INTERNAL EXAM
2016-2017
MATH
COMPULSORY
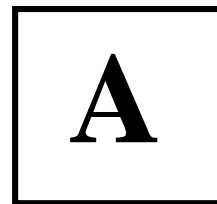**PART**
PAPER 1

A

ST.PAUL'S COLLEGE

Internal Examination 2016-2017

MATHEMATICS   Compulsory Part
PAPER 1

Section A

Question-Answer Book

($2^{1}/4$ hours)

This paper must be answered in English

Name	
Group	Circle the respective group: G1 PSK G2 KWN G3 FBL G4 TH G5 PCC G6 TH G7 LMW G8 PSK
Class	( )

	Marker's Use Only	**Examiner's Use Only**
	Marker No.	Examiner No.
Question No.	Marks	Marks
1–2	/6	
3–4	/8	
5–6	/9	
7–8	/8	
9	/4	
10	/7	
11	/7	
12	/8	
13	/7	
14	/6	
Total	/70	

INSTRUCTIONS

1.   Write your Name, Class and Class Number in the spaces provided on Page 1.

2.   This paper consists of THREE sections, A(1), A(2) and B.

3.   Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4.   Graph paper and supplementary answer sheets will be supplied on request. Write your Name and mark the question number box on each sheet, and fasten them with string INSIDE this book.

5.   Unless otherwise specified, all working must be clearly shown.

6.   Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

7.   The diagrams in this paper are not necessarily drawn to scale.

SECTION A(1) (35 marks)

1. If $a^n = 4$ and $b^m = 7$, find $\dfrac{(2a^{-n})^{-3}(b^m)^2}{(8a^n)^2(2b^m)^{-1}}$, where a, b, m and n are positive numbers.

(3 marks)

$$\frac{(2a^{-n})^{-3}(b^m)^2}{(8a^n)^2(2b^m)^{-1}} = \frac{2^{-3}(a^n)^3(b^m)^2}{8^2(a^n)^2 2^{-1}(b^m)^{-1}}$$

$$= \frac{2^{-3}(4)^3 \cdot 7^2}{8^2(4)^2 2^{-1}(7^{-1})}$$

$$= 7^3/8^2 = 343/64 \text{ (or } \approx 5.36)$$

2. Evaluate $1001_2 - 111_2$. Express your answer in both binary numeral and decimal numeral.

(3 marks)

$1001_2 - 111_2 = 10_2$

$10_2 = 1 \times 2^1 + 0 \times 2^0 = 2$

Page total

3. (a) Factorize $4x^2 + 15x - 4$.
   (b) Factorize $4(y + 1)^4 + 15(y + 1)^2 - 4$.

   (4 marks)

(a) $4x^2 + 15x - 4 = (4x - 1)(x + 4)$

(b)   From (a) $4((y+1)^2)^2 + 15(y + 1)^2 - 4$

        $= (4(y+1)^2 - 1)((y+1)^2 + 4)$

        $= (2(y+1) - 1)(2(y+1) + 1)((y+1)^2 + 4)$

        $= (2y + 1)(2y + 3)(y^2 + 2y + 5)$

4. The cost of an item X is $100. To attract the customers, Peter makes a 20% discount on the marked price, but he still wants to have a 10% profit. What should be the marked price?

   (4 marks)

Let $y be the marked price

$y(0.8) = 100 \times 1.1$

$y = 137.5$

The marked price is $137.5.

Page total

5. Jo's pocket contains 20 coins. They are either $10 coins or $1 coins. Jo knows that all his coins in his pocket cannot exceed one hundred and twenty dollars. Find the maximum number of $10 coins in his pocket.

(4 marks)

Let there are n $10 coins.

$10n + (20 − n) \leq 120$

$9n \leq 100$

$n \leq 100/9$

The max. number of $10 coins is 11.

6. A conical cone has base radius r cm and the height of it is h cm. The volume of the cone equals to the volume of a sphere with radius r cm.
   (a) Find the ratio of h to r.
   (b) Join claims that the total surface area of the cone will be more than the surface area of the sphere by at least 25%. Do you agree? Explain your answer.
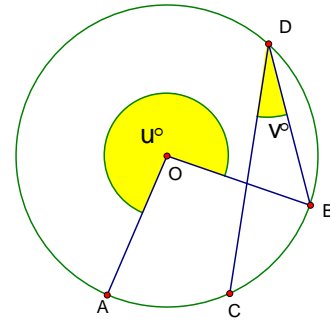
(5 marks)

(a)   Vol. of the cone = $(1/3)\pi r^2 h$

Vol. of the sphere = $(4/3)\pi r^3$

$(1/3)\pi r^2 h = (4/3)\pi r^3$

$h = 4r$

$h : r = 4 : 1$

(b)   Total surface area of the cone = $\pi r^2 + \pi r^2\sqrt{17}$

Surface area of the sphere = $4\pi r^2$

$\dfrac{\pi r^2(1+\sqrt{17}) - 4\pi r^2}{4\pi r^2} \times 100\% \approx 28.1\%$

Yes.   I agree

Page total

7. In the figure, O is the centre of the circle. A, C, B and D are points on the circle. C is the mid-point on the arc ACB. If $\angle CDB = v°$, major $\angle AOB = u°$ and u = 6v, find u and v.

(4 marks)

$\angle AOB = 360° - u°$ (∠. at a point)

$\angle AOC = \angle COB$ (equal arc equal ∠)

$\angle AOC = 2\angle CDB$ (∠ at centre twice ∠ at circumference)

$360° - u° = 4v°$

$360 - 6v = 4v$

$v = 36$

$u = 216$

8. F(x) = A – Bcosx where A, B are positive constants. If the maximum and minimum values of F(x) are 7 and 3 respectively, find A and B. (4 marks)

Max. of F(x) = A + B

Min. of F(x) = A - B

A + B = 7

A – B = 3

A = 5, B = 2

Page total

9. When $2x^3 + Ax^2 + 6x + B$ is divided by $2x$, the remainder is 3 and the quotient is $x^2 - x + 3$, find A and B.

(4 marks)

$2x^3 + Ax^2 + 6x + B \equiv 2x(x^2 - x + 3) + 3$
$\equiv 2x^3 - 2x^2 + 6x + 3$

By comparing coef., A = -2, B = 3

SECTION A(2) (35 marks)

10. Peter is a salesman and his monthly salary consists of two parts. One part is the basic salary which is a constant and the other part is his commission which is proportional to the income from his selling. Peter's salary in June was $15000. In July, Peter sold twice as much as what he had sold in the month before.   His salary was $18000 in July. In August, Peter could only sell half of what he had sold in June. Find the salary of Peter in August.

(7 marks)

Let C be the basic salary, I is the income from selling and S be the monthly salary

$S = C + kI$, where k is a constant

If Peter income from his selling is I_1 in June, then in July is $2I_1/2$

$15000 = C + kI_1$

$18000 = C + K(2I_1)$

$kI_1 = 3000, C = 12000$

$C + (kI_1)/2 = 12000 + 3000/2 = 13500$

His salary in Aug. is $13500.

Page total

11. $y = g(x)$ is a quadratic function. The y-intercept of $g(x)$ is 4 and the x-intercepts are –2 and 6. Paul claims that $g(x)$ must have a maximum value at $x = 2$ without sketching the graph of $y = g(x)$. Do you agree with him? Explain your answer. Find the maximum or the minimum value of y.

(7 marks)

$y = A(x+2)(x-6)$, where A is a constant

when $x=0$, $y=4$

$\therefore$ $4 = A(2)(-6)$

$A = -1/3$

$f(x) = -1/3 (x+2)(x-6)$

$f(x) = -1/3 (x-2)^2 + 16/3$

$\because$ coefficient of $x^2 = -1/3 < 0$

$\therefore$ $f(x)$ will obtain maximum value at $x=2$

I agree with him

Page total

12. (a) Show that $\frac{1}{2}$ is a root of the equation : $2x^3 + 7x^2 + 2x - 3 = 0$ and

hence factorize $2x^3 + 7x^2 + 2x - 3$.

(4 marks)

(b) Someone claims that the trigonometric equation
$2\cos^3\theta + 4\cos^2\theta + 2\cos\theta = 3\sin^2\theta$, $0° < \theta < 360°$, has exactly 3 roots.
Do you agree? Explain your answer.

(4 marks)

Let $f(x) = 2x^3+7x^2+2x-3$

$f(1/2) = 2(1/2)^3+7(1/2)^2+2(1/2)-3=0$

By factor theorem, 1/2 is a root of $2x^3+7x^2+2x-3=0$

$2x^3+7x^2+2x-3 \equiv (2x-1)(x^2+Ax+B)$

By comparing coefficient, B=3, A=4

$\therefore 2x^3+7x^2+2x-3 \equiv (2x-1)(x^2+4x+3)$

$2x^3+7x^2+2x-3 \equiv (2x-1)(x+3)(x+1)$

$2\cos^3\theta+4\cos^2\theta+2\cos\theta=3(1-\cos^2\theta)$

$2\cos^3\theta+7\cos^2\theta+2\cos\theta-3=0$

$(2\cos\theta-1)(\cos\theta+3)(\cos\theta+1)=0$

$\theta = 60°, 300°$ or $180°$

$\therefore$ it has exactly 3 solution

Page total

13. (a) Find the equation of a circle centred at C(0, 3) and passes through the points
    A(-4, 0) and B(4, 0). (3 marks)
    (b) The circle is drawn as shown in the diagram below. D is a point on the y-axis and the
    circle. If P is a moving point within the circle and its distance from D and the chord
    AB are the same. Describe and sketch the locus on the diagram below.
    (The equation of the locus is not required.) (4 marks)
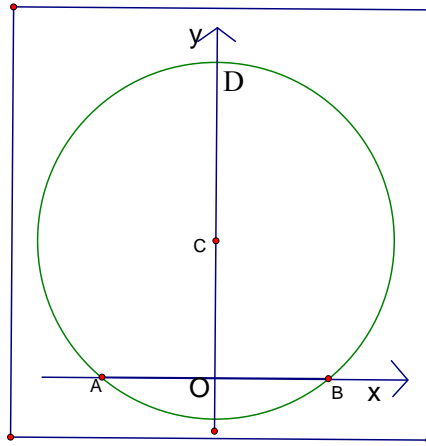
Radius of the circle=$\sqrt{(0-4)^2+(3-0)^2}$=5
Equation of circle:
$(x-0)^2+(y-3)^2=5^2$
$x^2+y^2-6y-16=0$

it will be part of a parabola with the two end points on the circle
and passing through (0,4)

Page total

Page total

14. A game is played by tossing a fair coin first and then a fair die. If a player tosses a tail, he will not be allowed to toss the die and has to leave; if the player gets a head, he can choose one of the two options:
    (i) He receives 10 dollars and leaves.
    (ii) He does not receive any money but can toss the die. If he gets the number 5, he receives 100 dollars; otherwise he receives no money.
    The player has to make a decision before playing the game. If you are the player, which option will you take? Explain your answer.

(6 marks)

The expected value when he chooses (1)
$=(0)(1/2)+10(1/2)$
$=5$

The expected value when he chooses (2)
$=(0)(1/2)+(1/2)[(5/6)(0)+(1/6)(100)]$
$=100/12$
$\because 100/12 > 5$
$\therefore$ I should choose (2)

Page total

END OF SECTION A

Page total