ST. PAUL'S COLLEGE

F.6 Internal Examination 2016 - 2017

Mathematics - Compulsory Part

Paper 1 Section B

B

Name:	
Class:	
Class Number:	
Group:	
Score of Section B:	**/ 35**

INSTRUCTIONS

1. Write your Name, Class, Class Number and Group in the spaces provided on Page 1.

2. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

3. Graph paper and supplementary answer sheets will be supplied on request. Write your Name and mark the question number box on each sheet, and fasten them with string INSIDE this book.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

6. The diagrams in this paper are not necessarily drawn to scale.

SECTION B (35 marks)

15. The mean of the test scores obtained by a class of students in a Mathematics test is 50 marks. The overall result is not satisfactory, so the test score of each student is adjusted such that each score is increased by 10% and then 5 marks are added.

(a) Find the mean of the test scores after the score adjustment.

(1 mark)

(b) Is the standard score of each student changed due to the score adjustment? Explain your answer.

(2 marks)

(a) the mean of the test scores after the score adjustment

$= 50 \times (1 + 10\%) + 5$

$= 60$

(b) Let x be the test score and δ be the standard deviation before the score adjustment.

New standard score

$= \frac{x(1.1)+5-[50(1.1)+5]}{1.1\delta}$

$= \frac{x-50}{\delta}$

$=$Orignal standard score

$\therefore$ The standard score remains unchanged.

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16. Let $f(x) = 2x^2 + ax + b$, where a and b are real numbers.

The roots of $f(x) = 0$ are $2 \pm 3i$.

(a) Find the values of a and b. (3 marks)

(b) The graph of $y = g(x)$ is obtained by reflecting the graph of $y = f(x)$ in the y-axis. Find the coordinates of the vertex of the graph of $y = g(x)$. (3 marks)

(a) $\because f(2+3i) = 0$

$2(2+3i)^2 + a(2+3i) + b = 0$

$2(-5+12i) + 2a + 3ai + b = 0$

$(-10+2a+b) + (24+3a)i = 0$

$\therefore 24 + 3a = 0$

$a = -8$

Sub $a = -8$ into $-10 + 2a + b = 0$

$-10 + 2(-8) + b = 0$

$b = 26$

(b) $f(x) = 2x^2 - 8x + 26$

$= 2(x-2)^2 + 18$

Vertex of the graph of $y = f(x)$ is $(2,18)$.

After reflecting the graph in the *y*- axis,

The vertex is $(-2,18)$

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17. Figure 17a shows an isosceles ΔABC with $AB = AC = 5$cm and $BC = 8$cm.

M is a mid-point on *BC*.

(a) Find *AM*. (2 marks)

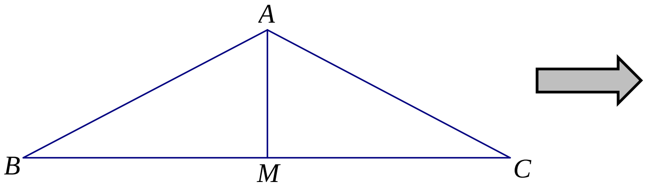

Figure 17a

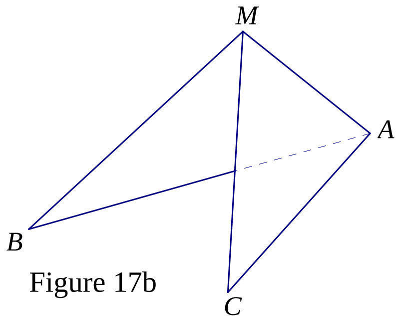

Figure 17b

The triangular paper card in (a) is folded along *AM* such than *AB* and *AC* lie on the horizontal ground as shown in Figure 17b. A tetrahedron is formed.

(b) Suppose $\angle BMC = 30°$.

(i) Find the area of ΔBMC.

(ii) Find the volume of the tetrahedron.

(4 marks)

(c) (i) Describe how the volume of the tetrahedron varies when $\angle BMC$ increases from $30°$ to $120°$. Explain your answer.

(ii) Find the angle between the planes BMC and AMB.

(3 marks)

(a) By prop. of Isos. Δ, $AM \perp BC$

$AM^2 + (\frac{8}{2})^2 = 5^2$ (Pyth. Thm.)

$AM = 3$ cm

(b) (i) area of ΔBMC

$= \frac{1}{2}(4)(4)\sin 30°$

$= 4$ cm^2

(ii) volume of the tetrahedron

$= \frac{1}{3}(4)(3)$ from (b)(i)

$= 4$ cm^3

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(c) (i) volume of the tetrahedron

$$= \frac{1}{3}\left(\frac{1}{2} \cdot 4 \cdot 4 \cdot \sin \angle BMC\right)(3)$$

$$= 8 \sin \angle BMC$$

As the arms of the base triangle and the height of the tetrahedron are constant, the volume of the tetrahedron varies as $\sin \angle BMC$.

When $\angle BMC$ increases from 30° to 90°, $\sin \angle BMC$ increases from $\frac{1}{2}$ to 1. When $\angle BMC$ increases from 90° to 120°, $\sin \angle BMC$ decreases from 1 to $\frac{\sqrt{3}}{2}$.

The volume of the tetrahedron increases from 4 cm^3 to the maximum 8 cm^3 then decreases to $4\sqrt{3}\text{cm}^3$.

(ii) the angle between the planes BMC and AMB is 90°.

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18. In Figure 18, a straight line L cuts the y-axis and the x-axis at points $A(0, 9)$ and B respectively. The inclination of L is $120°$. P is a point on AB such that $AP:PB = 2:1$.

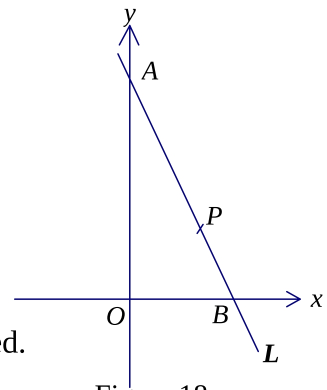

Figure 18

(a) (i) Find the equation of L.

(ii) Find the x-intercept of L.

(iii) Find the coordinates of P.

(5 marks)

(b) A circle passing through O, A and P is constructed.

(i) Find the equation of the circle.

(ii) Someone claims that $Q(-\frac{\sqrt{3}}{2}, \frac{9}{2})$ is the orthocenter of ΔAPO.

Is the claim correct? Explain your answer.

(6 marks)

(a) (i) slope of $L = \tan 120° = -\sqrt{3}$

The equation of L is $y = -\sqrt{3}x + 9$.

(ii) Put $y = 0$ in (a), $x = 3\sqrt{3}$

(iii) The coordinates of P

$= \left(\frac{0\times1+3\sqrt{3}\times2}{1+2}, \frac{9\times1+0\times2}{1+2}\right)$

$= (2\sqrt{3}, 3)$

(b) (i) Let $x^2 + y^2 + Dx + Ey + F = 0$ be the equation of the circle.

Sub $(0,0)$, $(0,9)$ and $(2\sqrt{3}, 3)$ into the equation.

$F = 0$

$9^2 + 9E + 0 = 0$

$E = -9$

$(2\sqrt{3})^2 + (3)^2 + D(2\sqrt{3}) - 9(3) = 0$

$D = \sqrt{3}$

$\therefore x^2 + y^2 + \sqrt{3}x - 9y = 0$

(ii) slope of $QP = \frac{3-\frac{9}{2}}{2\sqrt{3}+\frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{5} \neq 0$which is not perpendicular to OA

$\therefore Q$ is not the orthocenter of ΔAPO.

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19. For the Anniversary Carnival, 6 different stalls will be set up by 18 student helpers, 3 students from F6A, 6 from F6B and 9 from F6C. The students will be divided into 6 groups, each of 3 students, for each stall.

(a) How many different groups can be formed?

(2 marks)

(b) Find the probability that the 3 students from F6A are in the same group?

(2 marks)

(c) Find the probability that the 6 students from F6B are in different groups?

(2 marks)

(a) No of groups $= C_3^{18} \cdot C_3^{15} \cdot C_3^{12} \cdot C_3^{9} \cdot C_3^{6} \cdot C_3^{3}$

$= 1.37 \times 10^{11}$

(b) the probability that the 3 students from F6A are in the same group

$= \frac{C_1^6 \cdot C_3^3 \cdot C_3^{15} \cdot C_3^{12} \cdot C_3^9 \cdot C_3^6 \cdot C_3^3}{C_3^{18} \cdot C_3^{15} \cdot C_3^{12} \cdot C_3^9 \cdot C_3^6 \cdot C_3^3}$

$= \frac{1}{136}$

(c) the probability that the 6 students from F6B are in different groups

$= \frac{6! \cdot C_2^{12} \cdot C_2^{10} \cdot C_2^8 \cdot C_2^6 \cdot C_2^4 \cdot C_2^2}{C_3^{18} \cdot C_3^{15} \cdot C_3^{12} \cdot C_3^9 \cdot C_3^6 \cdot C_3^3}$

$= \frac{243}{6188}$

End of Paper

Answers written in the margins will not be marked.