St. Paul's Secondary School

Form 6 **Final Examination 2021 – 2022**

Mathematics Paper 1

SECTION A(1) (35 marks)

1. Simplify $\dfrac{a^4b^{-9}}{(a^2b^{-2})^3}$ and express your answer with positive indices. (3 marks)

2. Factorize
   (a) $2a^2+ab-3b^2$,
   (b) $4a^2+2ab-6b^2-2a-3b$. (3 marks)

3. Make y the subject of the formula $\dfrac{y-1}{2y-5}=\dfrac{1}{x}$. (3 marks)

4. (a) Find the range of values of x which satisfy both $\dfrac{13-15x}{4}\geq 1-3x$ and $2-x<7$.
   (b) Write down the least odd number satisfying both inequalities in (a). (4 marks)

5. A watch is sold at a discount of 20% on its marked price. The selling price of the watch is \$400.
   (a) Find the marked price of the watch.
   (b) After selling the watch, the percentage profit is 25%. Find the cost of the watch. (4 marks)

6. In Figure 1, AC is a diameter of the circle $ABCD$. BD and AC intersect at E such that $\angle CBD=25°$ and $\angle AED=128°$. Find $\angle CDE$.

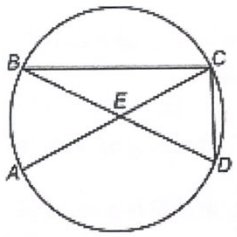

Figure 1 (4 marks)

7. In a polar coordinate system, O is the pole. The polar coordinates of the point A and B are $(10,15°)$ and $(10,135°)$ respectively. A is rotated anticlockwise about O through $300°$ to C.
   (a) Write down the polar coordinates of C.
   (b) Find $\angle OAC$.
   (c) Someone claims that ΔABC is a right-angled triangle. Do you agree? Explain. (5 marks)

8. It is given that $f(x)$ is the sum of two parts, one part varies as x and the other part varies as $\sqrt{x}$.
   Suppose $f(4)=46$ and $f(16)=188$.
   (a) Find $f(x)$.
   (b) If the value of x is decreased from 16 to 9, Write down the change in the value of $f(x)$. (4 marks)

Answers written in the margins will not be marked.

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9. The stem-and-leaf diagram shows the weights (in kg) of 14 scouts in a hiking group, where a and b are integers. The mean of the weights of the 14 scouts is 36 kg.

Stem (10 kg)	Leaf (1 kg)
2	1 3 4 7
3	a b 5 6 8
4	0 4 6
5	1 2

(a) Find a and b. (3 marks)

(b) Two scouts leave the hiking group. Find the smallest possible median of the weights of the remaining scouts. (2 marks)

SECTION A(2) (35 marks)

10. The following table shows the distribution of the numbers of films watched by a group of students last month, where $a > 5$, $b < 11$ and $c > 0$.

Number of films	0	1	2	3	4	5
Number of students	$c + 1$	4	a	8	$b - a$	c

The median of the distribution is 2.5.

(a) Find a and b. (2 marks)

(b) Suppose $c = 1$. If a student is randomly selected from the group, find the probability that the selected student watched more than 3 films last month. (2 marks)

(c) It is given that the mode of the distribution is greater than 2. Find the standard deviation of the distribution when c attains its greatest value. (2 marks)

11. Let $f(x) = 2x^3 + ax^2 + bx - 6$, where a and b are constants. $f(x)$ is divisible by $x + 2$. The remainder obtained when $f(x)$ is divided by $x - 1$ is 22 less than the remainder obtained when $f(x)$ is divided by $x + 1$.

(a) Find the values of a and b. (3 marks)

(b) Factorize $f(x)$. (2 marks)

(c) It is given that $g(x-2021) = f(x)$. Someone claims that all the roots of the equation $g(x) = 0$ are integers. Do you agree? Explain your answer. (2 marks)

12. In Figure 2, $ACED$ is a rectangle. BE intersects DC at F such that $BE \perp DC$ and $BC = AD$.

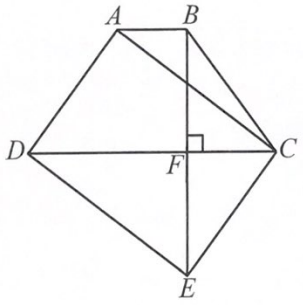

Figure 2

(a) Prove that $\Delta CEF \sim \Delta EDF$. (2 marks)

(b) It is given that $AD = 30$ cm and $BF = 24$ cm.

(i) Is F the mid-point of BE? Explain your answer.

(ii) Find ED.

(5 marks)

13. The coordinates of points X and Y are $(-3, 7)$ and $(5, 1)$ respectively. P is a moving point in the rectangular coordinate plane such that $PX = PY$. Denote the locus of P by Γ.

(a) (i) Describe the geometric relationship between Γ and the line segment XY.

(ii) Find the equation of Γ.

(3 marks)

(b) It is given that a circle passes through X and Y. Denote the centre of the circle by R.

(i) Does Γ pass through R? Explain your answer.

(ii) The line $y = x$ divides the circle into two equal halves. Find the area of the circle.

(4 marks)

14. Figure 3 shows a vessel which consists of two parts with a common base. The upper part of the vessel is a frustum of height 7.5 cm and the lower part is a right circular cylinder of height 8 cm. It is given that the frustum is made by cutting off a smaller inverted right circular cone of base radius a cm and height 5 cm from a larger inverted right circular cone of base radius 30 cm.

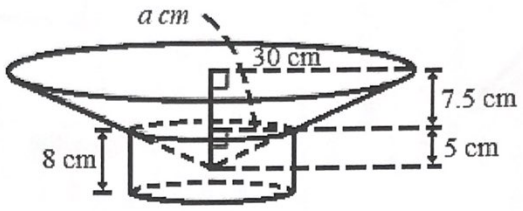

Figure 3

(a) Find a. (1 mark)

(b) Water of 1500π cm^3 is now poured into the vessel. Let R cm be the radius of the water surface and d cm be the depth of water.

(i) Show that $d > 8$ and $R = \dfrac{12(d-3)}{5}$.

(ii) Someone claims that the wet surface area of the frustum does not exceed 400 cm^2. Do you agree? Explain.

(7 marks)

SECTION B (35 marks)

15. In a box, there are 9 red balls, 4 yellow balls and 7 blue balls. If 5 balls are randomly selected from the box at the same time, find

(a) the probability that 5 balls of the same colour are selected;

(3 marks)

(b) the probability that at least 2 balls of different colours are selected.

(2 marks)

16. Let a and b be constants. Denote the graph of $\log_a y = 1 + \log_b x$ by G. G passes through the points $(4, 4)$ and $\left(\dfrac{1}{64}, \dfrac{1}{4}\right)$. Express x in terms of y. (4 marks)

Answers written in the margins will not be marked.

17. (a) It is given that ΔPQR is a right-angled triangle with $PQ = PR = 10$ and $\angle QPR = 90°$. Find the radius of the in-circle of ΔPQR. (Express the answer in surd form.) (4 marks)

(b) $A(-10, 0)$, $B(0, 10)$ and $O(0, 0)$ are the vertices of a triangle on the rectangular coordinate plane. Amy claims that the straight line $2x - \sqrt{2}y + 10 = 0$ passes through the in-centre of ΔOAB. Do you agree? Explain your answer. (3 marks)

18. In Figure 4(a), ABC is a triangular paper card. D is a point lying on AB. It is given that $\angle DAC = 90°$, $\angle BDC = 120°$, $AB = 11$ cm and $CD = 12$ cm.

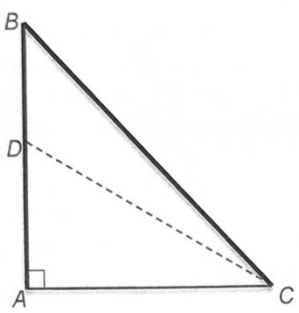

Figure 4(a)

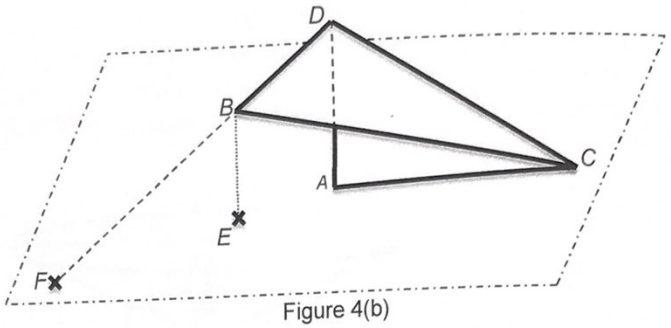

Figure 4(b)

In Figure 4(b), the triangular paper card is folded along CD such that AC lies on the horizontal ground, ΔACD is perpendicular to the horizontal ground and $BE = 4$ cm, where E is the point lying on the horizontal ground vertically below the point B. DB produced meets the horizontal ground at the point F.

(a) Find the distance between B and F. (2 marks)

(b) Find the area of ΔDCF. (2 marks)

(c) Find the inclination of the plane BCD to the horizontal ground. (3 marks)

19. Let $f(x) = 2x^2 - 16kx - 8x + 32k^2 + 31k + 18$, where k is a positive constant. On the same rectangular coordinate system, denote the vertex of the graph of $y = f(x)$ and the vertex of the graph of $y = f(2x - 22) - 14$ by Q and R respectively.

(a) Using the method of completing the square, express the coordinates of Q in terms of k. (2 marks)

(b) Write down the coordinates of R in terms of k. (1 mark)

(c) The coordinates of S are $(4k - 2, -k - 4)$. It is given that Q, R and S are three distinct points. Denote the circumcentre of ΔQRS by G.

(i) Express the equation of the perpendicular bisector of QS in terms of k.

(ii) Express the coordinates of G in terms of k.

(iii) Denote the circle passing through Q, R and S by C. Let A be a point outside C such that AQ and AS are the tangents to C at Q and S respectively. Is it possible that $AQGS$ is a square? Explain your answer. (9 marks)

Answers written in the margins will not be marked.

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There are 30 questions in Section A and 15 questions in Section B.
The diagrams in this paper are not necessarily drawn to scale.
Choose the best answer for each question.

Section A

1. $\frac{8^{n+1}}{2^{3n+1}} =$
   A. 1.
   B. 2.
   C. 4.
   D. 2^n.

2. If $\frac{1-x}{1+x} = 2y$, then $x =$
   A. $\frac{1-2y}{2}$.
   B. $\frac{1-2y}{2y}$.
   C. $\frac{1+2y}{1-2y}$.
   D. $\frac{1-2y}{1+2y}$.

3. Solve the equation $(x-1)(x-2) = x-1$.
   A. $x = 1$
   B. $x = 2$
   C. $x = 1$ or $x = 2$
   D. $x = 1$ or $x = 3$

4. $4rs + 6 - 3s - 8r =$
   A. $(4r+3)(s+2)$.
   B. $(rs+2)(s-1)$.
   C. $(4r-3)(s-2)$.
   D. $(4r-3)(s+2)$.

5. Let p and q be constants. If $x^2 + p(x+5) - q \equiv (x+3)(x-q)$, then $q =$
   A. 5.
   B. –2.
   C. –3.
   D. –5.

6. Which of the following is an identity / are identities?
   I. $4x^2 - 9 = (2x-3)^2$
   II. $4x^2 - 9 = (2x+3)(2x-3)$
   III. $4x^2 - 9 = 0$
   A. I only
   B. II only
   C. I and III only
   D. II and III only

7. Let $f(x) = x^{13} + mx^9 + n$, where m and n are constants. If $f(x)$ is divisible by $x+1$, find the remainder when $f(x)$ is divided by $x-1$.
   A. $2m$
   B. $-2m$
   C. $2m+2$
   D. $-2m+2$

8. Let $f(x) = 6 - 2x - x^2$. If $g(x-2) = f(2x)$, find the value of $g(-1)$.
   A. –9
   B. –2
   C. 3
   D. 6

9. Anna sells two diamond rings for \$67 200 each. She gains 40% on one and loses 40% on the other. After the two transactions, Anna
   A. loses \$25 600.
   B. gains \$4200.
   C. gains \$12 800.
   D. has no gain and no loss.

10. The solution of $\frac{2x-5}{3} \geq x-4$ or $7x+8<1$ is
    A. $x > -1$.
    B. $x < -1$.
    C. $x \leq 7$.
    D. $x \geq 7$.

11. If $bc : ca : ab = 1 : 2 : 3$, then $\frac{a}{bc} : \frac{b}{ca} =$
    A. 4 : 1.
    B. 3 : 1.
    C. 1 : 3.
    D. 1 : 4.

12. If z varies inversely as the square root of x and directly as the square of y, which of the following must be constant?

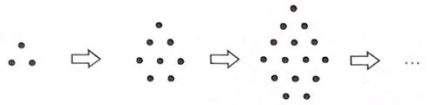
A. xy^4z^2

B. x^4yz^2

C. $\dfrac{xz^2}{y^4}$

D. $\dfrac{x^4z^2}{y}$

13. In the figure, the 1st pattern consists of 3 dot. For any positive integer n, the $(n+1)$th pattern is formed by adding $(2n+3)$ dots to the nth pattern. Which pattern has 99 dots?



A. 8th pattern

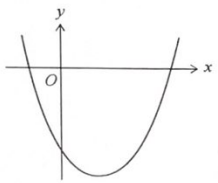
B. 9th pattern

C. 10th pattern

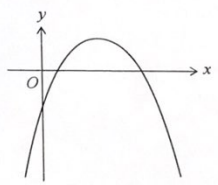
D. 11th pattern

14. If $p<0$ and $q>0$, which of the following may represent the graph of $y=(px+q)^2-3$?

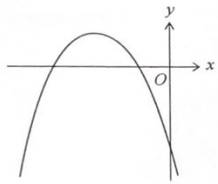
A.



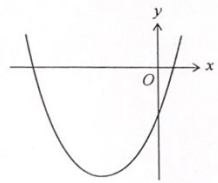
B.



C.



D.



15. The base of a solid right pyramid is a square. If the perimeter of the base is 72 cm and the length of each slant edge of the pyramid is 15 cm, then the total surface area of the pyramid is

A. 432 cm^2.

B. 756 cm^2.

C. 864 cm^2.

D. 1188 cm^2.

16. The volume of a solid right circular cone of radius R is three times the volume of a solid right circular cylinder of radius r. If the heights of these two solids are the same, then $R:r=$

A. 1 : 9.

B. 9 : 1.

C. 1 : 3.

D. 3 : 1.

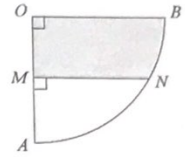
17. In the figure, OAB is a sector with centre O. M is the mid-point of OA and $\angle AOB=\angle AMN=90°$. If the radius of the sector is 12 cm, find the area of the shaded region.

A. $(12\pi+18\sqrt{2})\,cm^2$

B. $(12\pi+18\sqrt{3})\,cm^2$

C. $(18\pi+18\sqrt{2})\,cm^2$

D. $(18\pi+18\sqrt{3})\,cm^2$



18. In the figure, $PQRS$ is a rectangle where diagonals PR and QS intersect at D. $\triangle DFP$ is an equilateral triangle. DF and PS intersect at E, and RF and QS intersect at G. If $PF=RS$, which of the following must be true?

I. $DR=DF$

II. RF is the angle bisector of $\angle DRS$.

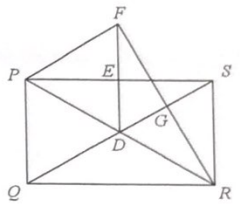
III. $\triangle PQR\cong\triangle PFR$

A. I and II only

B. I and III only

C. II and III only

D. I, II and III



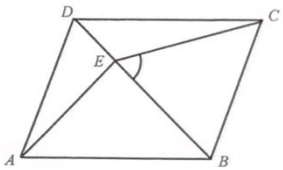
19. In the figure, $ABCD$ is a parallelogram. E is a point lying on BD such that $AE=BE$. If $\angle AEB=70°$ and $\angle DCE=20°$, then $\angle BEC=$

A. 60°.

B. 75°.

C. 85°.

D. 90°.



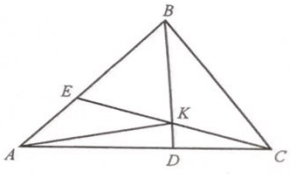
20. In the figure, D and E are the points lying on AC and AB respectively such that BD and CE intersect at K. It is known that $BE:AE=AD:DC=2:1$. Find $EK:KC$.

A. 4 : 3

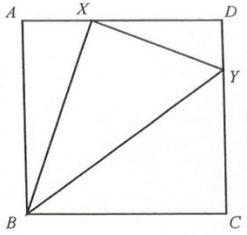
B. 5 : 3

C. 7 : 4

D. 8 : 5



21. In the figure, $ABCD$ is a square. X and Y are the points on AD and DC respectively. Given $BX = 42$ cm, $BY = 58$ cm and $XY = 40$ cm. Find the area of the square $ABCD$.



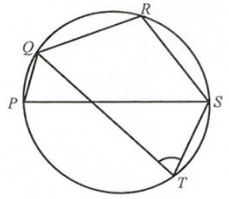
A. $\frac{37044}{221}\text{cm}^2$

B. $\frac{37044}{11}\text{cm}^2$

C. $\frac{388962}{11}\text{cm}^2$

D. $\frac{388962}{221}\text{cm}^2$

22. In the figure, PS is a diameter of the circle $PQRST$. If $QR = RS$ and $\angle PQR = 126°$, then $\angle QTS =$



A. 54°.

B. 60°.

C. 72°.

D. 78°.

23. If an interior angle of a regular n-sided polygon is greater than an exterior angle by 120°, which of the following are true?

I. The value of n is 10.

II. Each exterior angle of the polygon is 30°.

III. The number of axes of reflectional symmetry of the polygon is 12.

A. I and II only

B. I and III only

C. II and III only

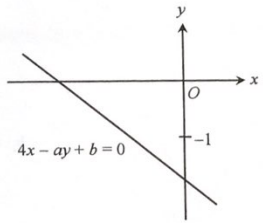
D. I, II and III

24. The figure shows the straight line $4x - ay + b = 0$. Which of the following are true?

I. $a < 0$

II. $b > 0$

III. $a + b > 0$



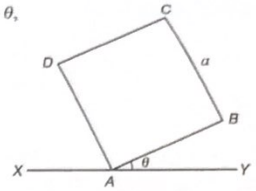
A. I and II only

B. I and III only

C. II and III only

D. I, II and III

25. The figure shows a square $ABCD$ with length equal to a. If $\angle BAY = \theta$, find the height of C above the horizontal line XAY.



A. $2a \sin \theta$

B. $2a \cos \theta$

C. $a(\sin \theta - \cos \theta)$

D. $a(\sin \theta + \cos \theta)$

26. If P is a moving point in the rectangular coordinate plane such that it maintains an equal distance from the point $F(0, -8)$ and the straight line L: $y = -4$, then the equation of the locus of P is

A. $y = -\frac{1}{8}x^2 - 6$.

B. $y = \frac{1}{8}x^2 - 6$.

C. $x^2 + (y + 8)^2 = 16$.

D. $y = -6$.

27. The equation of a circle is $4x^2 + 4y^2 - 15x + 24y + 36 = 0$. Which of the following is/are true?

I. The point $(2, -4)$ lies inside the circle.

II. The coordinates of the centre of the circle are $\left(\frac{15}{2}, -12\right)$.

III. The circle touches the y-axis.

A. I only

B. II only

C. I and III only

D. II and III only

28. The stem-and-leaf diagram below shows the distribution of the scores (in marks) of a class of students in an examination.

Stem (10 marks)	Leaf (1 mark)
4	8 8 9
5	0 0 2 3 7 9
6	1 3
9	4

A student is randomly selected from the class. Find the probability that the score of the selected student is not less than the mean mark.

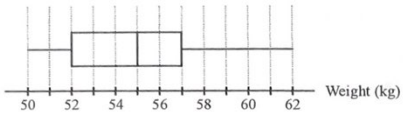
A. $\frac{1}{3}$

B. $\frac{5}{12}$

C. $\frac{1}{2}$

D. $\frac{7}{12}$

29. The box-and-whisker diagram below shows the distribution of the weights (in kg) of some students. Which of the following is/are true?



I. The weight of the lightest student is 52 kg.
II. The inter-quartile range of the distribution is 5 kg.
III. Less than half of the students are heavier than 54 kg.

A. I only
B. II only
C. I and III only
D. II and III only

30. Consider the following positive integers:

3 3 5 6 8 10 13 a b

If the mean of the above positive integers is 6, which of the following must be true?

I. Median = 5
II. Mode = 3
III. Range = 10

A. I only
B. II only
C. I and III only
D. II and III only

Section B

31. The L.C.M. of $3-a$, $9-a^2$ and $27-a^3$ is

A. $3-a$.
B. $(3-a)(3+a)(9+3a+a^2)$.
C. $(3-a)(3+a)(9-3a+a^2)$.
D. $(3-a)^3(3+a)(9+3a+a^2)$.

32. $3\times 2^{10}+2^6+4^4+6=$

A. 11000101011_2.
B. 11010100011_2.
C. 110001010110_2.
D. 110101000110_2.

33. Which of the following is the best estimate of 2022^{2021}?

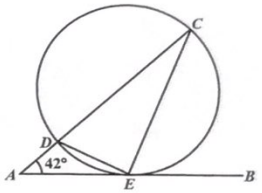
A. 10^{6681}
B. 10^{6682}
C. 10^{6683}
D. 10^{6684}

34. If $\begin{cases} 3\log y=\dfrac{x-2}{\log y} \\ x-5\log y=4 \end{cases}$, then $\log(10y)=$

A. 3 or $\frac{2}{3}$.
B. 2 or $-\frac{1}{3}$.
C. 100 or $\frac{1}{\sqrt[3]{10}}$.
D. 1000 or $\sqrt[3]{100}$.

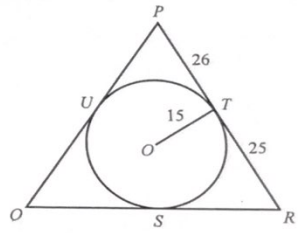
35. In the figure, ADC is a straight line. AB is the tangent to the circle at E. CE is the angle bisector of $\angle BED$ and $\angle CAE=42°$. Find $\angle CDE$.

A. 48°
B. 63°
C. 72°
D. 74°



36. In the figure, O is the centre of the circle STU. ΔPQR touches the circle at S, T and U. If $OT=15$ cm, $PT=26$ cm and $RT=25$ cm, then $QR=$

A. 49 cm
B. 50 cm
C. 51 cm
D. 52 cm



37. A and B are two different points on the line $x+2y+k=0$, where k is a non-zero constant. If $y=12$ is one of the altitudes of ΔOAB where O is the origin, find the x-coordinate of the orthocenter of ΔOAB.

A. 6
B. 12
C. 18
D. 24

38. Let $a=\frac{1}{1-2i}$ and $b=\frac{1}{1+2i}$. Which of the following must be true?

I. ab is a rational number.

II. The imaginary part of a is equal to the imaginary part of b.

III. The real part of $\frac{1}{a}$ is equal to the real part of $\frac{1}{b}$.

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

39. It is given that $\log a$, $\log b$ and $\log c$ is an arithmetic sequence, where a, b and c are positive numbers. Which of the following must be correct?

I. a^2, b^2, c^2 is a geometric sequence.

II. $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ is a geometric sequence.

III. a, b, c is an arithmetic sequence.

A. II only

B. III only

C. I and II only

D. I and III only

40. Consider the following system of inequalities: $\begin{cases} x \geq 0 \\ y \geq 0 \\ 3x+2y \leq 36 \\ x+4y \leq 32 \end{cases}$

Find the greatest value of $x + 5y + 3$ if (x, y) is a point lying in the region which represents the solution of the above system of inequalities.

A. 12

B. 15

C. 40

D. 43

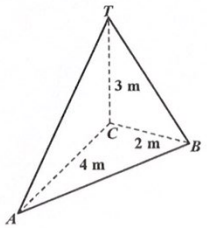
41. In the figure, CT is a vertical flagpole standing on the horizontal ground ABC, where $\angle ACB = 90°$. If the angle between the planes ABT and ACT is θ, then $\tan\theta =$

A. $\frac{5}{24}$.

B. $\frac{5}{6}$.

C. $\frac{6}{5}$.

D. $\frac{24}{5}$.



42. 5 students are selected from 9 boys and 11 girls to form a study group. If the group consists of at least one boy, how many different groups can be formed?

A. 2970

B. 12 534

C. 15 042

D. 15 504

43. Kim and Gloria are playing a shooting game. The probabilities that Kim and Gloria will hit the target are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. They shoot the target alternately until one of them hits the target. If Kim shoots first, find the probability that she will win the game.

A. $\frac{1}{2}$

B. $\frac{2}{3}$

C. $\frac{3}{4}$

D. $\frac{3}{5}$

44. In an ability test, the standard deviation is 12 marks. Lily gets 78 marks and her standard score is 1.5. If Ivana gets 57 marks in the test, then her standard score is

A. –2.

B. –0.25.

C. 0.25.

D. 0.5.

45. If two groups of numbers have the same mean, median and standard deviation, which of the following must be true?

I. They have the same range.

II. They have the same variance.

III. They have the same mode.

A. II only

B. III only

C. I and II only

D. I, II and III

End of Paper