ST. PAUL'S COLLEGE

FORM 6 INTERNAL EXAMINATION 2023 - 2024

MATHEMATICS Compulsory Part

PAPER 1

Section A2

Question-Answer Book

2¼ hours

This paper must be answered in English.

INSTRUCTIONS

1. Write your Name, Class and Class number in the spaces provided on the right. Circle your Group Number.

2. This paper consists of THREE sections, A(1), A(2) and B.

3. Attempt **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Graph paper and supplementary answer sheets will be supplied on request. Write your Name, Class and Class number in the spaces provided, mark the question number box, and fasten them with string INSIDE this book.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

A2

Name	
Class	( )
Group	G1 LMW G2 PSK G3 TMF G4 WHP G5 TMF G6 LMW G7 PSK

Question No.	Marks
10	
11	
12	
13	
14	
Total	

SECTION A(2) (35 marks)

10. The coordinates of the points A and B are $(5,8)$ and $(11,4)$ respectively. Let P be a moving point in the rectangular coordinate plane such that P is equidistant from A and B. Denote the locus of P by Γ.

(a) (i) Describe the geometric relationship between Γ and AB.

(ii) Find the equation of Γ.

(3 marks)

(b) It is given that the equation of a circle C is $x^2+y^2-12x-6y=0$. Someone claims that Γ divides C into two equal halves. Is the claim correct? Explain your answer.

(3 marks)

(a) (i) Γ is the perpendicular bisector of AB.

(ii) $\sqrt{(x-5)^2+(y-8)^2}=\sqrt{(x-11)^2+(y-4)^2}$ 1M

$3x-2y-12=0$ 1A

Thus, the equation of Γ is $3x-2y-12=0$.

The slope of AB

$=\dfrac{8-4}{5-11}$

$=-\dfrac{2}{3}$

The slope of Γ

$=\dfrac{3}{2}$

The mid-point of AB

$=\left(\dfrac{5+11}{2},\dfrac{8+4}{2}\right)$

$=(8,6)$

The equation of Γ is $y-6=\dfrac{3}{2}(x-8)$ 1M

Thus, the equation of Γ is $3x-2y-12=0$. 1A

(b) Centre of the circle $=(6,3)$ 1M

Sub $(6,3)$ into $3x-2y-12=0$,

L.H.S. $=3(6)-2(3)-12$ 1M

$=0$

$=$ R.H.S

Centre of the circle lies on Γ.

Γ divides C into two equal halves.

The claim is correct. 1Af.t.

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11. It is given that $f(x)$ partly varies as x^2 and partly varies as x. Suppose that $f(8) = -2$ and $f(-8) = -6$.

(a) Find $f(x)$. (3 marks)

(b) The graph of $y = f(x) + 12$ cuts the x-axis at $A(a, 0)$ and $B(b, 0)$, where $a < b$. If the graph of $y = f(x) + 12$ cuts the y-axis at C, find the shortest distance from A to BC. (4 marks)

(a) Let $f(x) = px^2 + qx$, where p and q are non-zero constants. 1M

$64p + 8q = -2 \ldots(1)$ 1M

$64p - 8q = -6 \ldots(2)$ for either substitution

(1)+(2),

$128p = -8$

$p = -\frac{1}{16}$

$q = \frac{1}{4}$

Thus, $f(x) = -\frac{1}{16}x^2 + \frac{1}{4}x$ 1A

(b) $y = -\frac{1}{16}x^2 + \frac{1}{4}x + 12$

The x-intercepts are -12 and 16. 1M

The y-intercept is 12.

Area of $\Delta ABC = \frac{1}{2}[16 - (-12)] \times 12$ 1M

$= 168$

$BC = \sqrt{(16-0)^2 + (0-12)^2}$ 1M

$= 20$

The shortest distance from A to BC

$= 168 \times \frac{2}{20}$

$= 16.8$ 1A

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12. 42 identical solid metal spheres with radius 6 cm are melted and recast into 2 similar solid right circular cones. The ratio of the height of the smaller circular cone to the height of the larger circular cone is 1 : 3.

(a) Find the volume of the larger circular cone in terms of π. (3 marks)

(b) If the base radius of the larger circular cone is 36 cm, find the total surface area of the smaller circular cone in terms of π. (4 marks)

(a) Let V_1 cm^3 be the volume of the larger circular cone and

V_2 cm^3 be the volume of the smaller circular cone.

$$\frac{V_2}{V_1} = \left(\frac{1}{3}\right)^3$$ 1M

$$V_2 = \frac{1}{27}V_1$$

Volume of the sphere $= \frac{4\pi}{3} \times 6^3 \times 42$ cm^3 1M

$$V_1 + V_2 = 12096\pi$$

$$V_1 + \frac{1}{27}V_1 = 10296\pi$$

$$\frac{28}{27}V_1 = 10296\pi$$

$$V_1 = 11664\pi$$

Thus, the volume of the larger circular cone is 11664π cm^3. 1A

(b) Let r cm, h cm and ℓ cm be the base radius, height and slant height of the smaller circular cone respectively.

$$\frac{r}{36} = \frac{1}{3}$$ 1M

$$r = 12$$

Volume of the smaller circular cone $= \frac{1}{3} \times \pi \times 12^2 \times h$ cm^3 1M

$$12096\pi - 11664\pi = \frac{144}{3}\pi h$$

$$432\pi = \frac{144}{3}\pi h$$

$$h = 9$$

$$\ell = \sqrt{12^2 + 9^2}$$

$$= 15$$

Total surface area of the smaller circular cone

$= (\pi \times 12^2 + \pi \times 12 \times 15)$ cm^2 1M

$= 324\pi$ cm^2 1A

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13. The stem-and-leaf diagram below shows the distribution of the weights (in kg) of the members of a handball team.

Stem (tens)	Leaf (units)						
4	2	4	5	5			
5	0	1	3	4	7	7	8
6	5	5	6	7	a		
7	1	3	3	b			

It is given that the interquartile range and the mean of the distribution are 17 kg and 59 kg respectively.

(a) Find the values of a and b. (3 marks)

(b) Two more members now join the handball team. It is found that both the mean and the range of the distribution of the weights are increased by 1 kg. Find the weight of each of these two members. (4 marks)

(a) Interquartile range = 17 kg

$$\frac{67+60+a}{2}-\frac{50+51}{2}=17$$ 1M

$$a=8$$ 1A

$$\frac{42+44+45+45+50+51+53+54+57+57+58+65+65+66+67+68+71+73+73+70+b}{20}=59$$

$$1174+b=1180$$

$$b=6$$ 1A

(b) Let x kg and y kg be the weights of these two members, where $x \le y$.

∵ The mean is increased by 1 kg.

∴ $\frac{x+y+59(20)}{22}=59+1$ 1M

$$x+y=140$$

∵ The range is increased by 1 kg.

∴ The new range is 35 kg.

There are two cases.

Case 1: $x = 41$, $41 \le y \le 76$ 1M

$41 + y = 140$

$b = 99$, which is impossible

Case 2: $42 \le x \le 77$, $y = 77$ (either one)

$x + 77 = 140$

$a = 63$

Thus, the weight of the two members are 63 kg and 77 kg. 1A + 1A

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14. The cubic polynomial $p(x)$ is divisible by $x-1$. When $p(x)$ is divided by x^2-1, the remainder is $cx+5$, where c is a constant.

(a) Find c. (3 marks)

(b) It is given that $x+2$ is a factor of $p(x)$. When $p(x)$ is divided by $x+3$, the remainder is -36. How many rational roots does the equation $p(x)=0$ have? (5 marks)

(a) Let $p(x)=(x^2-1)q(x)+(cx+5)$, where $q(x)$ is a polynomial. 1M

Since $p(1)=0$, $p(1)=(1^2-1)q(1)+(c+5)=0$ 1M

Thus, $c=-5$ 1A

(b) Let $p(x)=(x^2-1)(ax+b)+(cx+5)$, where a and b are constants. 1M

Since $p(-2)=0$, $p(-2)=((-2)^2-1)(-2a+b)+(-5(-2)+5)=0$ 1M

$-2a+b=-5\ldots(1)$

Since $p(-3)=-36$, $p(-3)=((-3)^2-1)(-3a+b)+(-5(-3)+5)=-36$ (either one)

$-3a+b=-7\ldots(2)$

(1)−(2), $a=2$ 1A

$b=-1$ 1A

$$\begin{aligned}p(x)&=(x^2-1)(2x-1)+(-5x+5)\\&=(x-1)(x+1)(2x-1)-5(x-1)\\&=(x-1)(2x+x-6)\\&=(x-1)(x+2)(2x-3)\end{aligned}$$

The roots of $p(x)=0$ are 1, -2 and $\frac{3}{2}$.

All the roots of $p(x)=0$ are rational.

Thus, there are 3 rational roots. 1Af.t.

End of Section A2

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