^2 + 15k + 13$

$= 4(x - 2k - 1)^2 + 9 - k$

∴ $P = (2k + 1, 9 - k)$

(b) $Q = (2k + 4, 13 - k)$

(c) $R = (2k + 1, 4 - k)$

Slope of $QR = \dfrac{13 - k - 4 + k}{2k + 4 - 2k - 1} = 3$

The required equation is

$$\frac{y - 9 + k}{x - 2k - 1} = -\frac{1}{3}$$

$x + 3y + k - 28 = 0$

y – coordinate of G = y – coordinate of Q = $13 - k$

Sub $y = 13 - k$ into $x + 3y + k - 28 = 0$

$x + 3(13 - k) + k - 28 = 0$

$x = 2k - 11$

∴ $G = (2k - 11, 13 - k)$.

(d) (i) $PQ = \sqrt{(2k+4-2k-1)^2 + (13-k-9+k)^2} = 5$

$PR = 9-k-4+k = 5$

$\therefore$ $PQ = PR$.

Thus, P, G and H are collinear.

(ii) y – coordinate of H = y – coordinate of mid-pt. of PR

$$= \frac{9-k+4-k}{2} = \frac{13-2k}{2}$$

Sub $y = \frac{13-2k}{2}$ into $x+3y+k-28=0$

$$x + 3(\frac{13-2k}{2}) + k - 28 = 0$$

$$x = \frac{4k+17}{2}$$

$\therefore$ $H = (\frac{4k+17}{2}, \frac{13-2k}{2})$.

Radius of C $= \sqrt{(2k+1-\frac{4k+17}{2})^2 + (9-k-\frac{13-2k}{2})^2}$

$$= \sqrt{\frac{125}{2}} = \frac{5\sqrt{10}}{2}$$

$\because$ C touches the x-axis

$\therefore$ $\frac{13-2k}{2} = \pm\frac{5\sqrt{10}}{2}$

$k = \frac{13-5\sqrt{10}}{2}$ or $\frac{13+5\sqrt{10}}{2}$

$\because \frac{13+5\sqrt{10}}{2} > 0$

$\therefore$ The claim is disagreed.

The End