SHUN TAK FRATERNAL ASSOCIATION LEUNG KAU KUI COLLEGE
S. 5 MATHEMATICS Compulsory Part - PAPER 1
Question - Answer Book

Date of examination: 10 – 6 – 2019

Time allowed: 2 hours 15 minutes

Name	
Class	
Class Number	

1. Write your name, Class and Class Number in the space provided on this cover.

2. Attempt **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

3. Graph paper and supplementary answer sheets will be supplied on request. Write your name, Class and Class Number on each sheet, and fasten them with string.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

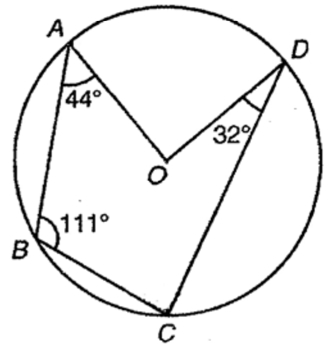
6. The diagrams in this paper are not necessarily drawn to scale.

1. The stem-and-leaf diagram below shows the distribution of the weights of 20 cats.

Stem (1 kg)	Leaf (0.1 kg)
2	5 7 7 8
3	0 0 1 2 3 4 6 7 8
4	1 2 2 5 7 8 9

Find the median, the range and the standard deviation of the distribution. (3 marks)

2. In the figure, $\angle ABC = 111°$, $\angle BAO = 44°$ and $\angle CDO = 32°$. Find $\angle BCD$. (3 marks)



3. (a) Solve the inequality $\frac{7x+26}{4} \le 2(3x-1)$.

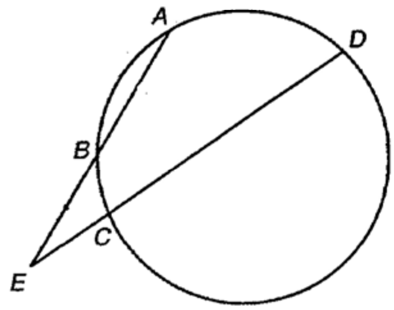
(b) Find the number of integers satisfying both inequalities $\frac{7x+26}{4} \le 2(3x-1)$ and $45 - 5x \ge 0$.

(4 marks)

4. If the line $L: y = 3x + 8$ and the circle $S: x^2 + y^2 - 5x - 4y + k = 0$ do not intersect, find the minimum integral value of k. (5 marks)

5. In the rectangular coordinate plane, the coordinates of the point A are $(-2, 3)$. A' is the reflection image of A with respect to the line $L: y = 1$.
   (a) Write down the coordinates of A'.
   (b) Let P be a moving point in the rectangular coordinate plane such that P is equidistant from A' and L. Find the equation of the locus of P and describe the locus of P. (5 marks)

6. In the figure, ABE and DCE are straight lines. If $AB = BE = 3$ cm and $CD = 7$ cm, find the length of CE. (5 marks)



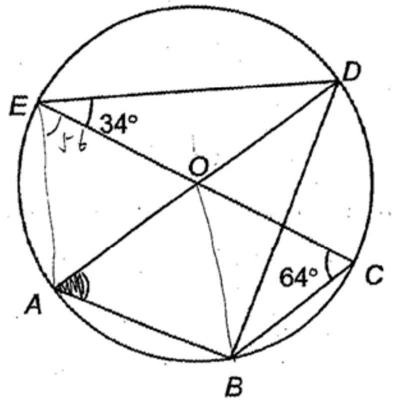
7. (a) How many 6-digit even numbers can be formed from the digits 1, 2, 3, 5, 7 and 9 if

   (i) there are no restrictions?

   (ii) no repetition of digits is allowed? (2 marks)

   (b) 2 teachers and 2 parents are selected from 5 teachers and 12 parents to sit in a row. In how many ways can the 2 teachers be arranged on the left of the 2 parents? (3 marks)

8.



In the figure, O is the centre of the circle. AD and CE are diameters of the circle. It is given that $\angle OCB = 64°$ and $\angle OED = 34°$.

(a) Find $\angle OAB$.

(b) Find $\frac{AB}{AD}$.

(5 marks)

9. The coordinates of points X and Y are $(-3, 7)$ and $(5, 1)$ respectively. P is a moving point in the rectangular coordinate plane such that $PX = PY$. Denote the locus of P by Γ.

(a) (i) Find the equation of Γ.

(ii) Describe the geometric relationship between Γ and the line segment XY.

(3 marks)

(b) It is given that a circle passes through X and Y. Denote the centre of the circle by R.

(i) Does Γ pass through R? Explain your answer.

(ii) The line $y = x$ divides the circle into two equal halves. Find the coordinates of R.

(4 marks)

10. In a Mathematics competition, Ray and Cindy score 68 marks and 50 marks respectively. The standard scores of Ray and Cindy are 0 and -1.5 respectively.

(a) Find the mean and the standard deviation of the scores in the competition. (3 marks)

(b) Later, Ray is found cheating in the competition and he is disqualified. Cindy claims that her standard score will increase after excluding Ray's score. Is Cindy's claim correct?

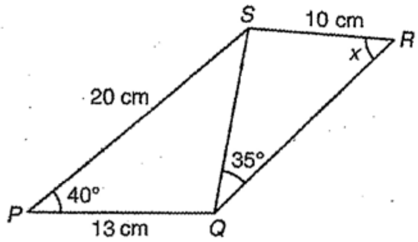
Explain your answer. (2 marks)

11. In the figure, $PQRS$ is a quadrilateral with $PQ = 13$ cm, $RS = 10$ cm, $PS = 20$ cm, Ð$QPS = 40°$ and $\angle RQS = 35°$.Find

(a) the length of QS,

(b) x.

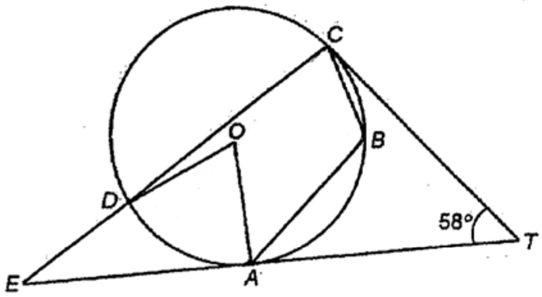
(Give your answers correct to 3 significant figures.) (5 marks)



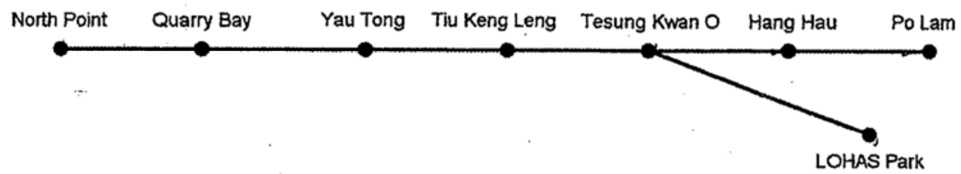
12. In the figure, TA and TC are tangents to the circle at A and C respectively. CD and TA are produced to meet at E. It is given that $\angle ATC = 58°$.

(a) Find $\angle ABC$. (3 marks)

(b) If $\angle AOD = 62°$, find $\angle AED$. (3 marks)



13. The figure shows the route map of the Tseung Kwan O line of MTR.



(a) A passenger gets on a train at a station and gets off at another station. Find the number of different journeys the passenger can travel along the Tseung Kwan O line. (2 marks)

(b) It is given that a passenger has to pay \$10 or above per journey if he / she travels through the cross-harbour tunnel which is located between Quarry Bay and Yau Tong. How many different journeys in (a) charge the passenger \$10 or above? (3 marks)

14. A developer plans to build a hotel with x single rooms and y double rooms subject to the conditions listed in the proposal below:

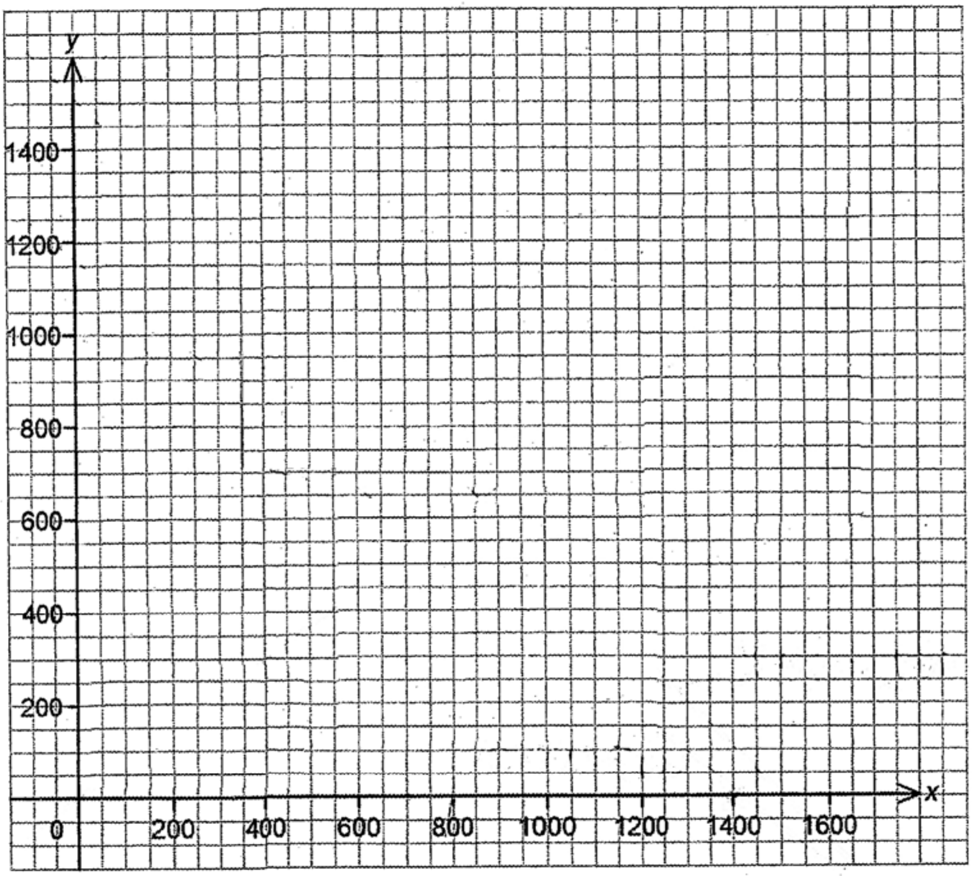
(1) The hotel should accommodate at least 1500 people.

(2) Each single room and double room must occupy an area of 15m^2 and $18\ \text{m}^2$ respectively. The total floor area available for the rooms is at most $21\ 000\ \text{m}^2$.

(3) The number of double rooms should be at most 75% of the number of single rooms.

(a) Write down all the constraints on x and y. (3 marks)

(b) On the graph paper provided, draw and indicate the solution that satisfies all the constraints in (a). (3 marks)



(c) It is known that the monthly maintenance costs for a single room and a double room are \$800 and \$1200 respectively. Using (b), find the values of x and y so that the hotel can be run at the minimum maintenance cost (3 marks)

15. The time taken for a group of people to finish a fitness test are normally distributed with a mean of 33 s and a standard deviation of 3.5 s.

(a) Find the percentage of people whose finishing time is between 29.5 s and 40 s. (2 marks)

(b) If 200 people can finish the test within 26 s, find the number of people in the group. (2 marks)

(c) Assume that people who finish the fitness test within 37.5 s will pass the test. If Peter is a person in the group and his standard score is 1.4, can he pass the test? Explain your answer. (3 marks)

(Assume that 68%, 95% and 99.7% of the data of a normal distribution lie within one, two and three standard deviations from the mean respectively.)

16. (a) In Figure (16a), the straight line $L_1 : x + y + 6 = 0$ cuts the x-axis and the y-axis at A and B respectively and the circumcircle of $\triangle OAB$ is drawn.

(i) Find the coordinates of A and B.

(ii) Find the equation of the circumcircle of $\triangle OAB$ in the general form. (5 marks)

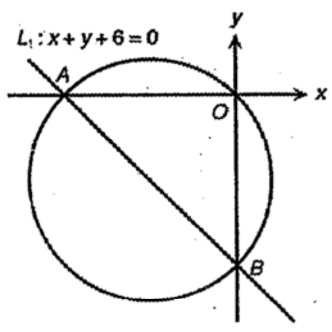


Figure (16a)

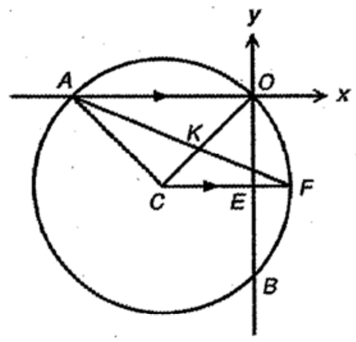


Figure (16b)

(b) In Figure (16b), C is the centre of the circle in Figure (1) and F is a point lying on the circle such that CF // AO. If CF cuts the y-axis at E and K is the intersection of OC and AF,

(i) prove that $\angle ACO = 90°$,

(ii) find the acute angle between OC and AF,

(iii) find the area of the segment AOF in terms of π.

(Leave your answer in surd form.) (8 marks)

17. Figure 17(a) shows a piece of paper card $ABCD$ with $AB = BC = 50$ cm, $AD = CD = 32$ cm and reflex $\angle ADC = 210°$.

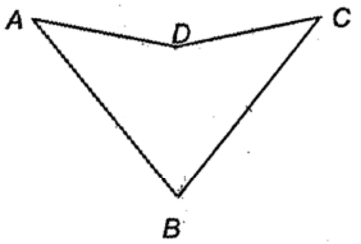


Figure 17(a)

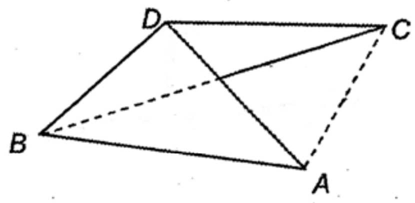


Figure 17(b)

(a) Find the length of BD. (3 marks)

(b) The paper card is then folded along BD such that AB and BC lie on the horizontal ground as shown in Figure 17(b). It is given that $\angle ADC = 40°$.

(i) Find the distance between A and C on the horizontal ground.

(ii) Find the angle between the plane ACD and the horizontal ground.

(iii) Find the volume of the pyramid $ABCD$.

(8 marks)

(c) Jerry claims that if P is the mid-point of BD, the volume of the pyramid $ABCP$ is half of the volume of the pyramid $ABCD$. Do you agree? Explain your answer. (2 marks)

End of paper

1. The stem-and-leaf diagram below shows the distribution of the weights of 20 cats.

Stem (1 kg)	Leaf (0.1 kg)
2	5 7 7 8
3	0 0 1 2 3 4 6 7 8
4	1 2 2 5 7 8 9

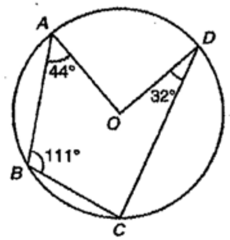
Find the median, the range and the standard deviation of the distribution. (3 marks)

The median $= \dfrac{3.4+3.6}{2}\text{kg} = \underline{3.5\text{ kg}}$

The range $= (4.9-2.5)\text{kg} = \underline{2.4\text{ kg}}$

The standard deviation $= \underline{0.738\text{ kg}}$ (cor. to 3 sig. fig.)

2. In the figure, $\angle ABC = 111°$, $\angle BAO = 44°$ and $\angle CDO = 32°$. Find $\angle BCD$. (3 marks)



Join AD

$\angle ABC + \angle ADC = 180°$ (opp. ∠s, cyclic quad.)

$111° + (\angle ODA + 32°) = 180°$

$\angle ODA = 37°$

In $\triangle OAD$,

$\because OA = OD$ (radii)

$\therefore \angle OAD = \angle ODA = 37°$ (base ∠s, isos. △)

$\angle BAD + \angle BCD = 180°$ (opp. ∠s, cyclic quad.)

$(44° + 37°) + \angle BCD = 180°$

$\angle BCD = \underline{99°}$

3. (a) Solve the inequality $\dfrac{7x+26}{4} \le 2(3x-1)$.

(b) Find the number of integers satisfying both inequalities $\dfrac{7x+26}{4} \le 2(3x-1)$ and $45-5x \ge 0$. (4 marks)

(a) $\dfrac{7x+26}{4} \le 2(3x-1)$

$7x+26 \le 24x-8$

$34 \le 17x$

$x \ge 2$

(b) $45-5x \ge 0$

$9 \ge x$

$\therefore 2 \le x \le 9$

$\therefore$ 8 integers satisfying both inequalities

4. If the line $L: y = 3x+8$ and the circle $S: x^2+y^2-5x-4y+k=0$ do not intersect, find the minimum integral value of k. (5 marks)

$$\begin{cases} y = 3x+8 & \text{......(1)} \\ x^2+y^2-5x-4y+k=0 & \text{......(2)} \end{cases}$$

By substituting (1) into (2), we have

$x^2+(3x+8)^2-5x-4(3x+8)+k=0$

$x^2+9x^2+48x+64-5x-12x-32+k=0$

$10x^2+31x+(32+k)=0$ ......(*)

$\because$ L and S do not intersect.

$\therefore$ For the equation (*),

$\Delta < 0$

$(31)^2-4(10)(32+k) < 0$

$961-1280-40k < 0$

$-319 < 40k$

$k > -\dfrac{319}{40} = -7.975$

$\therefore$ The minimum integral value of k is -7.

5. In the rectangular coordinate plane, the coordinates of the point A are $(-2, 3)$. A' is the reflection image of A with respect to the line $L: y = 1$.

(a) Write down the coordinates of A'.

(b) Let P be a moving point in the rectangular coordinate plane such that P is equidistant from A' and L. Find the equation of the locus of P and describe the locus of P. (5 marks)

(a) The coordinates of $A' = \underline{(-2,-1)}$

(b) Let (x, y) be the coordinates of P

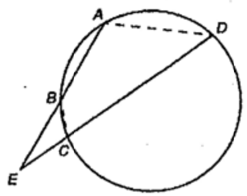
$\sqrt{[x-(-2)]^2+[y-(-1)]^2} = y-1$

$(x+2)^2+(y+1)^2 = (y-1)^2$

$y = -\dfrac{1}{4}x^2-x-1$

$\therefore$ The required equation is $y = -\dfrac{1}{4}x^2-x-1$, the locus of P is a parabola.

6. In the figure, ABE and DCE are straight lines. If $AB = BE = 3$ cm and $CD = 7$ cm, find the length of CE. (5 marks)



Join AD and BC.

In $\triangle EAD$ and $\triangle ECB$,
$\angle AED = \angle CEB$ (common angle)
$\angle ADE = \angle CBE$ (ext. $\angle$, cyclic quad.)
$\therefore \triangle EAD \sim \triangle ECB$ (AA)

Let $CE = x$ cm.

$\frac{AE}{CE} = \frac{DE}{BE}$ (corr. sides, $\sim\triangle$s)

$\frac{3+3}{x} = \frac{x+7}{3}$

$18 = x(x+7)$

$x^2 + 7x - 18 = 0$

$(x-2)(x+9) = 0$

$x = 2$ or -9 (rejected)

$\therefore CE = \underline{\underline{2\text{ cm}}}$

7. (a) How many 6-digit even numbers can be formed from the digits 1, 2, 3, 5, 7 and 9 if
   (i) there are no restrictions?
   (ii) no repetition of digits is allowed? (2 marks)

(b) 2 teachers and 2 parents are selected from 5 teachers and 12 parents to sit in a row. In how many ways can the 2 teachers be arranged on the left of the 2 parents? (3 marks)

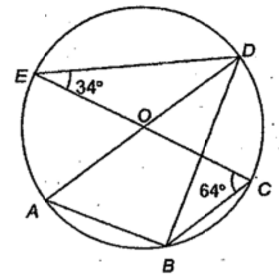
(a) (i) Only '2' is even among the digits, and therefore the last digit of the 6-digit even number must be 2.
The number of 6-digit even numbers $= 6^5 \times 1$
$= \underline{\underline{7776}}$

(ii) The number of 6-digit even numbers

$= (6-1)!$
$= \underline{\underline{120}}$

(b)
Number of ways of arranging the 2 teachers $= P_2^5$
Number of ways of arranging the 2 parents $= P_2^{12}$
$\therefore$ The required number of ways
$= P_2^5 \times P_2^{12}$
$= \underline{\underline{2640}}$

8.



In the figure, O is the centre of the circle. AD and CE are diameters of the circle. It is given that $\angle OCB = 64°$ and $\angle OED = 34°$.

(a) Find $\angle OAB$.

(b) Find $\frac{AB}{AD}$.

(5 marks)

(a) Join CD.

$\angle CDE = 90°$ ($\angle$ in semi-circle)

$\angle DCE + 34° + 90° = 180°$ ($\angle$ sum of Δ)

$\angle DCE = 56°$

$\angle OAB + \angle BCD = 180°$ (opp. $\angle$s, cyclic quad.)

$\angle OAB + (64° + 56°) = 180°$

$\angle OAB = \underline{\underline{60°}}$

(b) $\angle ABD = 90°$. ($\angle$ in semi-circle)

$\frac{AB}{AD} = \cos 60°$

$= \underline{\underline{\frac{1}{2}}}$

9. The coordinates of points X and Y are $(-3, 7)$ and $(5, 1)$ respectively. P is a moving point in the rectangular coordinate plane such that $PX = PY$. Denote the locus of P by Γ.

(a) (i) Find the equation of Γ.

(ii) Describe the geometric relationship between Γ and the line segment XY.

(3 marks)

(b) It is given that a circle passes through X and Y. Denote the centre of the circle by R.

(i) Does Γ pass through R? Explain your answer.

(ii) The line $y = x$ divides the circle into two equal halves. Find the coordinates of R.

(4 marks)

(a) (i) Let (x, y) be the coordinates of P.

$$\sqrt{[x-(-3)]^2+(y-7)^2} = \sqrt{(x-5)^2+(y-1)^2}$$
$$(x+3)^2+(y-7)^2=(x-5)^2+(y-1)^2$$
$$x^2+6x+9+y^2-14y+49=x^2-10x+25+y^2-2y+1$$
$$4x-3y+8=0$$

Thus, the equation of Γ is $4x-3y+8=0$.

(ii) Γ is the perpendicular bisector of the line segment XY.

(b) (i) Since Γ is the perpendicular bisector of the chord XY, R lies on Γ.
($\perp$ bisector of chord passes through centre)

Thus, Γ passes through R.

(ii) Since the line $y = x$ divides the circle into two equal halves, R also lies on the line $y = x$.

$$\begin{cases} 4x-3y+8=0 \\ y=x \end{cases}$$

$4x-3x+8=0$.
Solving, we have $x=-8$ and $y=-8$.

Thus, the coordinates of R are $(-8, -8)$.

10. In a Mathematics competition, Ray and Cindy score 68 marks and 50 marks respectively. The standard scores of Ray and Cindy are 0 and -1.5 respectively.

(a) Find the mean and the standard deviation of the scores in the competition. (3 marks)

(b) Later, Ray is found cheating in the competition and he is disqualified. Cindy claims that her standard score will increase after excluding Ray's score. Is Cindy's claim correct?
Explain your answer. (2 marks)

(a) Let m and σ be the mean and the standard deviation of the scores respectively.
Note that the standard scores of Ray and Cindy are 0 and -1.5 respectively.

Thus, we have $\frac{68-m}{\sigma}=0$ and $\frac{50-m}{\sigma}=-1.5$.
Solving, we have $m=68$ and $\sigma=12$.

Thus, the mean and the standard deviation of the scores are 68 marks and 12 marks respectively.

(b) Note that the score of Ray is equal to the mean of the scores.
So, the mean of the scores remains unchanged and the distribution of the scores is more dispersed. Therefore, the standard deviation of the scores is larger.

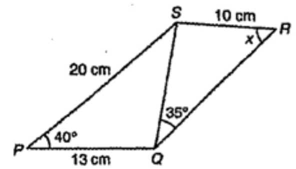
Hence, the standard score of Cindy will be less negative.
i.e. The standard score of Cindy will increase.
Thus, Cindy's claim is agreed.

11. In the figure, $PQRS$ is a quadrilateral with $PQ = 13$ cm, $RS = 10$ cm, $PS = 20$ cm, Ð$QPS = 40°$ and $\angle RQS = 35°$. Find

(a) the length of QS,

(b) x.

(Give your answers correct to 3 significant figures.) (5 marks)



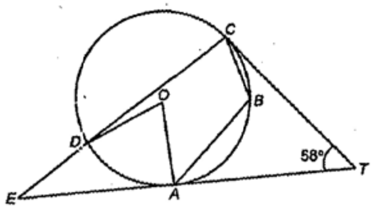
(a) In $\triangle PQS$, by the cosine formula,
$$QS^2 = PQ^2+PS^2-2(PQ)(PS)\cos\angle QPS$$
$$QS=\sqrt{13^2+20^2-2(13)(20)\cos 40°}\text{ cm}$$
$$\approx 13.0636\text{ cm}$$
$$=\underline{13.1\text{ cm}}\text{ (cor. to 3 sig. fig.)}$$

(b) In $\triangle QRS$, by the sine formula,
$$\frac{QS}{\sin\angle QRS}=\frac{RS}{\sin\angle RQS}$$
$$\frac{13.0636\text{ cm}}{\sin x}\approx\frac{10\text{ cm}}{\sin 35°}$$
$$\sin x\approx\frac{13.0636\sin 35°}{10}$$
$$x\approx 48.5295° \quad\text{or}\quad 180°-48.5295°$$
$$=\underline{48.5°}\text{ (cor. to 3 sig. fig.) or }\underline{131°}\text{ (cor. to 3 sig. fig.)}$$

12. In the figure, TA and TC are tangents to the circle at A and C respectively. CD and TA are produced to meet at E. It is given that $\angle ATC = 58°$.

(a) Find $\angle ABC$. (3 marks)

(b) If $\angle AOD = 62°$, find $\angle AED$. (3 marks)



(a) Join AC and AD.

$\angle TAC = \angle TCA$ (tangent prop.)

In $\triangle ACT$,

$\angle TAC + \angle TCA + \angle ATC = 180°$ ($\angle$ sum of $\triangle$)

$2\angle TAC + 58° = 180°$

$\angle TAC = 61°$

$\angle CDA = \angle TAC = 61°$ ($\angle$ in alt. segment)

$\angle ABC = \underline{\underline{119°}}$ (opp $\angle$, cyclic quad.)

(b)

$\angle ACD = \dfrac{\angle AOD}{2}$ ($\angle$ at centre twice $\angle$ at $\odot^{ce}$)

$= \dfrac{62°}{2} = 31°$

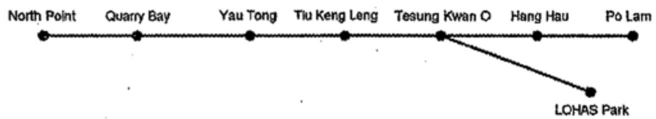
$\angle CAE = \angle ABC = 119°$ ($\angle$ in alt. segment)

$\angle AED + \angle CAE + \angle ACD = 180°$ ($\angle$ sum of $\triangle$)

$\angle AED + 119° + 31° = 180°$

$\angle AED = \underline{\underline{30°}}$

13. The figure shows the route map of the Tseung Kwan O line of MTR.



(a) A passenger gets on a train at a station and gets off at another station. Find the number of different journeys the passenger can travel along the Tseung Kwan O line. (2 marks)

(b) It is given that a passenger has to pay $10 or above per journey if he / she travels through the cross-harbour tunnel which is located between Quarry Bay and Yau Tong. How many different journeys in (a) charge the passenger $10 or above? (3 marks)

(a) The required number of different journeys

$= P_2^8 = \underline{\underline{56}}$

(b) There are two cases that can meet the condition.

Case 1: The journey starts at North Point or Quarry Bay and ends at one of the other 6 stations. The number of journeys $= C_1^2 \times C_1^6$

Case 2: The journey ends at North Point or Quarry Bay and starts at one of the other 6 stations. The number of journeys $= C_1^2 \times C_1^6$

$\therefore$ The required number of journeys

$= C_1^2 \times C_1^6 + C_1^2 \times C_1^6$

$= \underline{\underline{24}}$

14. A developer plans to build a hotel with x single rooms and y double rooms subject to the conditions listed in the proposal below:

(1) The hotel should accommodate at least 1500 people.

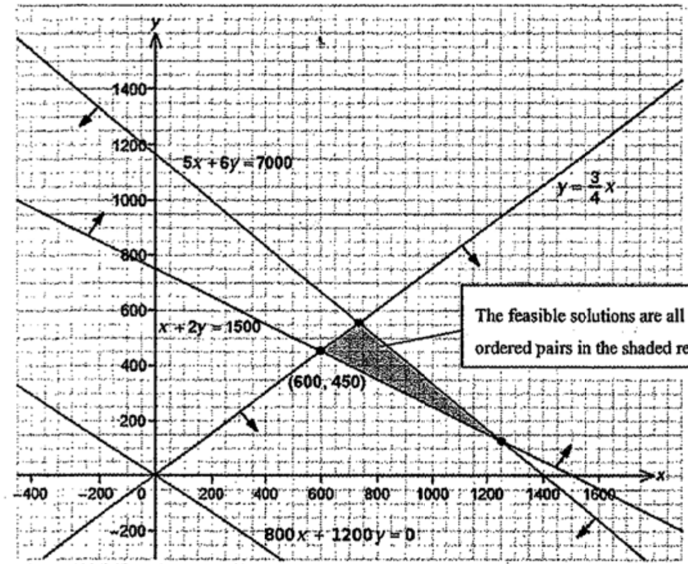
(2) Each single room and double room must occupy an area of 15m^2 and 18 m^2 respectively. The total floor area available for the rooms is at most $21\,000\text{ m}^2$.

(3) The number of double rooms should be at most 75% of the number of single rooms.

(a) Write down all the constraints on x and y. (3 marks)

$$\begin{cases} x + 2y \ge 1500 \\ 15x + 18y \le 21000 \\ y \le x \times 75\% \\ x \text{ and } y \text{ are non-negative integers.} \end{cases} \quad \text{i.e.,} \quad \begin{cases} x + 2y \ge 1500 \\ 5x + 6y \le 7000 \\ y \le \frac{3}{4}x \\ x \text{ and } y \text{ are non-negative integers.} \end{cases}$$

(b) On the graph paper provided, draw and indicate the solution that satisfies all the constraints in (a). (3 marks)



Marking Scheme	Marks
Correct line for $x + 2y = 1500$	1A for any correct line. 2A for all correct.
Correct line for $5x + 6y = 7000$	
Correct line for $y = \frac{3}{4}x$	
Correct ordered pairs/shaded region	1A

(c) It is known that the monthly maintenance costs for a single room and a double room are $800 and $1200 respectively. Using (b), find the values of x and y so that the hotel can be run at the minimum maintenance cost. (3 marks)

Let $C be the maintenance cost, then $C = 800x + 1200y$.

From the graph, C attains its minimum at (600, 450).

$\therefore$ $x = \underline{\underline{600}}$ and $y = \underline{\underline{450}}$

15. The time taken for a group of people to finish a fitness test are normally distributed with a mean of 33 s and a standard deviation of 3.5 s.

(a) Find the percentage of people whose finishing time is between 29.5 s and 40 s. (2 marks)
(b) If 200 people can finish the test within 26 s, find the number of people in the group. (2 marks)
(c) Assume that people who finish the fitness test within 37.5 s will pass the test. If Peter is a person in the group and his standard score is 1.4, can he pass the test? Explain your answer. (3 marks)

(Assume that 68%, 95% and 99.7% of the data of a normal distribution lie within one, two and three standard deviations from the mean respectively.)

(a) $\because$ $29.5\text{ s} = (33 - 3.5)\text{ s} = \bar{x} - \sigma$
$40\text{ s} = (33 + 2 \times 3.5)\text{ s} = \bar{x} + 2\sigma$
$\therefore$ The required percentage
$= \frac{68\%}{2} + \frac{95\%}{2}$
$= \underline{81.5\%}$

(b) $\because$ $26\text{ s} = (33 - 2 \times 3.5)\text{ s} = \bar{x} - 2\sigma$
$\therefore$ The percentage of people can finish the test within 26 s
$= \left(50 - \frac{95}{2}\right)\%$
$= 2.5\%$

$\therefore$ The number of people in the group
$= 200 \div 2.5\%$
$= \underline{8000}$

(c) Let x s be the finishing time of Peter.
$\frac{x - 33}{3.5} = 1.4$
$x = 37.9$

$\therefore$ The finishing time of Peter is 37.9 s > 37.5 s.
$\therefore$ Peter <u>cannot pass</u> the test.

16. (a) In Figure (16a), the straight line $L_1 : x + y + 6 = 0$ cuts the x-axis and the y-axis at A and B respectively and the circumcircle of $\triangle OAB$ is drawn.
(i) Find the coordinates of A and B.
(ii) Find the equation of the circumcircle of $\triangle OAB$ in the general form. (5 marks)

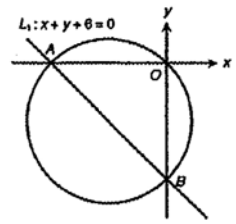


Figure (16a)

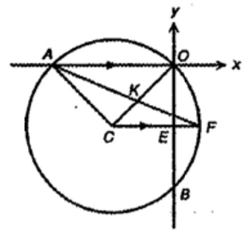


Figure (16b)

(b) In Figure (16b), C is the centre of the circle in Figure (1) and F is a point lying on the circle such that $CF // AO$. If CF cuts the y-axis at E and K is the intersection of OC and AF,
(i) prove that $\angle ACO = 90°$,
(ii) find the acute angle between OC and AF,
(iii) find the area of the segment AOF in terms of π.
(Leave your answer in surd form.) (8 marks)

(a) (i) $L_1 : x + y + 6 = 0$
x-intercept of $L_1 = -\frac{6}{1} = -6$ y-intercept of $L_1 = -\frac{6}{1} = -6$
$\therefore$ Coordinates of $A = \underline{(-6, 0)}$
Coordinates of $B = \underline{(0, -6)}$

(ii) $\because$ $\angle AOB = 90°$
$\therefore$ AB is a diameter of the circle. (converse of $\angle$ in semi-circle)
$\therefore$ The centre is the mid-point of AB.

$$\text{Centre} = \left(\frac{-6+0}{2}, \frac{0+(-6)}{2}\right) = (-3, -3)$$

$$Radius = \frac{AB}{2} = \frac{\sqrt{(-6-0)^2 + [0-(-6)]^2}}{2} = \frac{\sqrt{72}}{2}$$

$\therefore$ The equation of the circumcircle of $\triangle OAB$ is

$$[x - (-3)]^2 + [y - (-3)]^2 = \left(\frac{\sqrt{72}}{2}\right)^2$$
$$x^2 + 6x + 9 + y^2 + 6y + 9 = 18$$
$$x^2 + y^2 + 6x + 6y = 0$$

(b) (i) Centre $C = \left(-\frac{6}{2}, -\frac{6}{2}\right)$

$= (-3, -3)$

Slope of $CO = \frac{0-(-3)}{0-(-3)} = 1$ Slope of $AC = \frac{-3-0}{-3-(-6)} = -1$

$\because$ Slope of $AC \times$ slope of $CO = -1$

$\therefore$ $AC \perp CO$

$\therefore$ $\angle ACO = 90°$

(ii) $\angle CEO = 90°$ (corr. $\angle$s, $CF // AO$)

$\tan \angle OCE = \frac{OE}{CE}$

$= \frac{0-(-3)}{0-(-3)}$

$= 1$

$\therefore$ $\angle OCE = 45°$

$\angle OAF = \frac{1}{2}\angle OCE$ ($\angle$ at centre twice $\angle$ at $\odot^{ce}$)

$= \frac{1}{2} \times 45°$

$= 22.5°$

$\angle CFA = \angle OAF$ (alt.$\angle$s, $CF // AO$)

$= 22.5°$

$\angle AKC = \angle OCE + \angle CFA$ (ext. $\angle$ of Δ)

$= 45° + 22.5°$

$= 67.5°$

$\therefore$ The acute angle between OC and AF is 67.5°.

(iii) $CF = CO$ (radii)

$= \sqrt{(-3-0)^2 + (-3-0)^2}$

$= 3\sqrt{2}$

Area of $\Delta CAF = \frac{1}{2} \times CF \times OE$

$= \frac{1}{2} \times 3\sqrt{2} \times 3$

$= \frac{9\sqrt{2}}{2}$

Area of sector $CAOF = \pi(3\sqrt{2})^2 \times \frac{135°}{360°} = \frac{27}{4}\pi$

$\therefore$ Area of segment AOF

= area of sector $CAOF$ – area of ΔCAF

$= \frac{27}{4}\pi - \frac{9\sqrt{2}}{2}$

17. Figure 17(a) shows a piece of paper card $ABCD$ with $AB = BC = 50$ cm, $AD = CD = 32$ cm and reflex $\angle ADC = 210°$.

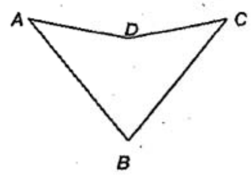


Figure 17(a)

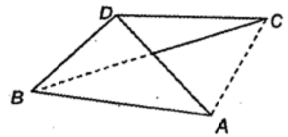


Figure 17(b)

(a) Find the length of BD. (3 marks)

(b) The paper card is then folded along BD such that AB and BC lie on the horizontal ground as shown in Figure 17(b). It is given that $\angle ADC = 40°$.

(i) Find the distance between A and C on the horizontal ground.

(ii) Find the angle between the plane ACD and the horizontal ground.

(iii) Find the volume of the pyramid $ABCD$.

(8 marks)

(c) Jerry claims that if P is the mid-point of BD, the volume of the pyramid $ABCP$ is half of the volume of the pyramid $ABCD$. Do you agree? Explain your answer. (2 marks)

(a) $\angle ADB = 105°$.

By the sine formula,

$\frac{\sin \angle ADB}{AB} = \frac{\sin \angle ABD}{AD}$

$\frac{\sin 105°}{50} = \frac{\sin \angle ABD}{32}$

$\angle ABD \approx 38.18426316°$ or $141.8157368°$ (rejected)

By the sine formula,

$\frac{\sin \angle BAD}{BD} = \frac{\sin \angle ADB}{AB}$

$\frac{\sin(180° - 105° - 38.18426316°)}{BD} \approx \frac{\sin 105°}{50}$

$BD \approx 31.01912633$ cm

$BD \approx 31.0$ cm

b) (i)

$$AC^2 = AD^2 + CD^2 - 2(AD)(CD)\cos\angle ADC$$
$$AC^2 = 32^2 + 32^2 - 2(32)(32)\cos 40^\circ$$
$$AC \approx 21.88928917 \text{ cm}$$

$\therefore$ The distance between A and C on the horizontal ground is 21.9 cm.

(ii) Note that $\triangle ABC$ and $\triangle ACD$ are isosceles triangles.
Let M be the mid-point of AC.
So, $BM \perp AC$ and $DM \perp AC$. (prop. of isos. $\triangle$)

$$BM^2 = AB^2 - AM^2$$
$$BM^2 = 50^2 - \left(\frac{21.88928917}{2}\right)^2$$
$$BM^2 \approx 2380.214755$$

$$DM^2 = AD^2 - AM^2$$
$$DM^2 = 32^2 - \left(\frac{21.88928917}{2}\right)^2$$
$$DM^2 \approx 904.2147549$$

By the cosine formula,
$$\cos\angle DMB = \frac{BM^2 + DM^2 - BD^2}{2(BM)(DM)}$$
$$\angle DMB \approx 37.67700411$$
$$\angle DMB \approx 37.7^\circ$$

Thus, the angle between the plane ACD and the horizontal ground is 37.7°.

(iii) The area of $\triangle ABC$
$= \frac{1}{2}(AC)(BM)$
$\approx 533.9612418 \text{cm}^2$

The height of the pyramid $ABCD$
$= DM \sin\angle DMB$
$\approx 18.37916772 \text{ cm}$

The volume of pyramid $ABCD$
$= \frac{1}{3}$(the area of $\triangle ABC$)(the height of pyramid $ABCD$)
$\approx 3271.254406 \text{ cm}^3$
$\approx \underline{\underline{3270 \text{ cm}^3}}$

(c) Let X and Y be the projections of D and P on the plane ABC respectively. Suppose that $\angle DBX = \alpha$.
When P is the mid-point of BD, we have $BP = \frac{1}{2}BD$.

The volume of pyramid $ABCD$
$= \frac{1}{3}$(the area of $\triangle ABC$)($BD \sin\alpha$), and

The volume of pyramid $ABCP$
$= \frac{1}{3}$(the area of $\triangle ABC$)($BP \sin\alpha$)
$= \frac{1}{3}$(the area of $\triangle ABC$)($\frac{1}{2}BD \sin\alpha$)
$= \frac{1}{2}$(the volume of pyramid $ABCD$)

Thus, the volume of the pyramid $ABCP$ is half of the volume of the pyramid $ABCD$.

Thus, the claim is agreed.

End of paper