ST. PAUL'S COLLEGE

FORM 6 INTERNAL EXAMINATION 2023 - 2024

MATHEMATICS Compulsory Part

PAPER 1

Section B

Question-Answer Book

2¼ hours

This paper must be answered in English.

INSTRUCTIONS

1. Write your Name, Class and Class number in the spaces provided on the right. Circle your Group Number.

2. This paper consists of THREE sections, A(1), A(2) and B.

3. Attempt **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Graph paper and supplementary answer sheets will be supplied on request. Write your Name, Class and Class number in the spaces provided, mark the question number box, and fasten them with string INSIDE this book.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

B

Name	
Class	( )
Group	G1 LMW G2 PSK G3 TMF G4 WHP G5 TMF G6 LMW G7 PSK

Question No.	Marks
15	
16	
17	
18	
19	
Total	

SECTION B (35 marks)

15. A group of 5 people are randomly selected from 5 boys and 4 girls.

(a) Find the probability that a group of exactly 2 boys and 3 girls are selected. (2 marks)

(b) Find the probability that a group consisting of at most 2 boys are selected. (2 marks)

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Page total

16. The first term of a geometric sequence is 16 and the sum of the second term and the third term of the sequence is 60. It is given that all terms of the sequence are positive.

(a) Find the common ratio of the sequence. (2 marks)

(b) Find the least value of n such that the sum of the $(n+1)^{th}$ term to the $(2n+1)^{th}$ term of the sequence is greater than 4.8×10^{12}. (3 marks)

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17. (a) Let a and b be real constants. If the roots of the equation $x^2 + ax + b = 0$ are $2p$ and $5p$, prove that $10a^2 = 49b$. (2 marks)

(b) It is given that the x-axis is a tangent to the circle C: $x^2 + y^2 - 8x + 12y + r = 0$, where r is a real constant. Find the real constant m such that the straight line $y = mx$ cuts C at the points Q and R with $OQ : OR = 2 : 5$, where O is the origin. (4 marks)

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18. In Figure 3(a), the diagonals of cardboard *ABCD* intersect at *E*. It is given that $AB = AD = 50$ cm, $BC = DC = 62$ cm and $\angle ABD = 53°$.

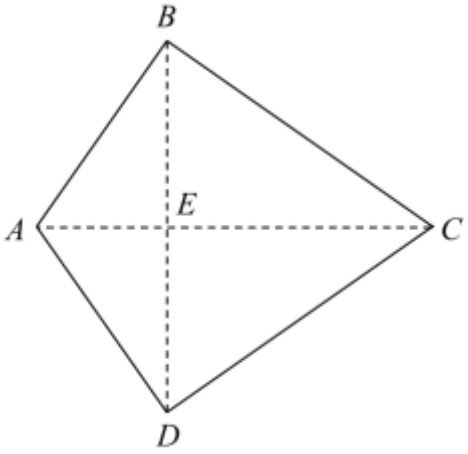

Figure 3(a)

(a) Find $\angle BCD$.

(2 marks)

(b) The cardboard *ABCD* in Figure 3(a) is folded along *BD* such that *A* is vertically above the line *EC*. Two extra triangular cardboards are placed to form the tetrahedron *ABCD* as shown in Figure 3(b). It is given that the total surface area of the tetrahedron *ABCD* is 4335 cm^3.

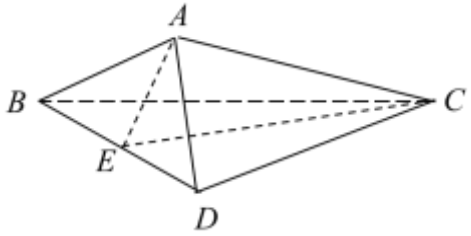

Figure 3(b)

(i) Does the angle between $\triangle ABD$ and $\triangle BCD$ exceed 20°? Explain your answer.

(ii) Find the volume of the tetrahedron *ABCD*.

(7 marks)

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19. The coordinates of the points A and B are $(0\ ,\ 9)$ and $(32\ ,\ -15)$ respectively. C is a point in the rectangular coordinate plane such that BC is a vertical line. H is a point in the same rectangular coordinate plane such that $\angle AHB = \angle AHC$ and $BH = CH$.

(a) Prove that $\triangle ABH \cong \triangle ACH$. (2 marks)

(b) Find the coordinates of C. (2 marks)

(c) Find the coordinates of the circumcentre of $\triangle ABC$. (2 marks)

(d) Suppose that $AH = BH = CH$. Denote the in-centre of $\triangle ABC$ by I.

(i) Are A, H and I collinear? Explain your answer.

(ii) Someone claims that the area of the circumcircle of $\triangle ABC$ is greater than 4 times the area of the inscribed circle of $\triangle ABC$. Do you agree? Explain your answer.

(5 marks)

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End of Paper

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