Candidate Number

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION
2020

MOCK EXAMINATION
MATHEMATICS Compulsory Part
PAPER 1
Question-Answer Book

17 February, 2020

8.25 am – 10.40 am (2¼ hours)
This paper must be answered in English

INSTRUCTIONS

(1) After the announcement of the start of the examination, you should first write your Candidate Number in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.

(2) This paper consists of THREE sections, A(1), A(2) and B.

(3) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

(4) Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number and mark the question number box on each sheet, and fasten them with string INSIDE this book.

(5) Unless otherwise specified, all working must be clearly shown.

(6) Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

(7) The diagrams in this paper are not necessarily drawn to scale.

(8) No extra time will be given to candidates for writing Candidate Number after the 'Time is up' announcement.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

SECTION A(1) (35 marks)

1. Simplify $\dfrac{(m^{\frac{-1}{2}}n^3)^2}{(n^{\frac{-2}{3}}m^4)^{-3}}$ and express your answer with positive indices. (3 marks)

2. Make x the subject of the formula $k(2kx-h)=2x-h$. (3 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Candidate Number									

3. Factorize

(a) $3x^3 - 13x^2 + 12x$,

(b) $(3x-4)^2 + 3x^3 - 13x^2 + 12x$.

(4 marks)

4. (a) Solve the inequality $\frac{2(2-x)}{-3} < 4x + 5$.

(b) Find all integers satisfying both $\frac{2(2-x)}{-3} < 4x + 5$ and $5x - 17 \leq 0$.

(4 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

5. The volume of juice in a jar is measured as 0.87 L correct to the nearest 0.01 L.
   (a) Find the least possible volume of the juice.
   (b) Is it possible to share the juice among 18 children, such that each of them can have 50 mL of juice correct to the nearest 5 mL? Explain your answer.

(4 marks)

6. Wine *A* contains 10% alcohol while wine *B* contains 40% alcohol. In what ratio must wine *A* and wine *B* be mixed to form a new wine *C* containing 20% alcohol? (4 marks)

Answers written in the margins will not be marked.

Candidate Number									

7. The coordinates of the points A and B are $(-3, -2)$ and $(-4, -6)$ respectively. A is rotated anticlockwise about the origin O through $90°$ to A'. B' is the reflection image of B with respect to the x-axis.

   (a) Write down the coordinates of A' and B'.

   (b) Prove that A', O and B' are collinear.

(4 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

8. A sector of radius s cm and angle θ is folded to form a right circular cone of base radius 7 cm and height 24 cm.

(a) Find the value of s. (1 mark)

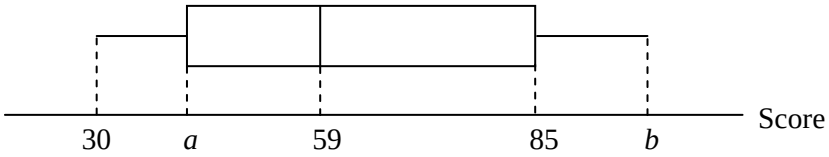
(b) Hence, find the value of θ. (3 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Candidate Number									

9. The box-and-whisker diagram below shows the distribution of the scores of the students in Class *X* in a mathematics examination. There are 25 students in Class *X*. It is given that the range and the inter-quartile range of the distribution are 67 and 42 respectively.



(a) Find the values of *a* and *b*.

(b) A student in Class *Y* who scored 59 in the mathematics examination joins Class *X* and the student in Class *X* who obtained the lowest score leaves Class *X*. The teacher of Class *X* claims that the median of the scores of the students in Class *X* will increase. Do you agree? Explain your answer.

(5 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

SECTION A(2) (35 marks)

10. It is given that f(x) partly varies as x^2 and partly varies as x. Suppose that $f(3)=15$ and $f(-5)=55$.

(a) Find f (x). (3 marks)

(b) Using the method of completing the square, find the coordinates of the vertex of the graph of $y=f(x)+3$. (2 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Candidate Number									

11. Let f (x) be a cubic polynomial. When f (x) is divided by $x-2$, the remainder is 30 . When f (x) is divided by $2x+3$, the remainder is 51 . It is given that f (x) is divisible by $3x^2+4x-5$.

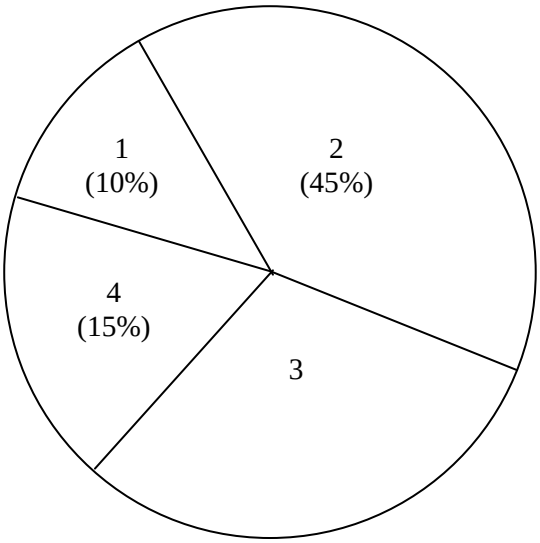
(a) Find the quotient when f (x) is divided by $3x^2+4x-5$. (3 marks)

(b) How many rational roots does the equation $f(x)=0$ have? Explain your answer. (3 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

12. The pie chart below shows the distribution of the number of calculators owned by a group of students.



Distribution of the number of calculators owned by a group of students

(a) Write down the mode of the distribution. (1 mark)

(b) Find the mean of the distribution. (2 marks)

(c) Find the standard deviation of the distribution. (2 marks)

(d) It is given that there are 200 students in the group. If two students are selected from the group, find the probability that both of them have at least 3 calculators. (3 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Candidate Number									

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

13. Let O be the origin of a rectangular coordinate plane. The coordinates of two points Q and R are $(6, 0)$ and $(0, -10)$ respectively. Let C be the circle passing through O, Q and R.

(a) Find the equation of C. (3 marks)

(b) Find the equation of the tangent to the circle at Q. (3 marks)

(c) Let P be a moving point such that the length of tangent from P to the circle C is 4. Describe the locus of P. (2 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

14. In Figure 1, S denotes the circle $AOCD$ (solid curve). S' denotes the circle centred at O and passing through A and C (dotted curve). B is the point of intersection of S' and OD.

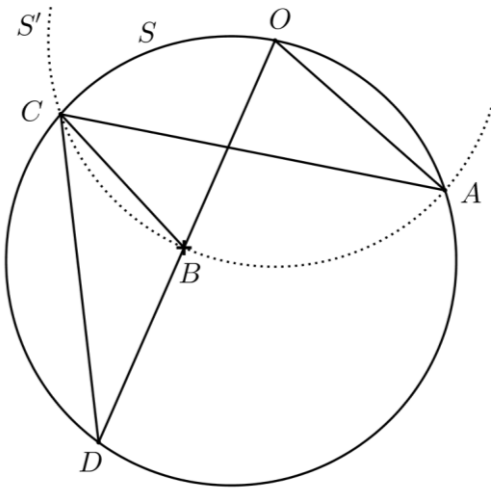


Figure 1

(a) (i) Prove that $\angle DCB = \angle BCA$.

(ii) Hence, prove that B is the in-centre of ΔACD.

(4 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

14. (b) CB in Figure 1 is extended to cut the circle S at E as shown in Figure 2. CE and AD intersect at F. S'' denotes the circle CDF (dashed curve). G is the point of intersection of AC and the circle S''.

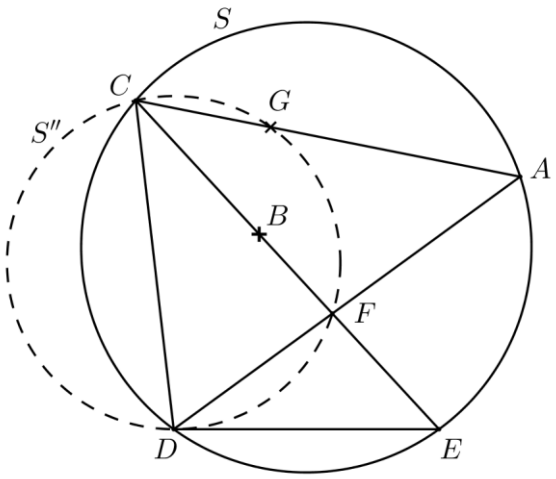


Figure 2

(i) Prove that DE is tangent to S'' at D.

(ii) If CD is a diameter of S'' and $\angle CDB = 20°$, by using (a)(ii) and(b)(i), find $\angle ADE$.

(4 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

SECTION B (35 marks)

15. 4 people are selected from 7 couples in a party.

(a) Find the number of possible ways such that there are no couples. (2 marks)

(b) Given that there is at least one couple, find the probability that there are exactly two couples.

(2 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

16. Let b and c be constants, where $c > 0$ and $c \neq 1$. Denote the graph of $y = b + \log_c x$ by G. The x-intercept of G is 64 and G passes through the point $(256, 1)$. Express x in terms of y.

(4 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

17. In Figure 3, $A_1B_1C_1D_1$ is a square of side 10 cm. A second square $A_2B_2C_2D_2$ is formed inside $A_1B_1C_1D_1$ such that $A_1A_2D_2$, $B_1B_2A_2$, $C_1C_2B_2$ and $D_1D_2C_2$ are straight line segments, $\angle B_2B_1C_1 = 60°$ and $\Delta A_1D_1D_2$, $\Delta D_1C_1C_2$, $\Delta C_1B_1B_2$ and $\Delta B_1A_1A_2$ are congruent. This process is continued to form an infinite number of squares $A_nB_nC_nD_n$, where n is a positive integer, $n \geq 2$.

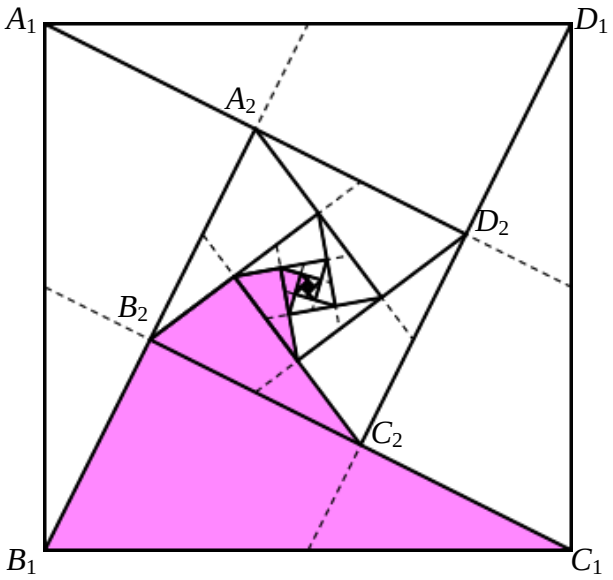


Figure 3

(a) Express the area of $\Delta C_1B_1B_2$ in surd form. (2 marks)

(b) Express the length of B_2C_2 in surd form. (2 marks)

(c) Let K_1 be the area of $\Delta C_1B_1B_2$. For any integer $n > 1$, let K_n be the area of $\Delta C_nB_nB_{n+1}$. Find the sum of the areas of the shaded regions. (3 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

18. In Figure 4, ΔABC is a piece of triangular paper lying on a horizontal plane, where $AB = BC = 10$ cm and $AC = 8$ cm. D and E are the mid-points of AB and AC respectively.

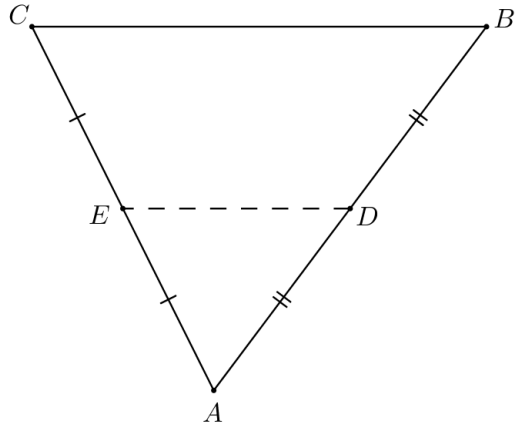


Figure 4

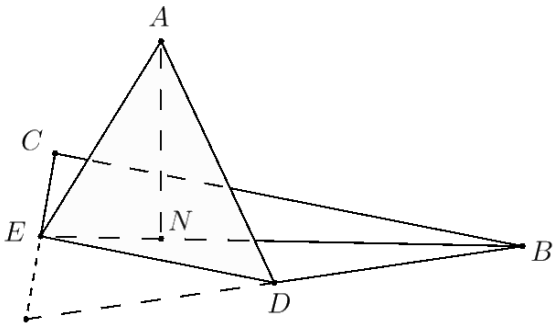


Figure 5

(a) (i) Show that $\cos\angle B = \frac{17}{25}$.

(ii) Hence, show that the length of the altitude of ΔABC which passes through A is $\frac{8}{5}\sqrt{21}$ cm.

(3 marks)

(b) ΔADE in Figure 4 is folded along DE such that A lies vertically above BE as shown in Figure 5. N denotes the projection of A onto the horizontal plane.

(i) Find AN.

(ii) Hence, find the angle between ΔADE and the horizontal plane.

(5 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

19. (a) Figure 6 shows the circumcircle of an obtuse-angled triangle ABC with centre G. AD is a diameter of the circle. $AB = c$, $BC = a$, $CA = b$, $\angle ACB = \theta$ and the radius of the circle is R.

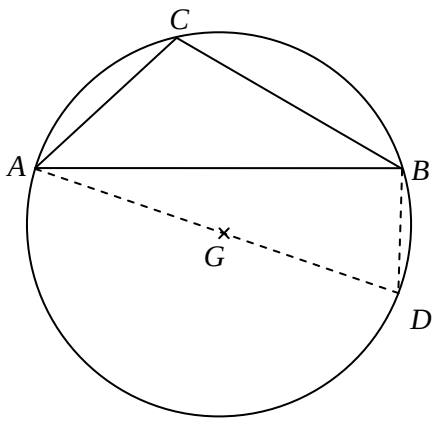


Figure 6

(i) Prove that $\sin\theta = \dfrac{c}{2R}$.

(ii) Prove that the area of ΔABC is $\dfrac{abc}{4R}$.

(3 marks)

(b) The coordinates of point E are $(18, 24)$.

(i) Find the equation of the perpendicular bisector of OE, where O is the origin.

(ii) F is a point such that $OF = 14$ and $EF = 40$. It is given that ΔOEF is an obtuse-angled triangle.

(I) Using (a)(ii), or otherwise, find the radius of the circumcircle of ΔOEF.

(II) Let $N(a, b)$ be the circumcentre of ΔOEF. Show that $a^2 + b^2 = 625$.

Hence find the values of a and b.

(III) Let Q be a moving point such that the areas of ΔOEQ and ΔOEF are equal. Find the minimum value of $QO \times QE$.

(9 marks)

Answers written in the margins will not be marked.
Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

END OF PAPER