Solutions for F.6 Math Compulsory Part Paper 1 Mock Exam 2019-2020

1. $\dfrac{(m^3n^{-2})^3}{m^{13}n^{-7}} = m^{3(3)-13}n^{-2(3)-(-7)}$ **1M+1M**

$= \dfrac{n}{m^4}$ **1A**

3 marks

2. $\dfrac{x+3}{2} = \dfrac{x}{y}$ **1M**

$y(x+3) = 2x$

$x(y-2) = -3y$ **1M**

$x = \dfrac{3y}{2-y}$ **1A**

3 marks

3.(a) $27p^3 - 8 = (3p-2)(9p^2+6p+4)$ **1A**

(b) $27p^3 - 3pq + 2q - 8$

$= (3p-2)(9p^2+6p+4) - q(3p-2)$ **1M**

$= (3p-2)(9p^2+6p-q+4)$ **1A**

3 marks

4.(a) $\dfrac{3(x-2)}{2} \le 11 - 5x$

$x \le \dfrac{28}{13}$ **1A**

(b) $x \ge \dfrac{-9}{2}$ and $x \le \dfrac{28}{13}$

$\therefore \dfrac{-9}{2} \le x \le \dfrac{28}{13}$ **1M+1A**

$\therefore$ There are 7 integers. **1A**

4 marks

5. The new length $= 40(1+20\%) = 48$ **1A**

The new width $= (40+30) - 48$

$= 22$ **1M**

The required percentage change

$= \dfrac{48(22)-(40)(30)}{40(30)} \times 100\%$ **1M**

$= -12\%$ **1A**

4 marks

6. Let \$$x$ be the cost of a bottle of water.

$3(3x) + 5x = 112$ **1M+1A**

$x = 8$ **1A**

$\therefore$ The cost of a bottle of orange juice is \$24. **1A**

4 marks

7. $\angle AOB = 210° - 120° = 90°$ **1A**

$AB = \sqrt{8^2 + 15^2}$ **1M**

$= 17$

The required distance $= \dfrac{8(15)}{17}$ **1M**

$= \dfrac{120}{17} \approx 7.06$ **1A**

4 marks

8.(a) Let $f(x) = ax + b$, where a and b are constants. **1A**

$\begin{cases} 2 = 2a + b \\ -7 = -a + b \end{cases}$ **1M**

$a = 3$ and $b = -4$

$\therefore f(x) = 3x - 4$ **1A**

(b) $x(3x-4) = 15$ **1M**

$3x^2 - 4x - 15 = 0$

$(3x+5)(x-3) = 0$

$x = 3$ or $x = \dfrac{-3}{5}$ **1A**

5 marks

9.(a) $\angle BCD = 180° - 75°$ **1A**

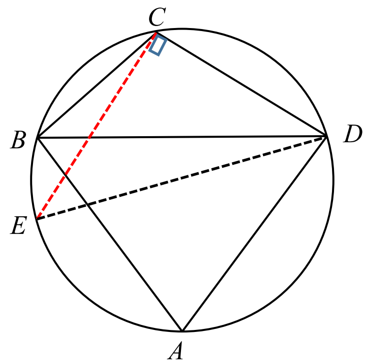
$= 105°$

$\overset{\frown}{BC} : \overset{\frown}{CD} = \angle BDC : \angle CBD$

$= 2 : 3$

$\angle DBC = (180° - 105°)(\dfrac{3}{2+3})$ **1M**

$= 45°$ **1A**



(b) Let E be a point on the circle such that DE is a diameter.

$\angle DEC = \angle DBC = 45°$

$DE = \dfrac{4}{\sin 45°}$ **1M**

$= 4\sqrt{2}$

The required radius $= 2\sqrt{2}$ cm **1A**

Alternative

Let O be the centre of the circle.

$\angle COD = 2\angle DBC$

$= 90°$

$CO^2 + DO^2 = CD^2$ **1M**

$2r^2 = 16$

$r = 2\sqrt{2}$

$\therefore$ The radius of the circle is $2\sqrt{2}$ cm. **1A**

5 marks

10.(a) $\dfrac{2}{3}\pi r^3 = \dfrac{1}{3}\pi r^2 h$ **1A**

$\dfrac{r}{h} = \dfrac{1}{2}$

$r : h = 1 : 2$ **1A**

(b) Let the height of the upper part circular cone be x cm.

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$\frac{1}{3}\pi(\frac{x}{2})^2 x = 144\pi$ **1M**

$x^3 = 1728$

$x = 12$ **1A**

Let the height of the frustum be y cm.

$(\frac{12}{12+y})^3 = \frac{1}{2}$

$\frac{12}{12+y} = \frac{1}{\sqrt[3]{2}}$ **1M**

$y = 12(\sqrt[3]{2}-1) \approx 3.12\text{cm}$ **1A**

6 marks

11.(a) In ΔBAE and ΔADC,

$\angle CAD + \angle BAF = 90°$ (given)

$\angle ABE + \angle BAF = 90°$ (ext. $\angle$ of Δ)

$\angle CAD = \angle ABE$

$AB = AD$ (given)

$\angle BAD = \angle ADC = 90°$ (given)

$\therefore$ $\Delta BAE \cong \Delta ADC$ (ASA)

3: correct proof w/ reasons **2**: correct proof w/o reasons **1**: one line of correct proof

(b) $\angle BAD = \angle ADC = 90°$ (given)

$\angle BAD + \angle ADC = 180°$

$\therefore AB // DC$ (int. $\angle$s supp.)

$\angle DCG = \angle BAG$ (alt. $\angle$s, AB//DC)

$\angle CDG = \angle ABG$ (alt. $\angle$s, AB//DC)

$\therefore \Delta CDG \sim \Delta ABG$ (AA)

3: correct proof w/ reasons **2**: correct proof w/o reasons **1**: one line of correct proof

(c) $\frac{DG}{BG} = \frac{CD}{AB} = \frac{1}{2}$ (corr. sides, ~Δs)

The centroid lies on BE. GN intersects BE at H.

$\frac{EH}{HB} = \frac{DG}{BG} = \frac{1}{2}$ (Equal Ratios Theorem) **1M**

$\therefore H$ is the centroid of ΔABD.

I agree. **1A**

8 marks

12.(a) $42 + ... + (50+m) + ... + (70+n) + ...95 = 22(63)$ **1A**

$m + n = 7$

IQR = $Q_3 - Q_1$

$(70+n) - (50+m) = 23$ **1A**

$n - m = 3$

$\therefore m = 2$ and $n = 5$ **1A**

(b) $\because$ There are 2 modes for the remaining data.

$\therefore$ One of the deleted data must be 65kg. **1A**

Another deleted datum $= 63(22) - 61.3(20) - 65$ **1M**

$= 95\text{kg}$

$\text{IQR}_{new} = \frac{1}{2}(74+75) - (51+52)$ **1M**

$= 23\text{kg}$

$\therefore$ No, I do not agree. **1A**

7 marks

13.(a) $b = 15$ **1A**

(b) Let d and r be the common difference and common ratio respectively.

$\begin{cases} 15 - d + \frac{14}{r} = 32, \frac{14}{r} - d = 17 \\ 15 + d + 14r = 33, 14r + d = 18 \end{cases}$ **1M** **1M**

$\frac{14}{r} + 14r = 35$

$14r^2 - 35r + 14 = 0$ **1M**

$2r^2 - 5r + 2 = 0$

$(2r-1)(r-2) = 0$

$r = \frac{1}{2}$ or $r = 2$ **1A**

$a + \frac{14}{\frac{1}{2}} = 32$

$a = 4$

$a + \frac{14}{2} = 32$

$a = 25$

$\therefore$ $a = 4$ or 25 **1A**

6 marks

14.(a) $a^2 - c^2 = 3$ **1M**

$(a-c)(a+c) = 3$

$\therefore a + c = 3$ and $a - c = 1$

$\therefore a = 2$ and $c = 1$ **1A+1A**

(b) $f(-1) = 9$ **1M**

$[(-1)^2 + 2]^2 - [b(-1)+1]^2 = 9$

$(1-b)^2 = 0$

$b = 1$ **1A**

(c) $f(x) = (x^2+2)^2 - (x+1)^2 = 0$

$(x^2 + 2 + x + 1)(x^2 + 2 - x - 1) = 0$ **1M**

$(x^2 + x + 3)(x^2 - x + 1) = 0$

For $x^2 + x + 3 = 0$,

$\Delta = 1^2 - 4(3)(1)$ **1M**

$= -11 < 0$

For $x^2 - x + 1 = 0$,

$\Delta = 1^2 - 4(1)(1)$

$= -3 < 0$

$\therefore$ $f(x) = 0$ has no real roots. I agree. **1A**

8 marks

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15. $\frac{2^m + 9}{2^{2m} + 9} = \frac{1}{3}$ **1A**

$(2^m)^2 - 3(2^m) - 18 = 0$ **1M**

$(2^{2m})^2 - 3(2^{2m}) - 18 = 0$

$(2^m - 6)(2^m + 3) = 0$

$2^m = 6$ or $2^m = -3$ (rejected)

$m = \log_2 6 \approx 2.58$ **1M+1A**

4 marks

16(a) $2OP = PQ$

$4OP^2 = PQ^2$

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$4(x^2+y^2)=(x-q)^2+y^2$ **1M**

$3x^2+3y^2+2qx-q^2=0$ **1A**

(b) Γ is a circle.

The greatest distance is the diameter. **1M**

Let the radius be r.

$(\frac{-q}{3})^2+\frac{q^2}{3}=4^2$ **1M**

$\frac{4q^2}{9}=16$

$q=\pm 6$ **1A**

5 marks

17.(a) The required probability $=\frac{C_4^4 C_3^8}{C_7^{12}}$ **1A**

$=\frac{7}{99}$ **1A**

(b) The required probability $=\frac{C_7^8}{C_7^{12}}$ **1A**

$=\frac{1}{99}$ **1A**

(c) The required probability

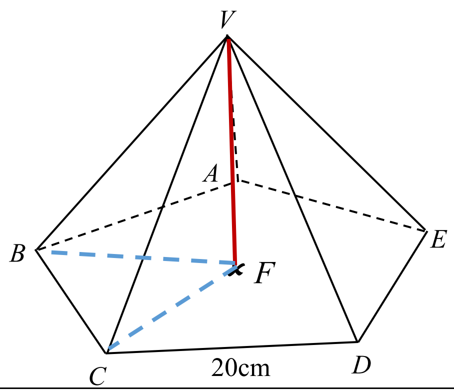
$=\frac{\text{P(3B4G)}+\text{P(2B5G)}+\text{P(1B6G)}}{1-\text{P(All Girls)}}$

$=\frac{1-\frac{1}{99}-\frac{7}{99}}{1-\frac{1}{99}}$ **1M + 1M**

$=\frac{13}{14}$ **1A**

7 marks

18.



18.(a) $\angle BFC=\frac{360°}{5}=72°$

$\angle FBC=\frac{1}{2}(180°-\frac{360°}{5})=54°$

$\frac{CF}{\sin 54°}=\frac{20}{\sin 72°}$ **1M**

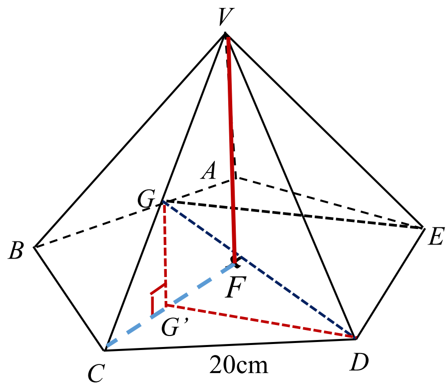
$CF=17.0$ cm (17.013) **1A**

b(i) F is the projection of V on $ABCDE$.

The required height = VF **1M**

$VF=\sqrt{26^2-CF^2}$

$=19.7$ cm (19.66106) **1A**



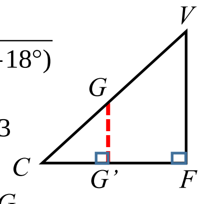
b(ii) Let G' be the projection of G to $ABCDE$.

The required $\angle = \angle GDG'$ **1M**

$\angle G'DE=90°$

$\angle G'DC=108°-90°=18°$

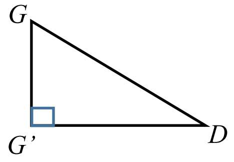
$\frac{CG'}{\sin 18°}=\frac{20}{\sin(180°-54°-18°)}$



$CG'=6.4983939$

$DG'=DF=CF=17.013$

$\frac{GG'}{VF}=\frac{CG'}{CF}$ **1M**



$GG'=7.5099$

$\tan\angle GDG'=\frac{GG'}{DG'}$ **1M**

$\angle GDG'=23.8°$ **1A**

8 marks

19.(a) $f(x)=k(x^2-2x)-2$

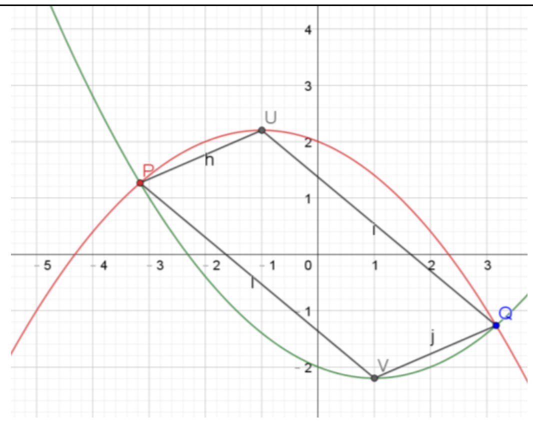
$=k(x^2-2x+1^2-1^2)-2$ **1M**

$=k(x-1)^2-k-2$ **1A**

$\therefore V=(1,-k-2)$ **1A**

(b) $U=(-1,k+2)$ **1A**

(ci)



$f(x)=g(x)$

$k(x-1)^2-k-2=-k(x+1)^2+k+2$ **1M**

$k[(x-1)^2+(x+1)^2]=2k+4$

$k(2x^2+2)=2k+4$

$kx^2=2$

$x=\pm\sqrt{\frac{2}{k}}=\pm\frac{\sqrt{2k}}{k}$

$\therefore P=(\frac{-\sqrt{2k}}{k},2\sqrt{2k})$ **1A**

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c(ii)	$Q=(\frac{\sqrt{2k}}{k},-2\sqrt{2k})$	
	The mid-point of PQ = (0,0)	**1A**
	And the mid-point of UV = (0,0)	
	$\therefore OP=OQ$ and $OU=OV$	**1M**
	$\therefore PUQV$ is a parallelogram. (diagonals bisect each other)	**1**
c(iii)	When $PUQV$ is a rectangle then it is a cyclic quadrilateral. i.e. $QV\perp QU$. PQ is a diameter and $(0,0)$ is the centre.	**1A**
	$OP=OU$	
	$(\frac{-\sqrt{2k}}{k})^2+(2\sqrt{2k})^2=(-1)^2+(k+2)^2$	
	$\frac{2}{k}+8k=1+k^2+4k+4$	
	$k^3-4k^2+5k-2=0$	**1M**
	$(k-1)(k^2-3k+2)=0$	
	$(k-1)(k-1)(k-2)=0$	
	$k=1$	**1A**
c(iii)	Alternative Method:	
	When $PUQV$ is a rectangle then it is a cyclic quadrilateral. i.e. $QV\perp QU$.	
	slope of $QV=\frac{-2\sqrt{2k}-(-k-2)}{\frac{\sqrt{2k}}{k}-1}$ $=\frac{-2\sqrt{2k}+k+2}{\frac{\sqrt{2k}}{k}-1}$	
	slope of $QU=\frac{-2\sqrt{2k}-(k+2)}{\frac{\sqrt{2k}}{k}-(-1)}$ $=\frac{-2\sqrt{2k}-(k+2)}{\frac{\sqrt{2k}}{k}+1}$	
	slope of $PV\times$slope of $PU=-1$	**1M**
	$\frac{-2\sqrt{2k}+(k+2)}{\frac{\sqrt{2k}}{k}-1}(\frac{-2\sqrt{2k}-(k+2)}{\frac{\sqrt{2k}}{k}+1})=-1$	
	$(-2\sqrt{2k})^2-(k+2)^2=-[(\frac{\sqrt{2k}}{k})^2-1^2]$	
	$8k-k^2-4k-4=1-\frac{2}{k}$	**1M**
	$k^3-4k^2+5k-2=0$	
	$(k-1)(k^2-3k+2)=0$ $(k-1)(k-1)(k-2)=0$	**1A**
	$k=1$	
	11 marks	