ST. PAUL'S COLLEGE
FORM 6 INTERNAL EXAMINATION 2022 - 2023
MATHEMATICS Compulsory Part
PAPER 2 SOLUTION

Section A

1. $\dfrac{(9^{2n-1})(81^{n+2})}{27^{2n}} =$

A. 3^2.

B. 3^{n+1}.

C. 3^{2n+2}.

D. 3^{2n+6}.

Method 1

$$\frac{(9^{2n-1})(81^{n+2})}{27^{2n}}$$
$$= \frac{(3^{2(2n-1)})(3^{4(n+2)})}{3^{3(2n)}}$$
$$= 3^{2(2n-1)+4(n+2)-3(2n)}$$
$$= 3^{2n+6}$$

Method 2 (true for all n)

Putting $n = 0$, using a calculator

Q $= \dfrac{(9^{-1})(81^2)}{27^0} =$ **729**

A = 9
B = 3
C = 9
D = 729

2. It is given that x is a real number. If x is rounded up to 3 significant figures, then the result is 234. Find the range of values of x.

A. $233 < x \le 234$

B. $234 \le x < 235$

C. $233.5 \le x < 234.5$

D. $233.5 \le x \le 234$

Note that *A includes D*

Left end when rounded up	Right end when rounded up
234	234
234	235
234	235
234	234

3. If $y = \dfrac{2x+3}{2-x}$, then $x =$

A. $\dfrac{2y-3}{y+2}$.

B. $\dfrac{3-2y}{y+2}$.

C. $\dfrac{2y+3}{2-y}$.

D. $\dfrac{2y-3}{2-y}$.

Method 1

$$y = \frac{2x+3}{2-x}$$
$$y(2-x) = 2x+3$$
$$2y - xy = 2x+3$$
$$2y-3 = 2x+xy$$
$$2y-3 = x(2+y)$$
$$x = \frac{2y-3}{y+2}$$

Method 2 (true for all x, y)

When $x = 1$,

$y = \dfrac{2+3}{2-1} = 5$

Putting $y = 5$,

$A = \dfrac{2(5)-3}{5+2} = 1$

$B = \dfrac{3-2(5)}{5+2} = -1$

$C = \dfrac{2(5)+3}{2-5} = -\dfrac{13}{3}$

$D = \dfrac{2(5)-3}{2-5} = -\dfrac{7}{3}$

4. $x^2 - y^2 - 2xz + z^2 =$

A. $(x+y-z)(x+y+z)$.

B. $(x+y-z)(x-y-z)$.

C. $(x-y+z)(x+y-z)$.

D. $(x-y+z)(x-y-z)$.

Method 1	Method 2 (true for all x, y, z)
$x^2-y^2-2xz+z^2$ $=(x^2-2xz+z^2)-y^2$ $=(x-z)^2-y^2$ $=[(x-z)+y][(x-z)-y]$ $=(x+y-z)(x-y-z)$	When $y=0$ $Q=x^2-y^2-2xz+z^2$ $=x^2-2xz+z^2=(x-z)^2$ $=$ B

5. $\dfrac{1}{x^2-2x+1}-\dfrac{1}{x^2-3x+2}=$

A. $-\dfrac{3}{(x-1)(x+1)}$.

B. $-\dfrac{3}{(x-1)^2(x-2)}$.

C. $-\dfrac{1}{(x-1)^2(x-2)}$.

D. $\dfrac{2x-3}{(x-1)^2(x-2)}$.

Method 1	Method 2 (true for all x)
$\dfrac{1}{x^2-2x+1}-\dfrac{1}{x^2-3x+2}$ $=\dfrac{1}{(x-1)^2}-\dfrac{1}{(x-1)(x-2)}$ $=\dfrac{(x-2)-(x-1)}{(x-1)^2(x-2)}$ $=\dfrac{-1}{(x-1)^2(x-2)}$	When $x=0$ $Q=\dfrac{1}{1}-\dfrac{1}{2}=\dfrac{1}{2}$ $A=-\dfrac{3}{(-1)(1)}=3$ $B=-\dfrac{3}{(-1)^2(-2)}=\dfrac{3}{2}$ $C=-\dfrac{1}{(-1)^2(-2)}=\dfrac{1}{2}$ $D=\dfrac{-3}{(-1)^2(-2)}=\dfrac{3}{2}$

6. If $f(x) = 3x^2 - 2x + k$. If k is a constant, then $f(k+1) - f(k-1) =$

A. -4.

B. $k + 8$.

C. $10k$.

D. $12k - 4$.

Method 1 (true for any k)	Method 2
Putting $k = 1$, $f(x)=3x^2-2x+1$ $f(k+1)-f(k-1)=f(2)-f(0)$ $f(2)=3(2)^2-2(2)+1=9$ $f(0)=3(0)^2-2(0)+1=1$ $f(k+1)-f(k-1)=9-1=8$ A = –4 B = 9 C = 10 D = 8	$f(k+1)=3(k+1)^2-2(k+1)+k$ $=3(k^2+2k+1)-2k-2+k$ $=3k^2+6k+3-2k-2+k$ $=3k^2+5k+1$ $f(k-1)=3(k-1)^2-2(k-1)+k$ $=3(k^2-2k+1)-2k+2+k$ $=3k^2-6k+3-2k+2+k$ $=3k^2-7k+5$ $f(k+1)-f(k-1)$ $=3k^2+5k+1-3k^2+7k-5$ $=12k-4$

7. If A and B are constants such that $Ax(x+4)+B(x+4)+C \equiv 3x^2+11x-40$, then $C =$

A. -96.

B. -48.

C. -40.

D. -36.

Method 1 (identity, true for any x)	Method 2
Putting $x=-4$, $A(0)+B(0)+C \equiv 3(-4)^2+11(-4)-40$ $C=-36$	$Ax(x+4)+B(x+4)+C$ $= Ax^2+4Ax+Bx+4B+C$ Comparing corresponding terms, $A=3$, $4A+B=11$, $4B+C=-40$ $B=-1$ $C=-36$

8. Let $g(x)=8x^3+ax^2-5$, where a is a constant. When $g(x)$ is divided by $(2x-1)$, the remainder is -2, then $g(1) =$

A. -2.

B. 8.

C. 11.

D. 19.

$$g\left(\frac{1}{2}\right)=-2$$
$$8\left(\frac{1}{2}\right)^3+a\left(\frac{1}{2}\right)^2-5=-2$$
$$1+\frac{a}{4}-5=-2$$
$$a=8$$
$$g(x)=8x^3+8x^2-5$$
$$g(1)=8+8-5=11$$

9. The solution of $\frac{5y+3}{2} \le 3y+2 < 2y+5$ is

A. $-1 \le y < 3$.

B. $-1 < y \le 3$.

C. $3 \le y < 7$.

D. $3 < y \le 7$.

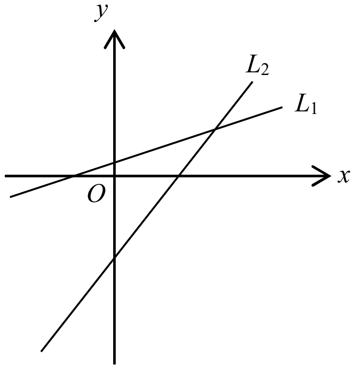
$$\frac{5y+3}{2} \le 3y+2 < 2y+5$$
$$\frac{5y+3}{2} \le 3y+2 \quad \text{and} \quad 3y+2 < 2y+5$$
$$5y+3 \le 6y+4 \quad \text{and} \quad y<3$$
$$-1 \le y \quad \text{and} \quad y<3$$
$$-1 \le y < 3$$

10. In the figure, the equations of the straight lines L_1 and L_2 are $x+ay+b=0$ and $x+cy+d=0$ respectively. Which of the following are true?

I. $a<c$

II. $b>0$

III. $ad>bc$



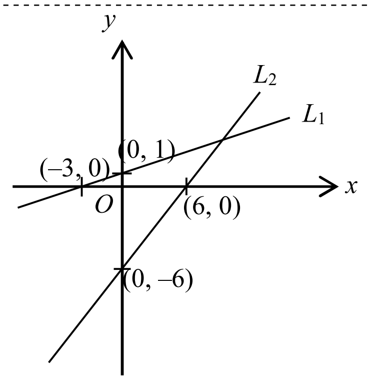
A. I and II only

B. I and III only

C. II and III only

D. I, II and III

Method 1



L_1: $y=\frac{1}{3}x+1$

$x-3y+3=0$ vs. $x+ay+b=0$

$a=-3$, $b=3$

L_2: $y=x-6$

$x-y-6=0$ vs. $x+cy+d=0$

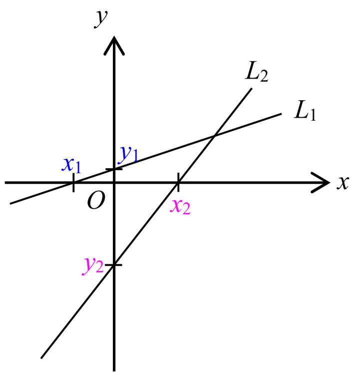
$c=-1$, $d=-6$

I. ($-3<-1$?) True

II. ($3>0$?) True

III. ($(-3)(-6)>(3)(-1)$?) True

Method 2



L_1: $x+ay+b=0$

x_1 = x-intercept $=-b<0$

y_1 = y-intercept $=-\frac{b}{a}>0$

m_1 = slope $=-\frac{1}{a}>0$

$\therefore a<0$, $b>0$ **(II is true)**

L_2: $x+cy+d=0$

x_2 = x-intercept $=-d>0$

y_2 = y-intercept $=-\frac{d}{c}<0$

m_2 = slope $=-\frac{1}{c}>0$

$\therefore c<0$, $d<0$

$m_2>m_1$ $\therefore -\frac{1}{c}>-\frac{1}{a}$

$\frac{1}{c}<\frac{1}{a}$

$a<0$ and $c<0$

$ac>0$

$\therefore a<c$ **(I is true)**

$ad>0>bc$ **(III is true)**

11. A sum of \$$x$ is deposited at an interest rate of 6% per annum for 4 years, compounded monthly. If the interest is \$12000, find the value of x (correct to the nearest integer).

A. 44 323

B. 44 364

C. 45 718

D. 50 000

$$x\left[\left(1+\frac{6\%}{12}\right)^{48}-1\right]=12000$$

$x = 44364$

12. The actual area of a field is 0.54 km^2. If the area of the field on a map is 216 cm^2, then the scale of the map is

A. 1 : 250 .

B. 1 : 500 .

C. 1 : 5 000 .

D. 1 : 25 000 000 .

Areas = $216\text{ cm}^2 : 0.54\text{ km}^2$

Dividing by 6,

Areas = $36\text{ cm}^2 : 0.09\text{ km}^2$

Taking square roots,

Lengths = 6 cm : 0.3 km

= 6 : 0.3 × 1000 × 100

= 1 : 5 000

13. If y varies directly as the square of x and inversely as the square root of w, which of the following is a constant?

A. $\dfrac{x^4y^2}{w}$

B. $\dfrac{x^4}{wy^2}$

C. $\dfrac{x^2y}{w^2}$

D. $\dfrac{x^2}{w^2y}$

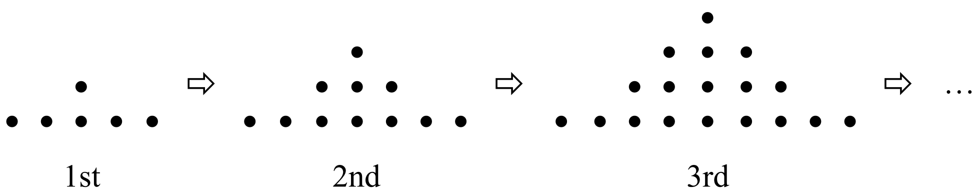
$$y=\frac{kx^2}{\sqrt{w}}$$

$$y^2=\frac{k^2x^4}{w}$$

$$\frac{wy^2}{x^4}=k^2$$

$$\frac{x^4}{wy^2}=\frac{1}{k^2}$$

14. In the figure, the 1st pattern consists of 6 dots. For any positive integer n, the $(n+1)^{\text{th}}$ pattern is formed by adding $(2n+3)$ dots to the n^{th} pattern. Find the number of dots in the 7^{th} pattern.



A. 42

B. 51

C. 66

D. 83

n	1	2	3	4	5	6
$2n+3$	5	7	9	11	13	15

T_1 T_2 T_3 T_4 T_5 T_6 T_7

6 + 5 + 7 + 9 + 11 + 13 + 15 = 66

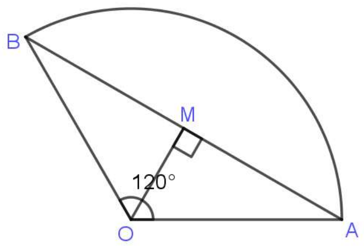
15. The area of the sector OAB is $3\pi \text{ cm}^2$, where O is the centre of the sector OAB. If the angle subtended is $120°$, which of the following is **not** true?

A. The radius of the sector OAB is 3 cm.

B. The length of the arc AB is 2π cm.

C. The area of ΔOAB is $\frac{9\sqrt{3}}{2}\text{ cm}^2$.

D. The length of AB is $3\sqrt{3}$ cm.



Area, $\pi r^2 \times \frac{120}{360} = 3\pi$

$r = 3$

Arc $AB = 2\pi(3)\times\frac{120}{360} = 2\pi$

Area of $\Delta OAB = \frac{1}{2}(3)(3)\sin 120° = \frac{1}{2}(3)(3)\left(\frac{\sqrt{3}}{2}\right) = \frac{9\sqrt{3}}{4}$

$AB = 2(3\sin 60°) = 2(3)\left(\frac{\sqrt{3}}{2}\right) = 3\sqrt{3}$

16. If a is a negative constant, which of the following statements about the graph of $y=(ax-3)(ax+4)+15$ must be true?

I. The x-coordinate of the vertex is positive.
II. The line $y=3$ cuts the graph at two distinct points.
III. The graph cuts the x-axis.

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

Method 1 (a can be any negative number)

Putting $a=-1$, $y=(-x-3)(-x+4)+15$

$y = x^2 - x + 3$

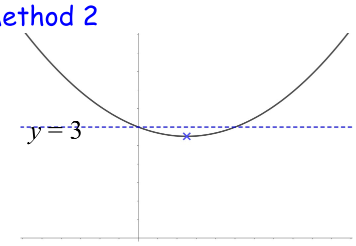
y-intercept $= 3$

When $y=3$, $3 = x^2 - x + 3$, $x = 0$ or 1 II is true

By symmetry, x-coordinate of vertex $= \frac{0+1}{2} = 0.5$ I is true

When $y=0$, $0 = x^2 - x + 3$ has no real roots. III is false

Method 2



$y=(ax-3)(ax+4)+15$

$y = a^2x^2 + ax + 3$

Coefficient of $x^2 = a^2 > 0$

y-intercept $= 3$

When $y=3$, $3 = a^2x^2 + ax + 3$, $x = 0$ or $-\frac{1}{a}$ II is true

x-coordinate of the vertex $= \frac{0+(-\frac{1}{a})}{2} = -\frac{1}{2a} > 0$ I is true

y-coordinate of the vertex $= a^2\left(-\frac{1}{2a}\right)^2 + a\left(-\frac{1}{2a}\right) + 3 = \frac{1}{4} - \frac{1}{2} + 3 = \frac{11}{4} > 0$

The vertex is above the x-axis. $\therefore$ $y=0$ has no real roots. III is false

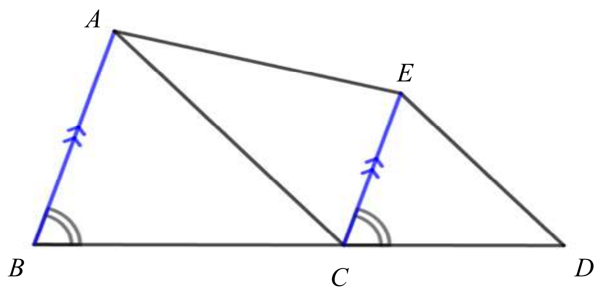
17. In the figure, BCD is a straight line, ΔABC is similar to ΔECD and $BC : CD = 5 : 4$. If the area of ΔABC is 50 cm^2, find the area of ΔACE.

A. 32 cm^2

B. 40 cm^2

C. 45 cm^2

D. 48 cm^2



$\Delta ABC \sim \Delta ECD$

$\therefore \dfrac{AB}{EC} = \dfrac{BC}{CD} = \dfrac{5}{4}$

$\angle ABC = \angle ECD$

$AB \,//\, EC$

$\therefore$ Height of ΔABC = Height of ΔACE

$\therefore$ Area of ΔABC : Area of ΔACE $= AB : EC$

$= 5 : 4$

Area of $\Delta ACE = 50 \times \dfrac{4}{5} = 40\text{ cm}^2$

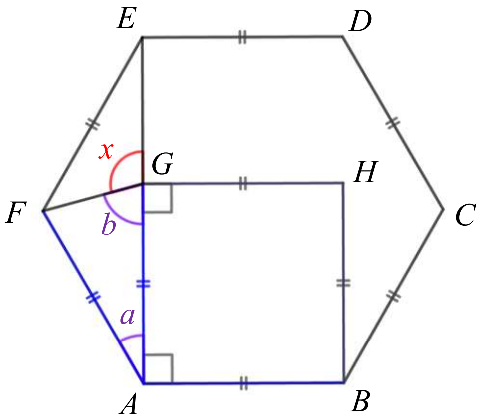
18. In the figure, $ABCDEF$ is a regular hexagon, $ABHG$ is a square. Find $\angle FGE$.

A. $100°$

B. $105°$

C. $110°$

D. $120°$



Note that EGA should be a straight line.

Ext. $\angle$ of a regular hexagon $= \dfrac{360°}{6} = 60°$

$\angle FAB = 180° - 60° = 120°$

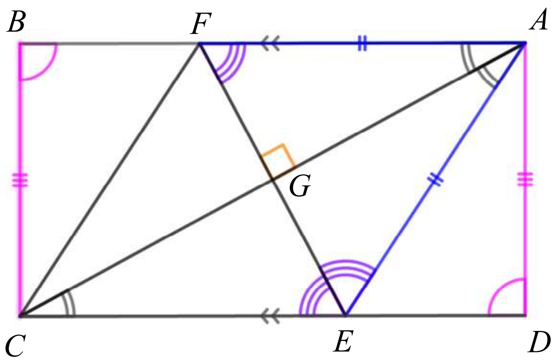
$a = 120° - 90° = 30°$

$AF = AG\ (= AB)$

$b = \dfrac{180° - 30°}{2} = 75°$

$x = 180° - 75° = 105°$

19. In the figure, $ABCD$ is a rectangle. Let E be a point on CD such that $\angle CAE = \angle CAB$, F be a point on AB such that $AE = AF$ and G be the intersection of AC and EF. Which of the following must be true?



I. $\Delta BCF \cong \Delta DAE$

II. $\Delta FAG \sim \Delta ECG$

III. $AECF$ is a rhombus.

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

Given that $\angle CAE = \angle CAB$ and $AE = AF$

$\Delta FAG \cong \Delta EAG$ (SAS)

$\therefore \angle AFG = \angle AEG$

$\angle CEG = \angle AFG = \angle AEG$ and

$\angle ECG = \angle FAG = \angle EAG$

$\Delta CEG \cong \Delta AEG$ (AAS)

… $\Delta FAG \cong \Delta ECG$

$\therefore \Delta FAG \sim \Delta ECG$ II is true

$CE = AE = AF$, …

$\therefore$ $AECF$ is a rhombus. III is true

$AD = BC$, $\angle B = \angle D = 90°$, $AE = CF$

$\Delta BCF \cong \Delta DAE$ (RHS) I is true

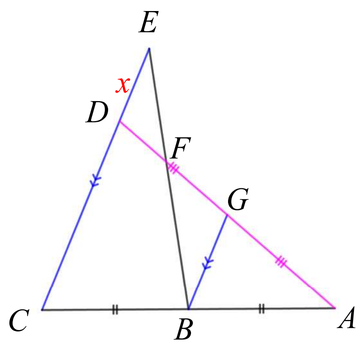
20. In the figure, ABC and CDE are straight lines, AD and BE intersect at F , $AB = BC$ and $AF : FD = 5:1$. Find $CD : DE$.

A. 3 : 1

B. 4 : 1

C. 5 : 1

D. 6 : 1



$AF : FD = 5:1$

Let $AF = 5a$, $FD = a$. $\therefore AD = 6a$

Construct $GB \parallel EC$

$DG = GA = 3a$ (intercept theorem)

$FG = 2a$

$GB \parallel EC$

$\therefore \Delta DEF \sim \Delta GBF$

$$\frac{DE}{GB} = \frac{DF}{GF} = \frac{a}{2a} = \frac{1}{2}$$

Let $DE = x$, $GB = 2x$

$CD = 2BG = 4x$ (mid-point theorem)

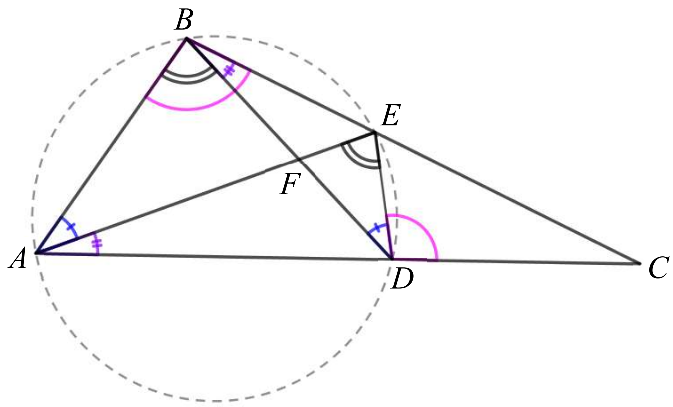
$\therefore CD : DE = 4 : 1$

21. In the figure, D and E are points lying on AC and BC respectively such that $ABED$ is a cyclic quadrilateral and F is the intersection of BD and AE. Which of the following must be true?

I. $\Delta ABF \sim \Delta DEF$

II. $\Delta ACE \sim \Delta BCD$

III. $\Delta ABC \sim \Delta DEC$



A. I and II only

B. I and III only

C. II and III only

D. I, II and III

$\angle ABF = \angle DEF$ and $\angle BAF = \angle EDF$

$\therefore\ \Delta ABF \sim \Delta DEF$ I is true

$\angle ACE = \angle BCD$ and $\angle CAE = \angle CBD$

$\therefore\ \Delta ACE \sim \Delta BCD$ II is true

$\angle ACB = \angle ECD$ and $\angle ABC = \angle EDC$

$\therefore\ \Delta ABC \sim \Delta EDC$ not ΔDEC III is false

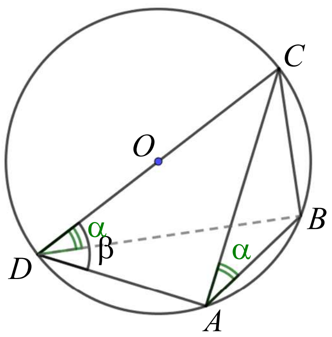
22. In the figure, $ABCD$ is a circle. If CD is a diameter, $\angle BAC = \alpha$ and $\angle ADC = \beta$, then $\dfrac{BC}{AD} =$

A. $\sin\alpha\tan\beta$.

B. $\dfrac{\cos\alpha}{\tan\beta}$.

C. $\dfrac{\sin\alpha}{\cos\beta}$.

D. $\dfrac{\cos\alpha}{\sin\beta}$.



$\angle CDB = \angle CAB$

$BC = CD\sin\alpha$

$AD = CD\cos\beta$

$\dfrac{BC}{AD} = \dfrac{CD\sin\alpha}{CD\cos\beta} = \dfrac{\sin\alpha}{\cos\beta}$

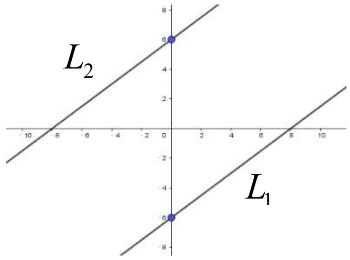
23. The equations of two distinct straight lines L_1 and L_2 are $3x-4y-24=0$ and $ax+by+96=0$ respectively. If they are parallel to each other and equidistant from the origin, find *a*.

A. -16

B. -12

C. 12

D. 16



$L_1 // L_2$ and equidistant from the origin.

$\therefore$ *x*-intercept of $L_1 = -$ *x*-intercept of L_2

$$8=-\left(-\frac{96}{a}\right)$$

$a=12$

24. Let *k* be a constant. Find the range of values of *k* such that $2x^2+5x+k=3$ has real roots.

A. $k \geq \frac{25}{8}$

B. $k \leq \frac{25}{8}$

C. $k \geq \frac{49}{8}$

D. $k \leq \frac{49}{8}$

Method 1	Method 2 (testing)
$2x^2+5x+k=3$ $2x^2+5x+k-3=0$ Real roots, $\Delta \geq 0$ $5^2-4(2)(k-3) \geq 0$ $25-8k+24 \geq 0$ $49 \geq 8k$ $k \leq \frac{49}{8}$	$k=\frac{25}{8}$ $2x^2+5x+\frac{25}{8}=3$ $2x^2+5x+\frac{1}{8}=0$ $x=-0.025255128$ or -2.474744871 Not equal roots, $\therefore$ A and B are rejected. When $k=0$, $2x^2+5x+0=3$ $2x^2+5x-3=0$ $x=0.5$ or -3 Two real roots, $\therefore$ D is correct (since $0 \leq \frac{49}{8}$).

25. The equation of the circle C is $(x-4)^2+(y-5)^2=9$. Which of the following must be true?

I. The radius of C is 3.

II. The straight line $3x+4y-32=0$ cuts C into two equal halves.

III. The origin O lies inside the circle C.

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

$(x-4)^2+(y-5)^2=9$

Centre = $G(4, 5)$, radius = 3 I is true

Diameters cut a circle into two equal halves.

Sub (4, 5) into $3x+4y-32=0$,

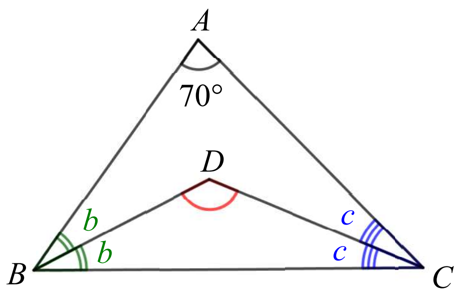
$\text{LHS}=3(4)+4(5)-32=0=\text{RHS}$

$\therefore$ the line passes through the centre. II is true

$OG=\sqrt{4^2+5^2}=\sqrt{41}>3$

$\therefore$ O lies outside the circle III is false

26. In the figure, DB and DC are the angle bisectors of $\angle ABC$ and $\angle ACB$ respectively. If $\angle BAC=70°$, find $\angle BDC$.



A. 110°

B. 125°

C. 140°

D. 145°

$2b+2c+70°=180°$

$b+c=55°$

$\angle BDC=180°-(b+c)$

$=180°-55°$

$=125°$

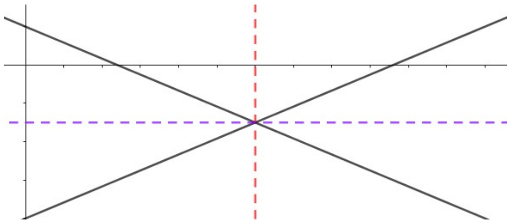
27. Given two intersecting straight lines L_1: $5x+12y-24=0$ and L_2: $5x-12y-96=0$. P is a moving point in the rectangular coordinate plane such that it is equidistant from L_1 and L_2. Find the equation(s) of the locus of P.

A. $5x+12y-60=0$

B. $x=12$ and $y=-3$

C. $12x+5y-129=0$ and $12x-5y-159=0$

D. $x^2+y^2-24x+6y=0$



The locus is the angle bisectors of L_1 and L_2.

Slope of $L_1=-\frac{5}{12}$

Slope of $L_2=\frac{5}{12}$

$\therefore$ the locus is a vertical line and a horizontal line.

28. Two numbers are randomly drawn at the same time from six cards numbered 2, 3, 5, 7, 8 and 9 respectively. Find the probability that the sum of the numbers drawn is divisible by 3.

A. $\frac{2}{15}$

B. $\frac{2}{9}$

C. $\frac{4}{15}$

D. $\frac{1}{3}$

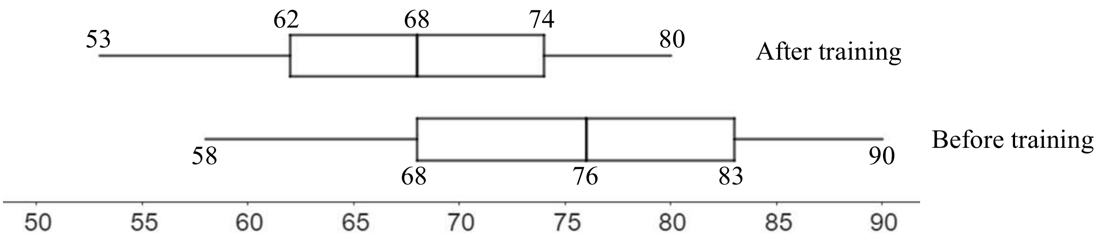
+	2	3	5	7	8	9
2		5	7	**9**	10	11
3	5		8	10	11	**12**
5	7	8		**12**	13	14
7	**9**	10	**12**		**15**	16
8	10	11	13	**15**		17
9	11	**12**	14	16	17	

By symmetry, we only need to count half of the table.

Required probability $=\frac{4}{15}$

29. The box-and-whisker diagrams show the weights of the members of a club before and after a training programme. Which of the following must be true?

I. All members have lost weight.
II. At least 25% of the members have lost 3 kg or more.
III. No one lost weight by more than 12 kg.



A. II only
B. I and II only
C. I and III only
D. I, II and III

(The diagram does not show the individual data.)

I is false. (58 kg may become 80 kg after training)

All members ≤ 80 kg after training and

before training, Q_3 = 83 (at least 25% ≥ 83 kg before training)

II is true.

III is false. (90 kg may become 53 kg after training)

30. The table below shows the distribution of the numbers of books some students read in a term.

Number of books read	5	6	7	8	9
Number of students (f)	3	4	23	50	40
Cumulative frequency	3	7	30	80	120

Which of the following is true?
A. The mode of the distribution is 50.
B. The mean of the distribution is 7.
C. The median of the distribution is 7.
D. The inter-quartile range of the distribution is 1.5.

Mode = 8

$$\text{Mean} = \frac{3(5)+4(6)+23(7)+50(8)+40(9)}{3+4+23+50+40} = 8$$

$$\text{120 data, median} = \frac{60^{\text{th}}+61^{\text{st}}}{2} = \frac{8+8}{2} = 8$$

$$Q_1 = \frac{30^{\text{th}}+31^{\text{st}}}{2} = \frac{7+8}{2} = 7.5$$

$$Q_3 = \frac{90^{\text{th}}+91^{\text{st}}}{2} = \frac{9+9}{2} = 9$$

IQR = 9 – 7.5 = 1.5

Section B

31. $11\times 8^4+2^7-3\times 2^4=$

A. 1011000001010000_2.

B. 1100000001010000_2.

C. 101100000101000_2.

D. 110000000101000_2.

$11\times 8^4+2^7-3\times 2^4 = 11\times 2^{12}+2^7-3\times 2^4$

Using a calculator, (MOD 3 11 DEC EXE BIN)

$11_{10}=1011_2 \quad (11=8+2+1)$

Only A and C are possible.

Start counting the power of 2 from the RHS with 0:

1	0	1	1	0	0	0	0	0	1	0	1	0	0	0	0
			12		10		8		6		4	3	2	1	0

The answer is A.

32. Which of the following is the least?

A. 543^{-375}

B. $(867)^{\frac{1}{4321}}$

C. $\left(\frac{1}{867}\right)^{349}$

D. $\left(\frac{2}{243}\right)^{492}$

Note that if $a>b$, $\log a>\log b$ and $\log a^n=n\log a$

Using a calculator, find

$\log A = -375\log 543 \approx -1025.549936$

$\log B = \frac{1}{4321}\log 867 \approx 0.0006799396199$

$\log C = 349\log\left(\frac{1}{867}\right) \approx -1025.368665$

$\log D = 492\log\left(\frac{2}{243}\right) \approx -1025.611529$

log D is the least.

33. If $\log_3 y=\log_{27}a+\log_9 x$, then $y=$

A. a^3x^2.

B. $a^{\frac{1}{3}}x^{\frac{1}{2}}$.

C. $\frac{a}{3}+\frac{x}{2}$.

D. $a^{\frac{1}{3}}+x^{\frac{1}{2}}$.

Method 1 (a, x and y can be any numbers)

Putting $a=27$, $x=9$, $\log_3 y=\log_{27}27+\log_9 9=2$

$$y=3^2=9$$

$A = 27^3 9^2 \neq 9$

$B = 27^{\frac{1}{3}}9^{\frac{1}{2}} = 9$

$C = \frac{27}{3}+\frac{9}{2}=\frac{27}{2} \neq 9$

$D = 27^{\frac{1}{3}}+9^{\frac{1}{2}}=6 \neq 9$

Method 2

Taking the logarithm of the options.

Obviously C and D are not possible.

A. If $y=a^3x^2$, $\log_3 y=3\log_3 a+2\log_3 x$

$$\log_3 y=3\left(\frac{\log_{27}a}{\frac{1}{3}}\right)+2\left(\frac{\log_9 x}{\frac{1}{2}}\right)$$

$$=9\log_{27}a+4\log_9 x$$

B. If $y=a^{\frac{1}{3}}x^{\frac{1}{2}}$, $\log_3 y=\frac{1}{3}\log_3 a+\frac{1}{2}\log_3 x$

$$\log_3 y=\frac{1}{3}\left(\frac{\log_{27}a}{\frac{1}{3}}\right)+\frac{1}{2}\left(\frac{\log_9 x}{\frac{1}{2}}\right)$$

$$=\log_{27}a+\log_9 x$$

The answer is B.

34. Let $z=(1-ai)i^{11}+\dfrac{10-ai}{3-4i}$, where a is a real number. If z is a real number, then $z=$

A. -5.

B. -3.

C. 3.

D. 5.

$i^2=-1$, $i^4=1$, $i^{11}=-i$

$$z=(1-ai)i^{11}+\frac{10-ai}{3-4i}$$

$$z(3-4i)=(1-ai)(-i)(3-4i)+10-ai$$

$$\begin{aligned}(1-ai)(-i)(3-4i)&=(1-ai)(-3i+4i^2)\\&=(1-ai)(-3i-4)\\&=-3i+3ai^2-4+4ai\\&=-3i-3a-4+4ai\end{aligned}$$

$$\begin{aligned}3z-4zi&=-3i-3a-4+4ai+10-ai\\&=-3a+6+i(-3+3a)\end{aligned}$$

Comparing the corresponding parts,

Real, $3z=-3a+6$

Imaginary, $-4z=-3+3a$

$\therefore\ z=-3$, $a=5$

35. Consider the following system of inequalities:

$$\begin{cases}x\ge 0\\ x+y\le 3\\ x+2y+3\ge 0\\ 3x-4y-16\le 0\end{cases}$$

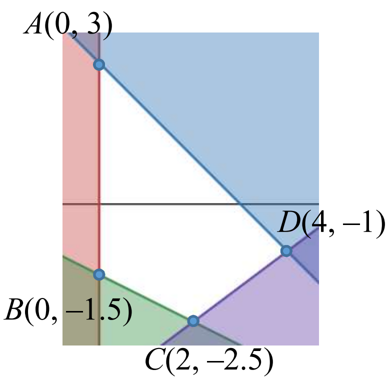
Let D be the region which represents the solution of the above system of inequalities. If (x, y) is a point lying in D, then the least value of $4x+2y+15$ is

A. 3.

B. 7.

C. 12.

D. 18.



The smaller the x and y, the smaller the value of $P=4x+2y+15$.

At $B(0,-1.5)$,

$P=4(0)+2(-1.5)+15=12$

At $C(2,-2.5)$,

$P=4(2)+2(-2.5)+15=18$

36. Let x_n be the nth term of an arithmetic sequence. If $x_4 = 8$ and $x_6 = 12$, which of the following must be true?

I. $x_{n+1} - x_n > 0$

II. $x_{92} + x_{100} = 2x_{96}$

III. $2^{-x_1} + 2^{-x_2} + 2^{-x_3} + \cdots + 2^{-x_n} < \frac{1}{2}$

A. I and II only
B. I and III only
C. II and III only
D. I, II and III

$a + 3d = 8$

$a + 5d = 12$

$a = 2, \quad d = 2$

$x_n = 2n$

The sequence is 2, 4, 6, 8, 10, 12, … I is true

$x_{92} + x_{100} = (a + 91d) + (a + 99d) = 2a + 190d$

$2x_{96} = 2(a + 95d) = 2a + 190d = x_{92} + x_{100}$ II is true

$2^{-x_1} + 2^{-x_2} + 2^{-x_3} + \cdots + 2^{-x_n}$

$= 2^{-2} + 2^{-4} + 2^{-6} + \cdots + 2^{-2n}$

$= \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \cdots + \frac{1}{2^{2n}}$

The sum of a geometric sequence with

$T_1 = \frac{1}{2^2} = \frac{1}{4}$ and $r = \frac{1}{2^2} = \frac{1}{4}$

$\frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \cdots + \frac{1}{2^{2n}} < \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \cdots$

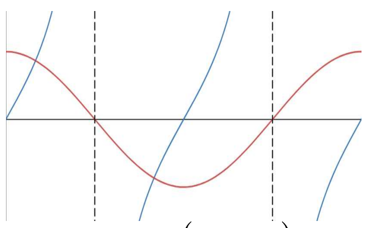
$= \frac{\frac{1}{4}}{1 - \frac{1}{4}} = \frac{1}{3} < \frac{1}{2}$ III is true

37. Given that $0° \le x < 360°$, how many roots does the equation $3\tan x = 2\sin(90° - x)$ have?

A. 2
B. 3
C. 4
D. 5

Method 1

$2\sin(90° - x) = 2\cos x$



Blue : $y = 2\sin(90° - x)$

Red : $y = 2\cos x$

There are 2 points of intersection.

$\therefore$ 2 roots

Method 2

$3\tan x = 2\sin(90° - x)$

$\frac{3\sin x}{\cos x} = 2\cos x$

$3\sin x = 2\cos^2 x$

$3\sin x = 2(1 - \sin^2 x)$

$2\sin^2 x + 3\sin x - 2 = 0$

$\sin x = 0.5$ or $\sin x = -2$ (rejected)

$\therefore$ 2 roots

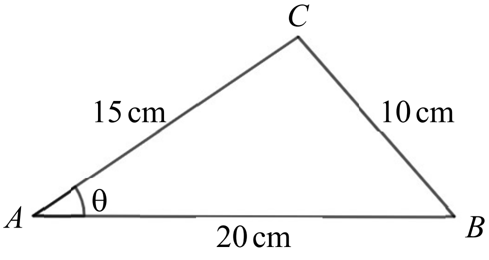
38. In the figure, $\tan\theta =$

A. $\frac{7}{\sqrt{15}}$.

B. $\frac{\sqrt{15}}{7}$.

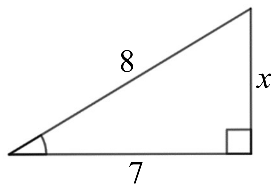
C. $\frac{8}{\sqrt{15}}$.

D. $\frac{\sqrt{15}}{8}$.



$$10^2 = 15^2 + 20^2 - 2(15)(20)\cos\theta$$

$$\cos\theta = \frac{15^2 + 20^2 - 10^2}{2(15)(20)} = \frac{7}{8}$$



$$x^2 + 7^2 = 8^2$$

$$x = \sqrt{15}$$

$$\tan\theta = \frac{\sqrt{15}}{7}$$

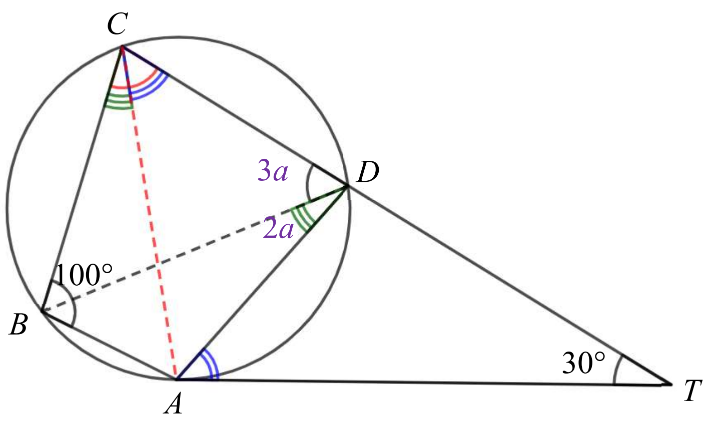
39. In the figure, AT is a tangent to circle $ABCD$ at A and CDT is a straight line. If $\angle ABC = 100°$, $\angle ATC = 30°$, $\overset{\frown}{AB} : \overset{\frown}{BC} = 2:3$, find $\angle BCT$.

A. 100°

B. 90°

C. 82°

D. 70°



arc AB : arc BC = 2 : 3	$\angle BCA = \angle BDA = 2a = 32°$
$\therefore \angle BDA : \angle CDB = 2 : 3$	$\angle DAT + 30° = 5a$ (ext. ∠ of Δ)
Let $\angle BDA = 2a$, $\angle CDB = 3a$	$\angle DAT = 50°$
$5a = 180° - 100° = 80°$	$\angle ACD = \angle DAT = 50°$ (∠ in alt. segment)
$a = 16°$	$\angle BCT = 32° + 50° = 82°$

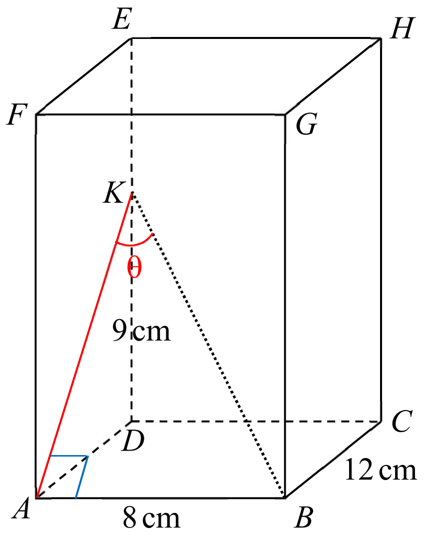
40. In the figure, $ABCDEFGH$ is a rectangular block. Let K be a point on DE such that $DK = 9$ cm and $EK = 5$ cm. Denote the angle between BK and the plane $ADEF$ by θ. Find $\cos\theta$.

A. $\frac{15}{17}$

B. $\frac{8}{17}$

C. $\frac{4}{5}$

D. $\frac{3}{5}$



AK is the projection of BK on the plane $ADEF$

$\angle AKB = \theta$

$AK = \sqrt{12^2 + 9^2} = 15$

$BK = \sqrt{15^2 + 8^2} = 17$

$\cos\theta = \frac{15}{17}$

41. Given that $G(10, 3)$ is the circumcentre of the ΔABC and the coordinates of B and C are $(18, -3)$ and $(2, 9)$ respectively, which of the following is **not** a possible y-coordinate of A?

A. 12

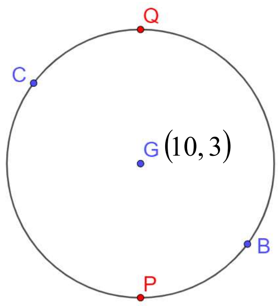
B. 3

C. -3

D. -8

$G(10, 3)$ is the circumcentre of the ΔABC

$\therefore$ G is the centre of the circle passing through A, B and C.



Radius = CG

$= \sqrt{(18-10)^2 + (-3-3)^2} = 10$

Uppermost position of A is at Q,
y-coordinate of $Q = 3 + 10 = 13$

Lowermost position of A is at P,
y-coordinate of $P = 3 - 10 = -7$
D is not possible.

42. A queue is formed by 6 boys and 5 girls. If the girls must be next to each other, how many different queues can be formed?

A. 604 800

B. 172 800

C. 86 400

D. 5 040

The 5 girls are bundled.
Together with the 6 boys, there are 7 objects to be rearranged.
(No. of permutation = 7!)
The 5 girls can be rearranged among themselves.
(No. of permutation = 5!)
No. of queues that can be formed
= (7!)(5!)
= 604 800

43. A bag contains 3 red balls, 7 yellow balls and 5 black balls. Peter repeats drawing one ball at a time randomly from the bag without replacement until a red ball is drawn. Find the probability that Peter needs at least three draws.

A. $\frac{64}{125}$

B. $\frac{44}{91}$

C. $\frac{22}{35}$

D. $\frac{16}{25}$

Peter needs at least three draws.

$\therefore$ He fails in the first 2 draws. (Not getting a red in the first 2 draws.)

Probability

$=\frac{C_2^{12}}{C_2^{15}}$ or $\frac{12}{15}\times\frac{11}{14}$

$=\frac{22}{35}$

44. The standard scores of Mary and Peter in a test are 1.5 and -1. If the test score of Peter is 12.5% less than that of Mary and the difference between their test scores is 10 marks, find the mean score of the test.

A. 84

B. 80

C. 74

D. 64

$x_P=(1-12.5\%)x_M=0.875x_M$ and $x_M-x_P=10$

$x_M=80$ and $x_P=70$

$z_M=1.5$ and $z_P=-1$

$\frac{80-\bar{x}}{\sigma}=1.5$ and $\frac{70-\bar{x}}{\sigma}=-1$

$\sigma=74$

45. It is given that T(n) is the nth term of an arithmetic sequence. Let x_1, y_1 and z_1 be the mean, the range and the variance of the group of numbers $\{T(1), T(3), T(5), \cdots, T(99)\}$ respectively while x_2, y_2 and z_2 be the mean, the range and the variance of the group of numbers $\{T(2), T(4), T(6), \cdots, T(100)\}$ respectively. Which of the following must be true?

I. $x_1 < x_2$

II. $y_1 = y_2$

III. $z_1 < z_2$

A. I only

B. II only

C. I and III only

E. II and III only

Both groups have 50 data.

If common difference < 0,

mean of $\{T(1), T(3), T(5), \cdots, T(99)\} > \{T(2), T(4), T(6), \cdots, T(100)\}$

I is false

Note that the common difference of $\{T(1), T(3), T(5), \cdots, T(99)\}$

= the common difference of $\{T(2), T(4), T(6), \cdots, T(100)\}$

Their variance should be the same. III is false

Their range should be the same. II is true

End of Paper