**ST. PAUL'S COLLEGE**

**FORM 6 INTERNAL EXAMINATION 2021 - 2022**

# MATHEMATICS Compulsory Part

## PAPER 1

## Section A2

## Marking Scheme

2¼ hours

This paper must be answered in English.

A2

| | |
|---|---|
| Name | |
| Class | ( ) |
| Group | G1 FBL G2 LMW G3 WHP<br>G4 TH G5 PSK G6 LTN<br>G7 HL |

**INSTRUCTIONS**

1. Write your Name, Class and Class number in the spaces provided on the right. Circle your Group Number.
2. This paper consists of THREE sections, A(1), A(2) and B.
3. Attempt **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your Name, Class and Class number in the spaces provided, mark the question number box, and fasten them with string INSIDE this book.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
7. The diagrams in this paper are not necessarily drawn to scale.

| Question No. | Marks |
|---|---|
| 10 | /6 |
| 11 | /6 |
| 12 | /7 |
| 13 | /7 |
| 14 | /9 |
| Total | /35 |

**SECTION A(2) (35 marks)**

10. The cost of design for an apartment of area $d\,\text{m}^2$ is \$$C$. $C$ is partly constant and partly varies as $(d+10)^2$. When $d=15$, $C=5500$. When $d=20$, $C=7700$.

 (a) Find the cost of design of an apartment of area $17.5\,\text{m}^2$. (4 marks)

 (b) For $d \ge 100$, a 10% discount will be offered. Peter has two apartments. He claims that the design cost for his apartment of area $100\,\text{m}^2$ is lower than that of the one of area $95\,\text{m}^2$. Do you agree with him? Explain your answer. (2 marks)

(a) Let $C = a + b(d+10)^2$ where $a$ and $b$ are non-zero constants. 1A

$$\begin{cases} a + b(25)^2 = 5500 \cdots (1) \\ a + b(30)^2 = 7700 \cdots (2) \end{cases}$$ 1M

$(2)-(1): b = 8$

$a = 500$

$\therefore C = 500 + 8(d+10)^2$

When $d = 17.5$, $C = 500 + 8(17.5+10)^2 = 6550$. 1M

Required Cost=\$6550 1A

(b)

Cost for $d=100$ :

$\left[500 + 8(d+10)^2\right](1-10\%) = \$87570$ 1M: (1+10%)

Cost for $d = 95$ :

$500 + 8(95+10)^2 = \$88700 > \$87570$

$\therefore$ Yes, I agree. 1A f.t.

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11. A sphere of radius 6 cm is melted and recast into two similar cones. The ratio of the base radii of the two cones is 1 : 2 .

(a) Find the volume of the larger cone, express your answer in terms of $\pi$ . (3 marks)

(b) If the ratio of the base radius to the height of the cones is 2 : 3, find the difference between the curved surface areas of the two cones. (3 marks)

(a) Let the volume of the smaller cone be $v$ cm$^3$.

Then the volume of the larger cone is $2^3 v$ cm$^3$ = $8v$ cm$^3$ 1M: ratio of volume

$$v+8v=\frac{4}{3}\pi 6^3$$ 1M: volume of sphere

$$v=32\pi$$

$\therefore$ The required volume is $32\pi(8)=256\pi$ cm$^3$. 1A

(b) Let $2r$ cm be the base radius of the larger cone.

Then the height of the larger cone is $3r$ cm.

$$\frac{1}{3}\pi(2r)^2(3r)=256\pi$$ 1M: volume of cone

$$r^3=64$$

$$r=4$$

Difference required

$$=\pi(8)\sqrt{12^2+8^2}-\pi(4)\sqrt{6^2+4^2}$$ 1M: $\pi rl$

$$=\pi(8)\sqrt{208}-\pi(4)\sqrt{52}$$

$$=24\pi\sqrt{13}$$

$$\approx 271.8521616\cdots$$

$$\approx 272 \text{ cm}^2$$ 1A

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12. Straight lines $L_1$ and $L_2$ are perpendicular to each other. They intersect at a point $P$. The equation of the straight line $L_1$ is $3x+4y+12=0$ and $L_2$ passes through $Q(3,26)$.

(a) Find the equation of $L_2$. (3 marks)

(b) Given that $L_2$ cuts the $y$-axis at $S$ and $R$ is a moving point such that $PR = RS$. Denote the locus of $R$ by $\Gamma$.

(i) Describe the geometric relationship between $\Gamma$ and $PS$.

(ii) Find the equation of $\Gamma$.

(4 marks)

(a)

$L_1 : 3x+4y+12=0$

$y=-\frac{3}{4}x-4$

Slope of $L_1 : -\frac{3}{4}$

Slope of $L_2 : \frac{4}{3}$ 1M

Equation of $L_2$ :

$y-26=\frac{4}{3}(x-3)$ 1M

$x-3y+66=0$ 1A

$y=\frac{4}{3}x+22$

(b) (i) $\Gamma$ is the perpendicular bisector of $PS$. 1A

(ii)

$L_2 : x-3y+66=0$

$y=\frac{4}{3}x+22$

$\therefore S(0,22)$ 1M: either

$\begin{cases} L_1 : 3x+4y+12=0 \\ L_2 : x-3y+66=0 \end{cases}$

$\therefore P(-12,6)$

Mid-point of $P$ and $S$: $\left(\frac{0-12}{2},\frac{22+6}{2}\right)=(-6,14)$. 1M

Equation of $\Gamma$ :

$y-14=-\frac{3}{4}(x+6)$

$3x+4y-38=0$ 1A

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Page total

13. The stem-and-leaf diagram below shows the distribution of the scores of some students in a test.

| Stem (tens) | Leaf (units) | | | | |
|---|---|---|---|---|---|
| 4 | $x$ | | | | |
| 5 | 3 | 6 | | | |
| 6 | 1 | 3 | 7 | | |
| 7 | 2 | 5 | 7 | 7 | |
| 8 | 0 | $n$ | $n$ | 6 | 8 |
| 9 | $y$ | | | | |

It is given that the inter-quartile range of the distribution is 18 marks.

(a) Find $n$. (2 marks)

(b) It is given that the mean of the distribution is 72.5 marks and the range of the distribution does not exceed 49 marks. Find

(i) $x$ and $y$,

(ii) the greatest possible standard deviation of the distribution. (5 marks)

(a)

$$80+n-\frac{61+63}{2}=18$$ 1M

$n=0$ 1A

(b) (i)

40+2(50)+3(60)+4(70)+5(80)+90+$x$+3+6+1+3+7+2+5+7+7+6+8+$y$=72.5$\times$16 1M

$\therefore x+y=15$

$90+y-(40+x)\le 49$ 1M

$\therefore y\le x-1$

$\therefore \begin{cases} x=8 \\ y=7 \end{cases}$ or $\begin{cases} x=9 \\ y=6 \end{cases}$ 1A both

b(ii) Note that $(48-72.5)^2+(97-72.5)^2>(49-72.5)^2+(96-72.5)^2$. 1M

The standard deviation when $\begin{cases} x=8 \\ y=7 \end{cases}$ is the greatest and

its value is 13.1 marks (cor. to 3 sig. fig.) 1A

The standard deviations when $\begin{cases} x=8 \\ y=7 \end{cases}$ and $\begin{cases} x=9 \\ y=6 \end{cases}$ are 13.1 marks and 12.9 marks respectively.

So the greatest possible standard deviation is 13.1 marks.

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14. Let $\mathrm{f}(x)$ be a cubic polynomial. When $\mathrm{f}(x)$ is divided by $2x^2+3x+1$ the remainder is $4x+4$.

(a) Find the remainder when $\mathrm{f}(x)$ is divided by $2x+1$. (2 marks)

(b) When $\mathrm{f}(x)$ is divided by $2x^2-x-1$, the remainder is $kx+12$.

(i) Find $k$.

(ii) When $\mathrm{f}(x)$ is divided by $x$, the remainder is $7$. Find $\mathrm{f}(x)$.

(iii) Someone claims that all the roots of $\mathrm{f}(x)+7x+7=0$ are real roots. Do you agree? Explain your answer. (7 marks)

(a) $\mathrm{f}(x)=Q(x)(2x^2+3x+1)+4x+4$ where $Q(x)$ is a polynomial.

i.e. $\mathrm{f}(x)=Q(x)(2x+1)(x+1)+4(x+1)$

Remainder

$=\mathrm{f}\left(-\frac{1}{2}\right)$ 1M

$=4\left(-\frac{1}{2}+1\right)$

$=2$ 1A

(b) (i) $\mathrm{f}(x)=P(x)(2x^2-x-1)+kx+12$ where $P(x)$ is a polynomial.

i.e. $\mathrm{f}(x)=P(x)(2x+1)(x-1)+kx+12$

$\mathrm{f}\left(-\frac{1}{2}\right)=2$

$k\left(-\frac{1}{2}\right)+12=2$

$k=20$ 1A

(ii) Let $Q(x)=ax+b$ and $P(x)=ax+c$ where $a,b$ and $c$ are constants.

$\mathrm{f}(x)=(2x^2+3x+1)(ax+b)+(4x+4)\equiv(2x^2-x-1)(ax+c)+(20x+12)$

$\mathrm{f}(0)=7$

$b+4=7=-c+12$ 1M for either

$b=3,\quad c=5$

Comparing coefficient of $x$ (or $x^2$), 1M

$a=1$

$\mathrm{f}(x)=(x+1)(2x^2+7x+7)$ or $f(x)=2x^3+9x^2+14x+7$ 1A

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(iii) $\mathrm{f}(x)+7x+7 = (x+1)(2x^2+7x+14)$

$\mathrm{f}(x)+7x+7=0$

$(x+1)(2x^2+7x+14)=0$ 1M

$x+1=0$ or $2x^2+7x+14=0$

For $2x^2+7x+14=0$,

Discriminant $=7^2-4(2)(14)$ 1M

$=-63<0$

The roots of $2x^2+7x+14=0$ are not real.

Not all the roots of $\mathrm{f}(x)+7x+7=0$ are real.

I disagree. 1M f.t.

**End of Section A2**

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