ST. PAUL'S COLLEGE

FORM 6 INTERNAL EXAMINATION 2022 - 2023

MATHEMATICS Compulsory Part

PAPER 1

Section A2

Question-Answer Book

2¼ hours

This paper must be answered in English.

INSTRUCTIONS

1. Write your Name, Class and Class number in the spaces provided on the right. Circle your Group Number.

2. This paper consists of THREE sections, A(1), A(2) and B.

3. Attempt **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Graph paper and supplementary answer sheets will be supplied on request. Write your Name, Class and Class number in the spaces provided, mark the question number box, and fasten them with string INSIDE this book.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

A2

Name	
Class	( )
Group	G1 LTN G2 PSK G3 LMW G4 HL G5 YKC G6 LTN G7 HL

Question No.	Marks
10	
11	
12	
13	
14	
Total	

SECTION A(2) (35 marks)

10. When Chris sells n handbags in a month, her income in that month is \$$S$. It is given that S is a sum of two parts, one part varies directly as n and the other part varies directly as n^2.
When $n=12$, $S=7\ 920$; when $n=16$, $S=12\ 160$.

(a) When Chris sells 24 handbags in a month, find her income in that month. (4 marks)

(b) If Chris's income in a month is \$17 200, find the number of handbags she sells in that month. (2 marks)

Solution

(a) Let $S=an^2+bn$, where a and b are non-zero constants. 1A

So, we have $a(12)^2+b(12)=7920$ and $a(16)^2+b(16)=12160$. 1M

Solving, we have $a=25$ and $b=360$. 1A

The required income
$=25(24)^2+360(24)$
$=\$23\ 040$ 1A

(b) $25n^2+360n=17200$ 1M

$5n^2+72n-3440=0$

$(5n+172)(n-20)=0$

$n=-34.4$ (rej.) or $n=20$ 1A

Thus, the number of handbags she sells in that month is 20.

Answers written in the margins will not be marked.

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11. The stem-and-leaf diagram below shows the distribution of the weights (in kg) of the students in a class.

Stem (tens)	Leaf (units)					
5	a	7	8			
6	a	a	8	9		
7	0	2	2	5	5	7
8	3	b				

It is given that the mean and the range of the above distribution are 69 kg and 35 kg respectively.

(a) Find the median and the inter-quartile range of the above distribution. (5 marks)

(b) A new student now joins the class. The mode of the distribution becomes 75 kg. Find the standard deviation of the distribution. (2 marks)

Solution

(a) The median

$= 70\,\text{kg}$ 1A

Note that $\dfrac{50+a+57+58+2\times(60+a)+68+69+70+72\times2+75\times2+77+83+80+b}{15}=69$. 1M

Therefore, we have $3a+b=9$.

Also note that $(80+b)-(50+a)=35$, we have $b-a=5$. 1M

Solving, we have $a=1$ and $b=6$. 1A

The inter-quartile range

$=75-61$

$=14\,\text{kg}$ 1A

(b) Newly added student's weight is 75 kg. 1M

New standard deviation

$\approx 9.33\,\text{kg}$ 1A

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12. The base radius of the solid hemisphere X and the height of the solid right circular cylinder Y are equal. The curved surface areas of X and Y are 648π cm^2 and 432π cm^2 respectively.

(a) Express, in term of π, the volume of Y. (3 marks)

(b) X and Y are melted and recast into two similar solid right circular cones. Denote these two circular cones by A and B. It is given that the base radii of A and B are 12 cm and 24 cm respectively. Henry claims that the curved surface area of A is at least 720 cm^2. Do you agree? Explain your answer. (4 marks)

Solution

(a) Let r cm and R cm be the base radius of X and Y respectively.

$2\pi r^2 = 648\pi$ 1M

$r = 18$

$2\pi rR = 432\pi$

$36R = 432$

$R = 12$

Volume of Y

$= \pi(12)^2(18)$ 1M

$= 2592\pi$ cm^3 1A

(b) Volume of X

$= \frac{2}{3}\pi(18)^3$

$= 3888\pi$

Let h cm and $2h$ cm be the height of A and B respectively. 1M

$\frac{1}{3}\pi(12)^2 h + \frac{1}{3}\pi(24)^2(2h) = 2592\pi + 3888\pi$ 1M

$h = 15$

Slant height of A

$= \sqrt{15^2 + 12^2} = 3\sqrt{41}$ cm

The curved surface area of A

$= \pi(12)(\sqrt{369})$ 1M

≈ 724.1762903 cm^2

> 720 cm^2

Thus, the claim is agreed. 1f.t.

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13. The polynomial $p(x)$ is divisible by x^2-1. When $p(x)$ is divided by x^2-2x+2, the quotient and the remainder are $10x^2+ax-19$ and $bx+41$ respectively, where a and b are constants.

(a) Find a and b. (4 marks)

(b) How many rational roots does the equation $p(x)=0$ have? Explain your answer. (3 marks)

Solution

(a) Let $p(x)=(x^2-2x+2)(10x^2+ax-19)+bx+41$. 1M

Note that $p(1)=p(-1)=0$ 1M(both)

Hence, we have $1(10+a-19)+b+41=0$ and $5(10-a-19)-b+41=0$

So, we have $a+b=-32$ and $5a+b=-4$, 1A(either one)

Solving, we have $a=7$ and $b=-39$. 1A

(b) $p(x)$

$=(x^2-2x+2)(10x^2+7x-19)-39x+41$ (by (a))

$=10x^4-13x^3-13x^2+13x+3$

$=(x-1)(10x^3-3x^2-16x-3)$ 1M

$=(x-1)(x+1)(10x^2-13x-3)$

$=(x-1)(x+1)(2x-3)(5x+1)$

$p(x)=0$

$(x-1)(x+1)(2x-3)(5x+1)=0$

$x=1$ or $x=-1$ or $x=\frac{3}{2}$ or $x=-\frac{1}{5}$ 1M

Thus, the equation $p(x)=0$ has 4 rational roots. 1A f.t.

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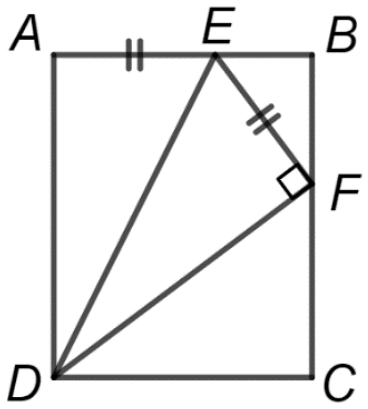
14. In the figure, $ABCD$ is a rectangle. E and F are points on AB and BC respectively such that $EA = EF$ and $\angle EFD = 90°$.



(a) Prove that

(i) $\Delta EAD \cong \Delta EFD$

(ii) $\Delta EBF \sim \Delta FCD$ (4 marks)

(b) Suppose that $AD = 30$ cm and $DC = 24$ cm.

(i) Find the length of EF.

(ii) Is there a point G lying on DE such that the distance between F and G is less than 13 cm? Explain your answer. (4 marks)

Solution

(ai)

$EA = EF$	(given)
$ED = ED$	(common side)
$\angle EAD = 90°$	(properties of rectangle)
$\angle EAD = \angle EFD = 90°$	
$\Delta EAD \cong \Delta EFD$	(RHS)

Marking Scheme:

Case 1	Any correct proof with correct reasons	2
Case 2	Any correct proof without reasons.	1

(aii)

$\angle EBF = \angle FCD = 90°$	(prroperties of rectangle)
$\angle BEF = 180° - \angle EBF - \angle EFB$	($\angle$ sum of Δ)
$\angle BEF = 90° - \angle EFB$	
$\angle EFD = 90°$	(given)
$\angle CFD = 180° - \angle EFD - \angle EFB$	(adj. $\angle s$ on st. line)
$\angle CFD = 90° - \angle EFB$	
$\angle BEF = \angle CFD$	
$\angle BFE = \angle CDF$	($\angle$ sum of Δ)
$\Delta EBF \sim \Delta FCD$	(AAA)

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Marking Scheme:

Case 1	Any correct proof with correct reasons	2
Case 2	Any correct proof without reasons.	1

(bi) By (ai), we have $FD = AD = 30\,\text{cm}$.

$FC = \sqrt{30^2 - 24^2} = 18\,\text{cm}$

$BF = DC - FC = 12\,\text{cm}$

By (aii), we have $\dfrac{EF}{FD} = \dfrac{BF}{CD}$ 1M

$\dfrac{EF}{30} = \dfrac{12}{24}$

$EF = 15\,\text{cm}$ 1A

(bii) DE

$= \sqrt{15^2 + 30^2}$

$= 15\sqrt{5}$

The shortest distance from F to DE

$= \dfrac{(15)(30)}{15\sqrt{5}}$ 1M

$\approx 13.41640786\,\text{cm}$

$> 13\,\text{cm}$

Thus, there is no point G lying on DE such that the distance between F and G is less than 13 cm. 1A f.t.

End of Section A2

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