ST. PAUL'S COLLEGE

FORM 6 INTERNAL EXAMINATION 2022 - 2023

MATHEMATICS Compulsory Part

PAPER 1

Section B

Question-Answer Book

2¼ hours

This paper must be answered in English.

INSTRUCTIONS

1. Write your Name, Class and Class number in the spaces provided on the right. Circle your Group Number.

2. This paper consists of THREE sections, A(1), A(2) and B.

3. Attempt **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Graph paper and supplementary answer sheets will be supplied on request. Write your Name, Class and Class number in the spaces provided, mark the question number box, and fasten them with string INSIDE this book.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

B

Name	
Class	( )
Group	G1 LTN G2 PSK G3 LMW G4 HL G5 YKC G6 LTN G7 HL

Question No.	Marks
15	
16	
17	
18	
19	
Total	

SECTION B (35 marks)

15. If 7 adults and 4 children randomly form a queue, find the probability that no children are next to each other. (3 marks)

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Page total

16. Let $f(x) = x^2 - 8kx + 6x + 16k^2 - 19k + 11$, where k is a constant.

(a) Using the method of completing the square, express, in terms of k, the coordinates of the vertex of the graph of $y = f(x)$. (2 marks)

(b) On the same rectangle coordinate system, O is the origin. Let P and Q be the vertex of the graph of $y = f(x)$ and the vertex of the graph of $y = f(x-9)+6$ respectively. Is it possible that $(-9,12)$ is the orthocentre of ΔOPQ? Explain your answer.

(3 marks)

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17. Let $A(n)$ be the nth term of a geometric sequence, where n is a positive integer.
It is given that $A(3) = 48$ and $A(6) = 3072$.

(a) Find $A(n)$. (2 marks)

(b) Let $B(n) = \log A(n)$. Find the least value of k such that the sum of the first k terms of the sequence $B(n)$ is greater than 2023. (4 marks)

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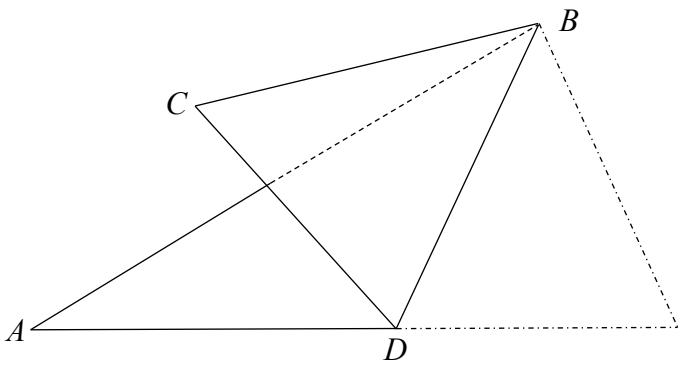
18. (a) A triangular paper card ABC with $AB = 20$ cm, $\angle ACB = 56°$ and $\angle ABC = 82°$. Find BC. (2 marks)

(b) The paper card ABC described in (a) is given. Let D be a point on AC such that BD is the angle bisector of $\angle ABC$. As shown, the paper card is folded along BD such that $AC = 10$ cm.



(i) Find $\angle ABC$.

(ii) Someone claims that the angle between ΔABD and ΔCBD is greater than $45°$. Do you agree? Explain your answer.

(6 marks)

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19. The equation of the circle C is $x^2+y^2-400x-300y+40000=0$ and the equation of the straight line L is $y=mx$, where m is a constant. It is given that C and L intersect at two distinct points P and Q. Denote the mid-point of P and Q by M.

(a) (i) Find the range of values of m.

(ii) Show that the y-coordinate of M is $\dfrac{50m(3m+4)}{1+m^2}$.

(4 marks)

(b) Two tangents are drawn from the origin to the circle C. The points of contact are A and B respectively, where the y-coordinate of A is positive.

(i) Find the equation of the perpendicular bisector of AB.

(ii) Find the equation of the perpendicular bisector of BM in terms of m. Hence, or otherwise, find the equation of the circle passing through A, B and M.

(iii) If m varies in the range in (a)(i), denote the locus of the moving point M by Γ. Someone claims that the length of Γ is greater than 320. Do you agree? Explain your answer.

(9 marks)

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End of Paper

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