St. Paul's Secondary School
2021 – 2022 F.6 Mock Examination Marking Scheme

Section A

1. $\dfrac{a^4b^{-9}}{(a^2b^{-2})^3}$

$=\dfrac{a^4b^{-9}}{a^6b^{-6}}$ 1M

$=a^{4-6}b^{-9-(-6)}$ 1M

$=\dfrac{1}{a^2b^3}$ 1A

2. (a) $2a^2+ab-3b^2$
$=(2a+3b)(a-b)$ 1A

(b) $4a^2+2ab-6b^2-2a-3b$
$=2(2a^2+ab-3b^2)-(2a+3b)$ 1M
$=2(2a+3b)(a-b)-(2a+3b)$
$=(2a+3b)(2a-2b-1)$ 1A

3. $\dfrac{y-1}{2y-5}=\dfrac{1}{x}$
$xy-x=2y-5$ 1M
$xy-2y=x-5$ 1M
$y(x-2)=x-5$
$y=\dfrac{x-5}{x-2}$ 1A or $\dfrac{5-x}{2-x}$

4 (a) $\dfrac{13-15x}{4}\geq 1-3x$
$13-15x\geq 4-12x$
$-3x\geq -9$
$x\leq 3$ 1A
$2-x<7$
$x>-5$ 1A
$\therefore -5<x\leq 3$ 1A

(b) -3 1A

5 (a) The marked price $=400/(1-20\%)$ 1M
$=\$500$ 1A

(b) Cost $=400/(1+25\%)$ 1M
$=\$320$ 1A

6. Join A and B
$\angle ABC=90^\circ$ ($\angle$ in semi-circle) 1M
$\angle ABD=90^\circ-25^\circ$
$=65^\circ$
$\angle ACD=\angle ABD$ ($\angle$s in the same segment) 1M
$=65^\circ$
In ΔCDE,
$\angle CDE+\angle ACD=\angle AED$ (ext. $\angle$ of Δ) 1M
$\angle CDE+65^\circ=128^\circ$
$\angle CDE=63^\circ$ 1A

7 (a) $(10, 315^\circ)$ 1A

(b) Note that $\angle AOC=60^\circ$ and $OA=OC$
So ΔOAC is an equilateral triangle. 1M
Hence $\angle OAC=60^\circ$ 1A

(c) Note that $\angle AOB=120^\circ$ and $OA=OB$
So $\angle OAB=(180^\circ-120^\circ)/2=30^\circ$ 1M
$\angle BAC=\angle OAB+\angle OAC=90^\circ$.
I agree the claim. 1A f.t.

8 (a) Let $f(x)=k_1x+k_2\sqrt{x}$, where k_1 and k_2 are non-zero constants 1A
$\because f(4)=46$
$\therefore 46=k_1(4)+k_2\sqrt{4}$ 1M
$2k_1+k_2=23$ .... (1)
$\because f(16)=188$
$\therefore 188=k_1(16)+k_2\sqrt{16}$
$4k_1+k_2=47$ .... (2)
$(2)-(1)$, $2k_1=24$
$k_1=12$
Substitute $k_1=12$ into (1),
$2(12)+k_2=23$
$k_2=-1$
$\therefore f(x)=12x-\sqrt{x}$ 1A

(b) $f(9)=12(9)-\sqrt{9}$
$=105$
$\therefore$ Decrease in the value of $f(x)=188-105$
$=83$ 1A

9 (a) $\frac{21+23+24+27+(30+a)+(30+b)+35+36+38+40+44+46+51+52}{14}=36$ 1M

$497+a+b=504$

$a+b=7$

$\because\ 0\le a\le b\le 5$

$\therefore\ \begin{cases}a=2\\b=5\end{cases}$ or $\begin{cases}a=3\\b=4\end{cases}$ 1A + 1A

(b) The weights of the two scouts who leave are 51 kg and 52 kg. 1A

When $a=2$ and $b=5$,

median of the weights of the remaining scouts $=\frac{35+35}{2}$ kg $=35$ kg

When $a=3$ and $b=4$,

median of the weights of the remaining scouts $=\frac{34+35}{2}$ kg $=34.5$ kg

$\therefore$ The smallest possible median of the weights is 34.5 kg. 1A

10 (a) $c+1+4+a=8+b-a+c$ 1M

$b=2a-3$

Note that $a>5$ and $b<11$.

When $a=6$, $b=2(6)-3=9$.

When $a=7$, $b=2(7)-3=11$ (*rejected*).

When $a\ge 8$, b must be greater than 11.

$\therefore\ a=6$ and $b=9$. 1A

(b) The required probability

$=\frac{(9-6)+1}{(1+1)+4+6+8+(9-6)+1}$ 1M

$=\frac{1}{6}$ 1A

(c) $\because$ The mode is greater than 2.

$\therefore\ c+1<8$

$c<7$

$\therefore$ The greatest possible value of c is 6. 1A

The standard deviation = 1.72 1A

11 (a) $f(1)=f(-1)-22$

$2(1)^3+a(1)^2+b(1)-6=2(-1)^3+a(-1)^2+b(-1)-6-22$ 1M

$a+b-4=a-b-30$

$2b=-26$

$b=-13$

$\therefore\ f(x)=2x^3+ax^2-13x-6$

$\because\ f(x)$ is divisible by $x+2$.

$\therefore\ f(-2)=0$

$2(-2)^3+a(-2)^2-13(-2)-6=0$ 1M

$4a=-4$

$a=-1$

$\therefore a=-1$ and $b=-13$ 1A

(b) $f(x)=2x^3-x^2-13x-6$

Using long division,

$$\begin{array}{r} 2x^2-5x-3 \\ x+2\overline{)2x^3-x^2-13x-6} \\ \underline{2x^3+4x^2} \\ -5x^2-13x \\ \underline{-5x^2-10x} \\ -3x-6 \\ \underline{-3x-6} \end{array}$$

$\therefore\ f(x)=(x+2)(2x^2-5x-3)$ 1M

$=(x+2)(x-3)(2x+1)$ 1A

(c) Let $t=x-2021$. Then $x=t+2021$.

Substitute $x=t+2021$ into $g(x-2021)=f(x)$. 1M

$g(t)=f(t+2021)$

$g(t)=(t+2021+2)(t+2021-3)[2(t+2021)+1]$

$g(x)=(x+2023)(x+2018)(2x+4043)$

$g(x)=0$

$(x+2023)(x+2018)(2x+4043)=0$

$x=-2023$ *or* -2018 *or* $-\frac{4043}{2}$

$\because\ -\frac{4043}{2}$ is not an integer.

$\therefore$ The claim is disagreed. 1A f.t.

12 (a) Let $\angle ECF = a$.

$\angle CFE = \angle EFD = 90°$ ($BE \perp DC$)

In $\triangle CEF$,

$$\begin{aligned} \angle ECF + \angle CEF + \angle CFE &= 180° \quad (\angle \text{ sum of } \triangle) \\ a + \angle CEF + 90° &= 180° \\ \angle CEF &= 90° - a \\ \angle DEF + \angle CEF &= 90° \quad (\text{property of rectangle}) \\ \angle DEF + 90° - a &= 90° \\ \angle DEF &= a \\ \therefore \quad \angle ECF &= \angle DEF \end{aligned}$$

$\therefore \quad \triangle CEF \sim \triangle EDF$ (AA)

Marking Scheme:	
Case 1 Any correct proof with correct reasons.	2
Case 2 Any correct proof without reasons.	1

(b)(i) $\because$ $AD = BC$ and $AD = CE$.

$\therefore$ $BC = CE = 30$ cm

$\because$ $BC = CE$ and $CF \perp BE$. 1M

$\therefore$ $BF = EF$

i.e. F is the mid-point of BE. 1A f.t.

(ii) $EF = BF$

$= 24$ cm

In $\triangle CEF$, by Pythagoras' theorem,

$CE^2 = CF^2 + EF^2$

$CF = \sqrt{30^2 - 24^2}$ cm 1M

$= 18$ cm

$\because$ $\triangle CEF \sim \triangle EDF$

$\therefore$ $\dfrac{CE}{ED} = \dfrac{CF}{EF}$ 1M

$\dfrac{30}{ED} = \dfrac{18}{24}$

$ED = 40$ cm 1A

13 (a)(i) Γ is the perpendicular bisector of XY 1A

(ii) $PX = PY$

$\sqrt{(x+3)^2 + (y-7)^2} = \sqrt{(x-5)^2 + (y-1)^2}$ 1M

$x^2 + 6x + 9 + y^2 - 14y + 49 = x^2 - 10x + 25 + y^2 - 2y + 1$

$16x - 12y + 32 = 0$

$4x - 3y + 8 = 0$ 1A

(b)(i) Yes, since the perpendicular bisector of XY passes through the centre 1A f.t.

(ii) Solving $4x - 3y + 8 = 0$ and $y = x$, 1M

$R = (-8, -8)$

Radius of circle $= \sqrt{(-8+3)^2 + (-8-7)^2} = \sqrt{250}$ 1M

Area of circle $= 250\pi$ sq. units 1A

14 (a) $\dfrac{a}{30} = \dfrac{5}{5+7.5}$

$a = 12$ 1A

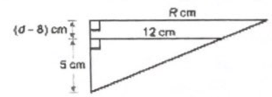
(b)(i) Volume of the cylindrical part $= \pi \times 12^2 \times 8$ cm^3

$= 1\,152\pi$ cm^3

$< 1\,500\pi$ cm^3

$\therefore$ $d > 8$ 1 f.t.

Let R cm be the radius of the water surface.



$\dfrac{R}{12} = \dfrac{d-8+5}{5}$ 1M

$R = \dfrac{12(d-3)}{5}$ 1 f.t.

(b)(ii) $1\,500\pi - 1\,152\pi = \dfrac{1}{3}\pi\left[\dfrac{12(d-3)}{5}\right]^2(d-8+5) - \dfrac{1}{3}\pi(12)^2 5$ 1A

$348\pi = \dfrac{1}{3}\pi(12)^2\left[\dfrac{(d-3)^3}{25} - 5\right]$

$(d-3)^3 = 306.25$

$d = \sqrt[3]{306.25} + 3$

$= 9.74$ (corr. to 3 sig. fig.) 1A

Area of wet surface on the frustum $= [\pi R\sqrt{(d-3)^2 + R^2} - \pi(12)\sqrt{5^2 + 12^2}]$ cm^2 1M

$= 401$ cm^2 (corr. to 3 sig. fig.)

So the claim is disagreed. 1A f.t.

15. (a) The required probability

$= \dfrac{C_5^9 + C_5^7}{C_5^{20}}$ 1M + 1M

$= \dfrac{147}{15504}$

$= \dfrac{49}{5168}$ 1A

(b) The required probability

$= 1 - \dfrac{49}{5168}$ 1M

$= \dfrac{5119}{5168}$ 1A

(5)

16. *Public Exam Reference: HKDSE 2017 Paper 1 Section B Q15*

$\begin{cases} \log_a 4 = 1 + \log_b 4 \quad \text{.........} (1) \\ \log_a \dfrac{1}{4} = 1 + \log_b \dfrac{1}{64} \quad \text{.........}(2) \end{cases}$ 1M

(1) + (2): $\log_a 4 + \log_a \dfrac{1}{4} = 2 + \log_b 4 + \log_b \dfrac{1}{64}$

$\log_a 1 = 2 + \log_b \dfrac{1}{16}$ 1M

$-2 = \log_b \dfrac{1}{16}$

$b^{-2} = \dfrac{1}{16}$

$b^2 = 16$

$b = 4$ or -4 (rejected) 1A

Substituting $b = 4$ into (1), we have $\log_a 4 = 2$

$a^2 = 4$

$a = 2$ or -2 (rejected)

$\therefore \quad \log_2 y = 1 + \log_4 x$

$= 1 + \dfrac{\log_2 x}{\log_2 4}$

$= 1 + \dfrac{\log_2 x}{2}$

$2\log_2 y = 2 + \log_2 x$

$\log_2 y^2 - \log_2 x = 2$

$\log_2 \left(\dfrac{y^2}{x} \right) = 2$

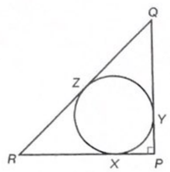
$\dfrac{y^2}{x} = 4$

$\therefore \quad x = \dfrac{y^2}{4}$ 1A

(4)

17. (a) Let r be the radius of the in-circle XYZ of ΔPQR.

Consider the figure.



$QZ = QY = 10 - r$ *(tangent properties)*

$RZ = RX = 10 - r$ *(tangent properties)* 1M

$QR^2 = PQ^2 + PR^2$ *(Pyth. theorem)*

$QR = \sqrt{10^2 + 10^2}$ 1M

$= 10\sqrt{2}$

$10 - r + 10 - r = 10\sqrt{2}$ 1M

$20 - 2r = 10\sqrt{2}$

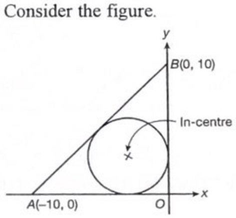
$r = \underline{\underline{10 - 5\sqrt{2}}}$ 1A

(4)

(b) From (a),

radius of the in-circle of $\Delta OAB = 10 - 5\sqrt{2}$

Consider the figure.



$\therefore$ Coordinates of the in-centre of ΔOAB

$= (5\sqrt{2} - 10, 10 - 5\sqrt{2})$ 1A

Substituting the coordinates of the in-centre into the equation of the line,

L.H.S. $= 2(5\sqrt{2} - 10) - \sqrt{2}(10 - 5\sqrt{2}) + 10$ 1M

$= 0$

$=$ R.H.S.

$\therefore$ The line passes through the in-centre of ΔOAB.

$\therefore$ The claim is agreed. 1A

(3)

18

Public Exam Reference: HKDSE 2017 Paper I Section B Q19

(a) AD

$= (CD \cos \angle ADC)$ cm

$= [12 \cos(180° - 120°)]$ cm

$= 6$ cm

$BD = (11 - 6)$ cm $= 5$ cm

Let x cm be the distance between B and F.

$\because \Delta FBE \sim \Delta FDA$

$\therefore \dfrac{BF}{DF} = \dfrac{BE}{AD}$ 1M

$\dfrac{x}{x+5} = \dfrac{4}{6}$

$6x = 4x + 20$

$x = 10$ 1A

$\therefore$ The distance between B and F is 10 cm.

(2)

(b) The area of ΔDCF

$= \dfrac{1}{2}(DF)(CD)\sin \angle BDC$ 1M

$= \left[\dfrac{1}{2}(10+5)(12)\sin 120°\right]$ cm^2

$= 45\sqrt{3}$ cm^2

$= \underline{\underline{77.9 \text{ cm}^2}}$ *(cor. to 3 sig. fig.)* 1A

(2)

(c) In ΔCDF, by the cosine formula,

$CF^2 = DF^2 + CD^2 - 2(DF)(CD)\cos \angle BDC$ 1M

$CF = \sqrt{15^2 + 12^2 - 2(15)(12)\cos 120°}$ cm

≈ 23.4307 cm

Let h cm be the perpendicular distance from D to CF.

$\dfrac{1}{2}(CF)h = 45\sqrt{3}$ 1M

$h \approx 6.6530$

Let the required inclination be θ.

$\sin\theta = \dfrac{6}{h}$

$\theta = 64.4°$ *(cor. to 3 sig. fig.)* 1A

$\therefore$ The inclination of the plane BCD to the horizontal ground is 64.4°

(3)

19. (a) $f(x) = 2x^2 - 16kx - 8x + 32k^2 + 31k + 18$

$= 2(x^2 - 8kx - 4x) + 32k^2 + 31k + 18$

$= 2[x^2 - 2(4k + 2)x + (4k + 2)^2 - (4k + 2)^2] + 32k^2 + 31k + 18$ 1M

$= 2[x - (4k + 2)]^2 - 2(16k^2 + 16k + 4) + 32k^2 + 31k + 18$

$= 2(x - 4k - 2)^2 - 32k^2 - 32k - 8 + 32k^2 + 31k + 18$

$= 2(x - 4k - 2)^2 - k + 10$

$\therefore$ The coordinates of Q are $(4k + 2, -k + 10)$. 1A

----------(2)

(b) $f(2x - 22) - 14 = 2(2x - 22 - 4k - 2)^2 - k + 10 - 14$

$= 2[2(x - 2k - 12)]^2 - k - 4$

$= 8(x - 2k - 12)^2 - k - 4$

$\therefore$ The coordinates of R are $(2k + 12, -k - 4)$. 1A

----------(1)

(c)(i) Coordinates of the mid-point of QS

$= \left(\frac{4k+2+4k-2}{2}, \frac{-k+10-k-4}{2}\right)$

$= (4k, -k + 3)$

Slope of $QS = \frac{-k+10+k+4}{4k+2-4k+2}$

$= \frac{14}{4}$

$= \frac{7}{2}$

Slope of the perpendicular bisector of $QS = -\frac{2}{7}$

$\therefore$ The required equation is

$y + k - 3 = -\frac{2}{7}(x - 4k)$ 1M

$7y + 7k - 21 = -2x + 8k$ 1A

$2x + 7y - k - 21 = 0$

(ii) $\because$ y-coordinate of R = y-coordinate of S

$\therefore$ RS is a horizontal line.

$\therefore$ x-coordinate of G = x-coordinate of the mid-point of RS

$= \frac{2k+12+4k-2}{2}$ 1M

$= 3k + 5$

Substitute $x = 3k + 5$ into the equation obtained in (c)(i).

$2(3k + 5) + 7y - k - 21 = 0$ 1M

$6k + 10 + 7y - k - 21 = 0$

$7y = 11 - 5k$

$y = \frac{11-5k}{7}$

$\therefore$ Coordinates of $G = \left(3k+5, \frac{11-5k}{7}\right)$ 1A

(iii) If $AQGS$ is a square, then $GQ \perp GS$. 1M

Slope of $GQ = \frac{\frac{11-5k}{7}+k-10}{3k+5-4k-2} = \frac{2k-59}{7(3-k)}$

Slope of $GS = \frac{\frac{11-5k}{7}+k+4}{3k+5-4k+2} = \frac{2k+39}{7(7-k)}$

Slope of GQ × slope of $GS = -1$

$\frac{2k-59}{7(3-k)} \times \frac{2k+39}{7(7-k)} = -1$

$4k^2 - 40k - 2\,301 = -49(k^2 - 10k + 21)$

$53k^2 - 530k - 1\,272 = 0$

$k = 12$ *or* -2 (*rejected*)

$\therefore$ When $k = 12$, we have $GQ \perp GS$. 1A

Note that $\angle AQG = \angle ASG = 90°$.

When $k = 12$, 1M

$\angle QAS = 360° - 90° - 90° - 90° = 90°$

and $QG = GS$ (*or* $AQ = AS$)

$\therefore$ When $k = 12$, $AQGS$ is a square.

$\therefore$ It is possible that $AQGS$ is a square. 1A

----------(9)

1. $\dfrac{8^{n+1}}{2^{3n+1}}$

$=\dfrac{2^{3(n+1)}}{2^{3n+1}}$

$=2^{3n+3-3n-1}$

$=2^2$

$=4$

[C]

2. $\dfrac{1-x}{1+x}=2y$

$1-x=2y(1+x)$

$1-x=2y+2xy$

$x+2xy=1-2y$

$x(1+2y)=1-2y$

$x=\dfrac{1-2y}{1+2y}$

[D]

3. $(x-1)(x-2)=x-1$

$(x-1)(x-2)-(x-1)=0$

$(x-1)[(x-2)-1]=0$

$(x-1)(x-3)=0$

$x=1$ or 3

[D]

4. $4rs+6-3s-8r$

$=4rs-8r-3s+6$

$=4r(s-2)-3(s-2)$

$=(4r-3)(s-2)$

[C]

5. Sub $x=-5$

$(-5)^2+p(0)-q=(-5+3)(-5-q)$

$25-q=10+2q$

$3q=15$

$q=5$

[A]

6. $4x^2-9$

$=(2x)^2-3^2$

$=(2x+3)(2x-3)$

(III) is an equation

[B]

7. $f(-1)=0$

$(-1)^{13}+m(-1)^9+n=0$

$-1-m+n=0$

$n=m+1$

Remainder

$=f(1)$

$=(1)^{13}+m(1)^9+n$

$=1+m+n$

$=1+m+(m+1)$

$=2m+2$

[C]

8. When $x-2=-1$, $x=1$

$\therefore$ $g(-1)$

$=f(2)$

$=6-2(2)-(2)^2$

$=-2$

[B]

9. Cost of the diamond ring that gains money

$=$ \$$67200\div(1+40\%)$

$=$ \$48000

Cost of the diamond ring that loses money

$=$ \$$67200\div(1-40\%)$

$=$ \$112000

Total gain/loss

$=$ \$$(67200\times2-112000-48000)$

$=$ −\$25600

[A]

10. $x\le7$ or $x<-1$

i.e. $x\le7$

[C]

11. $bc:ca:ab=1:2:3$

$bc:ca=1:2\Rightarrow b:a=1:2$

$ca:ab=2:3\Rightarrow c:b=2:3$

$\therefore\ a:b:c=6:3:2$

$\dfrac{a}{bc}:\dfrac{b}{ca}$

$=\dfrac{6k}{(3k)(2k)}:\dfrac{3k}{(2k)(6k)}$

$=4:1$

[A]

12. $z=\dfrac{ky^2}{\sqrt{x}}$

$k=\dfrac{\sqrt{x}\cdot z}{y^2}$

$k^2=\dfrac{xz^2}{y^4}$

[C]

13. 1st pattern = 3

2nd pattern = 3 + 5

3rd pattern = 3 + 5 + 7

…

By trial and error, $n=9$

(or solving $\dfrac{n}{2}[2(3)+(n-1)(2)]=99$)

[B]

14. When $px+q=0$, the function attains its minimum (−3).

$px+q=0$

$x=-\dfrac{q}{p}$, which is positive

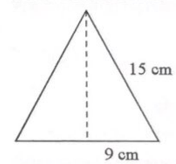
i.e. Coordinates of vertex = (+, −3)

[A]

15. Length of a side of the square base

$=72\div4=18$ cm

For each lateral face:



The height is therefore 12 cm.

Total surface area

$=4\times\dfrac{18\times12}{2}+18^2$

$=756$ cm^2

[B]

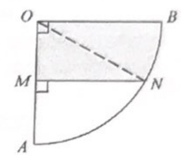
16. $\dfrac{1}{3}\pi R^2h=3\times\pi r^2h$

$\left(\dfrac{R}{r}\right)^2=9$

$R:r=3:1$

[D]

17.



$\cos\angle MON=\dfrac{OM}{ON}=\dfrac{OM}{OA}=\dfrac{1}{2}$

$\angle MON=60^\circ$

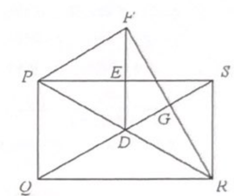
Area of shaded region

$=\pi(12)^2\times\dfrac{30^\circ}{360^\circ}+\dfrac{1}{2}(6)(12)\sin60^\circ$

$=(12\pi+18\sqrt{3})$ cm^2

[B]

18.



I. $DR=DP$ (prop. of rectangle)

$=DF$ (equil. Δ)

II. $\because\ PF=RS$

$\therefore\ PF=DR=DF=DR=DS=RS$

$\angle DFR=\angle DRF$ (base $\angle$s, isos. Δ)

$\angle DFR+\angle DRF=60^\circ$ (ext. $\angle$ of Δ)

$\angle DRG=\angle SRG=30^\circ$

III. $PF=PQ$

$\angle FPR=\angle QPR=60^\circ$ (equil. Δ)

$PR=PR$

$\Delta PQR\cong\Delta PFR$ (SAS)

[D]

19. $\because$ $PF = RS$

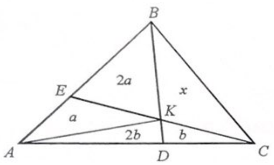
$\therefore$ $\angle EAB = \angle EBA = 55°$ (base $\angle$s, isos. Δ)

$\therefore$ $\angle CDE = \angle EBA = 55°$ (alt. $\angle$s, $DC // AB$)

$\angle BEC = 55° + 20° = 75°$ (ext. $\angle$ of Δ)

[B]

20.



$AD : DC = 2 : 1$

Area of ΔABD : Area of $\Delta CBD = 2 : 1$

Area of ΔAKD : Area of $\Delta CKD = 2 : 1$

$BE : AE = 2 : 1$

Area of ΔBKE : Area of $\Delta AKE = 2 : 1$

$\therefore$ $(3a + 2b) : (x + b) = 2 : 1$

$3a + 2b = 2x + 2b$

$x = 1.5a$

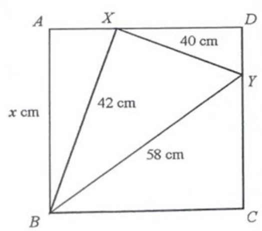
$\therefore$ $EK : KC$

$= 2a : 1.5a$

$= 4 : 3$

[A]

21.



By converse of Pyth. Theorem, $\angle BXY = 90°$

Therefore, $\triangle ABX \cong \triangle DXY$ (AAA)

Let $AB = x$ cm

$$\frac{x - AX}{x} = \frac{40}{42}$$

$$21x - 21(AX) = 20x$$

$$AX = \frac{x}{21}$$

Consider ΔAXB,

$$x^2 + (AX)^2 = 42^2$$

$$x^2 + \frac{x^2}{441} = 1764$$

$$441x^2 + x^2 = 777924$$

$$x^2 = \frac{388962}{221}$$

$\therefore$ The area of square is $\frac{388962}{221}$ cm^2.

[D]

22. Join QS

$\angle PQS = 90°$ ($\angle$ in semi-circle)

$\angle RQS = 126° - 90° = 36°$

$\because$ $QR = RS$

$\angle RQS = \angle RSQ = 36°$ (base $\angle$s, isos. Δ)

$\angle QRS = 108°$

$\angle QTS = 180° - 108°$ (opp $\angle$s, cyclic quad)

$= 72°$

[C]

23. For a regular n-sided polygon,

$$\frac{(n-2)\times 180°}{n} = \frac{360°}{n} + 120°$$

$$n = 12$$

II. Each exterior angle $= \frac{360°}{n} = 30°$

III. No. of axes of reflectional symmetry $= n$

[C]

24. x-intercept $= -\frac{b}{4}$; y-intercept $= \frac{b}{a}$

Slope $= \frac{4}{a}$

I. $\frac{4}{a} < 0 \Rightarrow a < 0$

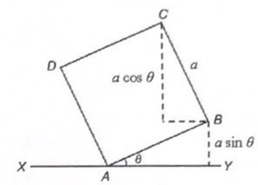
II. $-\frac{b}{4} < 0 \Rightarrow b > 0$

III. $\frac{b}{a} < -1 \Rightarrow b > -a$ $(a < 0!!)$

$\therefore$ $a + b > 0$

[D]

25.



[D]

26. Let $P = (x, y)$

$$\sqrt{(x-0)^2 + (y-(-8))^2} = -4 - y$$

$$x^2 + y^2 + 16y + 64 = 16 + 8y + y^2$$

$$8y = -x^2 - 48$$

$$y = -\frac{1}{8}x^2 - 6$$

[A]

27. Make it in general form:

$$x^2 + y^2 - \frac{15}{4}x + 6y + 9 = 0$$

Centre $= \left(\frac{15}{8}, -3\right)$

Radius $= \sqrt{\left(\frac{-15}{8}\right)^2 + \left(\frac{6}{2}\right)^2 - 9} = \frac{15}{8}$

[C]

28. Mean = 57

The required probability $= \frac{5}{12}$

[B]

29. I. Weight of the lightest student is 50 kg.

II. IQR = 57 − 52 = 5 kg

III. Median = 55 kg. i.e. More than or exactly half of the students are heavier than 54 kg.

[B]

30. $\frac{a + b + 48}{9} = 6$

$\therefore$ $a + b = 6$

$a < b \leq 5$

II. If $a = 1$ and $b = 5$, modes are 3 and 5

III. If $a = 1$ and $b = 5$, range = 13 − 1 = 12

[A]

31. $9 - a^2 = (3 + a)(3 - a)$

$27 - a^3 = (3 - a)(9 + 3a + a^2)$

$\therefore$ L.C.M. $= (3 - a)(3 + a)(9 + 3a + a^2)$

[B]

32. $3 \times 2^{10} + 2^6 + 4^4 + 6$

$= (2^1 + 1) \times 2^{10} + 2^6 + (2^2)^4 + (2^2 + 2^1)$

$= 1 \times 2^{11} + 1 \times 2^{10} + 0 \times 2^9 + 1 \times 2^8$

$+ 0 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4$

$+ 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

[D]

33. Let $10^x = 2022^{2021}$

$\log 10^x = \log 2022^{2021}$

$x = (2021)\log 2022$

$x = 6681$

[A]

34. From (2), $x = 4 + 5\log y$

$$3\log y = \frac{4 + 5\log y - 2}{\log y}$$

$$3(\log y)^2 = 5\log y + 2$$

$$3(\log y)^2 - 5\log y - 2 = 0$$

$$[\log y - 2][3\log y + 1] = 0$$

$\log y = 2$ or $\log y = -\frac{1}{3}$

$\log(10y)$

$= \log 10 + \log y$

$= 1 + 2$ or $1 - \frac{1}{3}$

$= 3$ or $\frac{2}{3}$

[A]

35. $\angle CED = \angle BEC$

$= \angle CDE$ ($\angle$ *in alt. segment*)

$\angle AED + \angle CED + \angle BEC = 180°$

$\angle AED = 180° - 2\angle CDE$

In $\triangle AED$,

$\angle CDE = \angle CAE + \angle AED$ (*ext.* $\angle$ *of* $\triangle$)

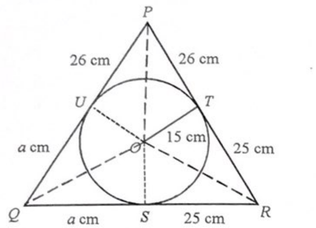
$\angle CDE = 42° + 180° - 2\angle CDE$

$3\angle CDE = 222°$

$\angle CDE = 74°$

[D]

36. Let $QS = QU = a$ cm



Area of ΔPQR

$=\dfrac{15\times 51}{2}+\dfrac{15\times(a+26)}{2}+\dfrac{15\times(a+25)}{2}$

$=15(a+51)$

By Heron's formula, $s = a + 51$

Area of ΔPQR

$=\sqrt{(a+51)[(a+51)-51][(a+51)-(a+26)][(a+51)-(a+25)]}$

$=\sqrt{(a+51)(a)(25)(26)}$

$15(a+51)=\sqrt{(a+51)(a)(25)(26)}$

$225(a+51)^2=650a(a+51)$

$9(a+51)=26a$

$17a=459$

$a=27$

$QR = 27 + 25 = 52$ cm

Alternative

$\angle OPT=\angle OPU=\tan^{-1}\dfrac{15}{26}$

$\angle ORT=\angle ORS=\tan^{-1}\dfrac{15}{25}$

$\angle OQS=\dfrac{180°-2\angle OPT-2ORT}{2}$

$\tan\angle OQS=\dfrac{15}{a}$

$a=27$

[D]

37. Another altitude L is the straight line passing through O and perpendicular to $x + 2y + k = 0$

Slope of $L=\dfrac{-1}{-\frac{1}{2}}=2$

$\therefore$ Equation of L is $y = 2x$

Point of intersection of two altitudes

$\begin{cases} y=12 \\ y=2x \end{cases}$

$\therefore$ x-coordinate of the orthocenter = 6

[A]

38. I. $\dfrac{1}{1-2i}\cdot\dfrac{1}{1+2i}$

$=\dfrac{1}{1-4i^2}$

$=\dfrac{1}{1-4(-1)}$

$=\dfrac{1}{5}$

II. $\dfrac{1}{1-2i}=\dfrac{1}{5}+\dfrac{2}{5}i$

$\dfrac{1}{1+2i}=\dfrac{1}{5}-\dfrac{2}{5}i$

III. $\dfrac{1}{a}=1-2i$

$\dfrac{1}{b}=1+2i$

[B]

39. $\log b-\log a=\log c-\log b=k$

$\log\dfrac{b}{a}=\log\dfrac{c}{b}=k$

i.e. $\dfrac{b}{a}=\dfrac{c}{b}$, $\therefore$ a, b and c is a G.S.

I. $\dfrac{b^2}{a^2}=\left(\dfrac{b}{a}\right)^2=\left(\dfrac{c}{b}\right)^2=\dfrac{c^2}{b^2}$

II. $\dfrac{\frac{1}{b}}{\frac{1}{a}}=\dfrac{a}{b}=\dfrac{b}{c}=\dfrac{\frac{1}{c}}{\frac{1}{b}}$

[C]

40. Consider $3x + 2y = 36$

x-intercept = 12; y-intercept = 18

Consider $x + 4y = 32$

x-intercept = 32; y-intercept = 8

Point of intersection of two lines = (8, 6)

Test the values at (12, 0), (0, 8) and (8, 6):

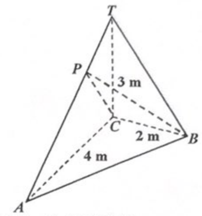
At (12, 0), the value of function = 15

At (0, 8), the value of function = 43

At (8, 6), the value of function = 41

[D]

41. The required angle is $\angle BPC$



$\Delta ACP \sim \Delta ATC$ (AA)

$\dfrac{CP}{TC}=\dfrac{AC}{AT}$

$\dfrac{CP}{3}=\dfrac{4}{5}$

$CP=\dfrac{12}{5}$

$\tan\theta=\dfrac{BC}{CP}=\dfrac{2}{\frac{12}{5}}=\dfrac{5}{6}$

[B]

42. The number of groups

$=C_5^{20}-C_5^{11}$

$=15042$

[C]

43. The required probability

= K✓ or K✗G✗K✓ or K✗G✗K✗G✗K✓ or…

$=\dfrac{1}{2}+\dfrac{1}{2}\times\dfrac{2}{3}\times\dfrac{1}{2}+\dfrac{1}{2}\times\dfrac{2}{3}\times\dfrac{1}{2}\times\dfrac{2}{3}\times\dfrac{1}{2}+\ldots$

$=\dfrac{\frac{1}{2}}{1-\frac{1}{2}\times\frac{2}{3}}$

$=\dfrac{3}{4}$

[C]

44. Let μ be the mean score

$\dfrac{78-\mu}{12}=1.5$

$\mu=60$

Ivana's standard score

$=\dfrac{57-60}{12}$

$=-0.25$

[B]

45. Counter example of I and III:

Group 1: 4 4 4 4 5 6 6 6 6

Group 2: 3 5 5 5 5 5 5 5 7

II. Variance = σ^2

[A]