Sing Yin Secondary School F.6 Mock Exam 2020 Mathematics CP Paper 2 Solution

1. B	2. C	3. C	4. D	5. C	6. A	7. D	8. C	9. B	10. B
11. B	12. C	13. A	14. B	15. D	16. D	17. B	18. D	19. C	20. B
21. A	22. A	23. C	24. A	25. D	26. D	27. D	28. A	29. C	30. A
31. C	32. A	33. B	34. B	35. A	36. B	37. D	38. A	39. D	40. D
41. C	42. A	43. C	44. D	45. B					

1. $8q^3 + 4pq^2 - 2p^2q - p^3$

$= 4q^2(2q+p) - p^2(2q+p)$

$= (4q^2 - p^2)(2q+p)$

$= (2q-p)(2q+p)(2q+p)$

$= (2q-p)(p+2q)^2$

2. $Y = \dfrac{1-X}{1+2X}$

$Y(1+2X) = 1-X$

$Y + 2XY = 1-X$

$2XY + X = 1-Y$

$X(2Y+1) = 1-Y$

$X = \dfrac{1-Y}{1+2Y}$

3. $\dfrac{(x^{3n})(x^{4n})}{(x^n)^2}$

$= \dfrac{x^{3n+4n}}{x^{2n}}$

$= x^{7n-2n}$

$= x^{5n}$

4. $\sqrt{\pi + 25}$

$\approx 5.3048\cdots$

So, it is 5.30 when correct to 2 d.p or 3 sig. fig. Hence, A and B are incorrect.

The number 5.305 has only 3 digits after the decimal place. Hence, C is incorrect.

5. $\dfrac{2x-1}{5} \le 1$ and $3 - 2x \le 4$

$2x - 1 \le 5$ and $-2x \le 1$

$x \le 3$ and $x \ge -0.5$

$\therefore -0.5 \le x \le 3$

x could be 0, 1, 2 or 3 if x is an integer.

Hence, the answer is 4 .

6. $\dfrac{f(-k)}{f(k)}$

$= \dfrac{(-k)^2 - 3k(-k) + k^2}{(k)^2 - 3k(k) + k^2}$

$= \dfrac{k^2 + 3k^2 + k^2}{k^2 - 3k^2 + k^2}$

$= \dfrac{5k^2}{-k^2}$

$= -5$

7. $g(x)$ is divisible by $2x - 1 \Rightarrow g(\frac{1}{2}) = 0$, hence

$\left(\frac{1}{2}\right)^6 + a\left(\frac{1}{2}\right)^3 + 1 = 0$

$8a + 65 = 0$

$a = \dfrac{-65}{8}$

$g(-2) = (-2)^6 + a(-2)^3 + 1$

$= -8a + 65$

$= -8\left(\dfrac{-65}{8}\right) + 65$

$= 130$

Therefore, the remainder is 130.

8. $y = b - (ax+1)^2 = -a^2\left(x + \dfrac{1}{a}\right)^2 + b$

Since $-a^2 < 0$, I is correct.

The equation of the line of symmetry is $x + \dfrac{1}{a} = 0$, II is incorrect.

As the vertex is at $\left(\dfrac{-1}{a}, b\right)$, which is above the x-axis. Together with the fact that the graph opens downwards, the graph must cuts the x-axis at two points, hence III is correct.

9. Method 1:

$(2x + m)(x + 3) + 6 \equiv (2x + n)(x + 1)$

$2x^2 + 6x + mx + 3m + 6 \equiv 2x^2 + 2x + nx + n$

$$\therefore \begin{cases} 6+m=2+n \\ 3m+6=n \end{cases}$$

Solving, $6 + m = 2 + (3m + 6) \Rightarrow m = -1$.

So, $n = 3(-1) + 6 = 3$.

Method 2:

Substitute $x = -3$ into the identity, we have

$(-6 + m)(-3 + 3) + 6 = (-6 + n)(-2)$

$6 = 12 - 2n$

$n = 3$

10. $x\left(1+\frac{8\%}{2}\right)^{2\times 2} - x = 66351$

$x(1.04^4 - 1) = 66351$

$x = 390625$

11. $r = k\frac{\sqrt{p}}{q} \Rightarrow p = \frac{1}{k^2}r^2q^2$

$p' = \frac{1}{k^2}(r')^2(q')^2 = \frac{1}{k^2}(r)^2(0.25q)^2 = 0.0625p$

Percentage decrease $= \frac{p-p'}{p} \times 100\% = 93.75\%$.

12. In one minute, pipe A and pipe B can fill $\frac{1}{60}$ and $\frac{1}{84}$ of the pool respectively.

Therefore, $\frac{1}{60}+\frac{1}{84}=\frac{1}{35}$ of the pool can be filled in one minute when both pipe A and pipe B are used at the same time. So, 35 minutes is needed.

13. Number of dots = 1 + 4 + 8 + 12 + … + 28

$= 1 + \frac{(4+28)\times 7}{2}$

= 113

14. $BC \times CE = GC \times CD \Rightarrow \frac{BC}{CD} = \frac{GC}{CE}$

Hence, $BC : CD = 3 : 4$.

Let $BC = 3k$ and $CD = 4k$, we have $(3k)(4k) = 300$ and so $k = 5$.

Now, $AF = AD - DF = 3(5) - 3 = 12$ (cm) ;

$AH = AB - HB = 4(5) - 4 = 16$ (cm) .

$FH = \sqrt{AF^2 + AH^2} = \sqrt{12^2 + 16^2} = 20$ (cm)

15. Let the radius and height of the right circular cylinder be r and h respectively. Then the radius of the sphere is $2r$.

$$4\pi(2r)^2 = 6\left(2\pi r^2 + 2\pi rh\right)$$

$$16\pi r^2 = 12\pi r^2 + 12\pi rh$$

$$4\pi r^2 = 12\pi rh$$

$r = 3h$

$$\frac{\text{The volume of the sphere}}{\text{The volume of the circular cylinder}} = \frac{\frac{4}{3}\pi(2r)^3}{\pi r^2(\frac{r}{3})}$$

$= 32$.

16. Let $CE : AD = 1 : r$, i.e. the ratio between the side lengths of the similar triangles CEF and ADF .

Observing ΔCDF and ΔCEF, we have $\frac{\text{The area of } \Delta CDF}{\text{The area of } \Delta CEF} = \frac{DF}{EF} = r$.

So, area of $\Delta CEF = \frac{100}{r}$ (cm^2).

Observing ΔCDF and ΔADF, we have $\frac{\text{The area of } \Delta CDF}{\text{The area of } \Delta ADF} = \frac{CF}{AF} = \frac{1}{r}$.

So, area of $\Delta ADF = 100r$ (cm^2).

Since Area of ΔACD = Area of ΔABC , we have

$$100r + 100 = 145 + \frac{100}{r}$$

$20r^2 - 9r - 20 = 0$

$(4r - 5)(5r + 4) = 0$

$r = \frac{5}{4}$ or $r = \frac{-4}{5}$ (rej.)

Thus, $r = \frac{5}{4}$ and $BE : EC = (AD - EC) : EC = (r - 1) : 1 = 1 : 4$.

17. Let $\angle CDB = \theta$, then $\angle DCB = \theta$ (base $\angle$s, isos. Δ) .

Also,

$\angle DAC = \angle ADC$ (base $\angle$s, isos. Δ)

$= 42° + \theta$

In ΔACD, $2(42° + \theta) + \theta = 180°$. Therefore, $\theta = 32°$.

$\angle ABD = \angle BCD + \angle BDC = 64°$ (ext. $\angle$s of Δ)

18. $DE = \sqrt{EF^2 - FD^2} = \sqrt{15^2 - 9^2} = 12$.

Notice that $\Delta ABF \sim \Delta FDE$, we have $AB : BF : AF = FD : DE : EF$. So,

$15 : CD : AF = 9 : 12 : 15$, and $CD = \frac{15 \times 12}{9} = 20$ cm .

19. Since $BCDE$ is a rhombus, we know that $\angle CBD = \angle DBE$. Now,

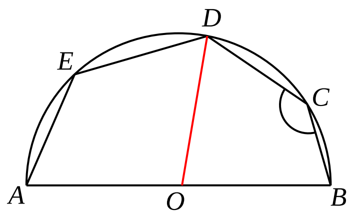
$\angle CBE = \angle AEB + \angle EAB$ (ext. $\angle$ of Δ)

$2\angle DBE = 90° + 40°$

$\angle DBE = 65°$

20. Method 1:

Denote the centre of the circle by O . Join OD .



Reflex $\angle AOD = 2\angle AED$ ($\angle$ at centre twice angle at circumference)

$360° - \angle AOD = 2(144°)$

$\angle AOD = 72°$

On the other hand,

Reflex $\angle BOD = 2\angle BCD$ ($\angle$ at centre twice angle at circumference)

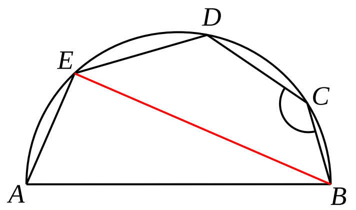
$180° + \angle AOD = 2\angle BCD$

$180° + 72° = 2\angle BCD$

$\angle BCD = 126°$

Method 2:

Join BE .



$\angle AED = \angle AEB + \angle BED$

$144° = 90° + \angle BED$ ($\angle$ at semi-circle)

$\angle BED = 54°$

Since $\angle BED + \angle BCD = 180°$ (opp. $\angle$s, cyclic quad.)

Therefore, $\angle BCD = 180° - 54° = 126°$

(Similarly, the question can be solved by joining AC.)

21. From the given information, we can show that $\Delta DAE \cong \Delta CAB$ (SAS) .

Hence, $DE = BC$ and I is correct.

Moreover, $\angle ADE = \angle ECF$ (corr. $\angle$s, $\cong\Delta$s) .

It can be observed that $\Delta ADE \sim \Delta FCE$ (AA) .

So, $\angle CFE = \angle DAE = 60°$ and II is correct.

22. $\begin{cases} \tan\alpha = \dfrac{AB}{BD} \\ \tan\beta = \dfrac{BD}{BC} \end{cases} \Rightarrow \tan\alpha\tan\beta = \dfrac{AB}{BD}\times\dfrac{BD}{BC}$

So, $\dfrac{\tan\alpha\tan\beta}{1} = \dfrac{AB}{BC}$

$AB : BC = \tan\alpha\tan\beta : 1$

23. If two straight lines are parallel, then the ratio of the coefficients of x to y are the same, i.e. 2 : 3 .

Hence, C is the answer.

24. $x + ay + b = 0 \Rightarrow y = \frac{-x}{a} + \frac{-b}{a}$ ($a \neq 0$ as L is not vertical)

By observing the slope of L, the x-intercept and the y-intercept, we have

$$\begin{cases} \frac{-1}{a} < 0 \\ -b > 1 \\ \frac{-b}{a} > 2 \end{cases}$$

Therefore, $a > 0$ and $b < -1$. So, I is true.

Since L is above the point P (1, 2) , it passes through a point (1, k) where $k > 2$.

So, $1 + ak + b = 0$.

Therefore $ak + b = -1$.

As $a > 0$ and $k > 2$,

$ak + b > 2a + b$

So, $2a + b < -1$ and III is false.

(Another way to determine the correctness of III:

Let the equation of the line S passing through P and parallel to L be $x + ay + c = 0$, then

$1 + 2a + c = 0 \Rightarrow c = -1 - 2a$.

Therefore the equation of S is $x + ay + (-1 - 2a) = 0$.

Consider the x-intercepts of L and S , we know that

$0 < 1 + 2a < -b$, so $2a + b < -1$.)

25. L_1 and L_2 are parallel. Hence, the locus of P is the straight line midway between L_1 and L_2 .

It can be observed that the x-intercept of the locus of P is the average of the x-intercepts of L_1 and L_2 .

Since the x-intercepts of L_1 and L_2 are $-a$ and $-b$ respectively, the x-intercept of the locus of P is $\frac{(-a)+(-b)}{2}$. That is, D .

26. Suppose the order of rotational symmetry is n .

The figure repeats itself after rotating about its centre of rotation through $\frac{360°}{n}$, $\frac{2 \times 360°}{n}$, $\frac{3 \times 360°}{n}$, …

Therefore, $\frac{k \times 360°}{n} = 108°$ for some integer k .

Now, $\frac{k}{n} = \frac{3}{10}$ implies that the least possible order of rotational symmetry, n, is 10 .

(Or by elimination: for example, a figure with order of rotational symmetry 5 repeats itself after rotation of $\frac{360°}{5} = 72°$.

As 108 is not a multiple of 72, 5 is not a possible answer.)

27. Rearranging terms, we have $x^2 + y^2 - 4x - 6y - 3 = 0$. By writing in standard form, $(x - 2)^2 + (y - 3)^2 = 4^2$.

Hence, the centre of C is (2, 3) and II is true.

The radius of C is 4 and I is false.

Since the distance from the centre (2, 3) to the origin is $\sqrt{13}$, less than the radius of the circle.

Therefore, the interior of C "covers" origin and C lies in all quadrants, including the third quadrant. So, III is true.

28. Required probability $= \text{P(both joined 3 act.)} + \text{P(both joined 4 act.)} + \text{P(both joined 5 act.)}$

$$= \frac{2}{9} \times \frac{1}{8} + \frac{3}{9} \times \frac{2}{8} + \frac{4}{9} \times \frac{3}{8}$$

$$= \frac{5}{18}$$

29. 75th percentile = Third quartile (Q3) = Upper quartile

From the diagram, 45 is the 75th percentile.

30. Since the range of the distribution is greater than 8, we know that $n > 20$ or $n < 12$.

If $n < 12$, the median becomes $(15 + 16) / 2 = 15.5$. Hence, b is false.

In both cases, the interquartile range is 4.5 , hence a is true.

The least possible value of $n = 1$ as the numbers are positive integers. Therefore,

the least possible value of c can be $\frac{12 + 14 + 15 + 16 + 17 + 18 + 20 + 1}{8} = 14\frac{1}{8}$ which is less than 16. III is false.

31. We have $\frac{1}{y} = mx + c$ where m and c are constant.

By observing the x-intercept, we have $0 = 2m + c$. So $c = -2m$.

Therefore, $\frac{1}{y} = mx - 2m$.

i.e. $1 = m(x - 2)y$

So, $(x - 2)y$ is a constant.

32. Method 1:

$FFF\ FFF\ FFF\ FFF\ FFF\ FFF_{16} = 15 + 15 \times 16 + 15 \times 16^2 + 15 \times 16^3 + \ldots + \ + 15 \times 16^{17}$

$$= \frac{15(16^{18} - 1)}{16 - 1}$$

$= 16^{18} - 1$

$= 2^{72} - 1$

Method 2:

$FFF\ FFF\ FFF\ FFF\ FFF\ FFF_{16} + 1_{16} = 1 \times 16^{18}$

Therefore, The number $= 16^{18} - 1$

$= 2^{72} - 1$

33. Method 1:

By "guessing" the possible transformation: $y = \mathrm{f}(x)$ can be transformed to $y = \mathrm{g}(x)$ by

Step 1: Reducing along the y-axis by $\frac{1}{2}$

Step 2: Translate $\frac{1}{2}$ units to the right

Therefore, $x \to 2x \to 2\left(x - \frac{1}{2}\right)$

Hence, $\mathrm{g}(x) = \mathrm{f}(2x - 1)$.

Method 2:

By "chasing points": From the graph of $y = \mathrm{f}(x)$, we know that $\mathrm{g}(1) = 0$, $\mathrm{g}(3) = 0$, $\mathrm{f}(1) = 0$ and $\mathrm{f}(5) = 0$.

Substitute $x = 1$ into the four choices.

For choice A,

LHS = $\mathrm{g}(1) = 0$.

RHS = $2\mathrm{f}(1) - 1 = 2(0) - 1 = -1 \neq$ LHS .

For choice C,

LHS = $\mathrm{g}(1) = 0$.

RHS = $2\mathrm{f}(1 - 1) = 2\mathrm{f}(0) > 0$

For choice D,

LHS = $\mathrm{g}(1) = 0$.

RHS = $\mathrm{f}(2(1)) - 1 = \mathrm{f}(2) - 1 < 0$

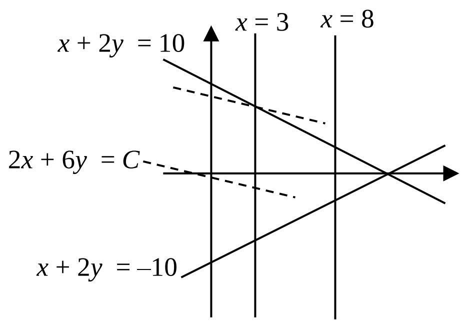
Hence, the only possible choice is B .

34. Ignore the condition of integral solution first.

By sliding the line $2x + 6y = C$, the maximum value of C is attained at the intersection of $x = 3$ and $x + 2y = 10$. Solving, we have $x = 3$ and $y = 3.5$.

Now, consider the integral solutions. The possible points are (3, 3) or (4, 3) .

Hence, the greatest value of $2x + 6y - 1 = 2(4) + 6(3) - 1 = 25$.



35. Method 1:

$\frac{i+a}{bi+2} = ki$ where k is a real number. Then

$a + i = -bk + 2ki$

Comparing the real parts and the imaginary parts, we have

$$\begin{cases} a = -bk \\ 1 = 2k \end{cases}$$

Hence, $k = 0.5$ and $a = -0.5b$.

Therefore, $2a + b = 0$.

Method 2:

$$\frac{i+a}{bi+2} = \frac{(a+i)(2-bi)}{(2+bi)(2-bi)}$$

$$= \frac{(2a+b)+(2-ab)i}{4+b^2}$$

Consider the real part, we know that $2a + b = 0$.

36. Let the general term of the sequence be $S(n)$.

$S(1) = \frac{9^1-3}{8} = \frac{3}{4}$.

For $n > 1$,

$$S(n) = \frac{9^n-3}{8} - \frac{9^{n-1}-3}{8}$$

$$= \frac{9^n-9^{n-1}}{8}$$

$$= \frac{9^{n-1}(9-1)}{8}$$

$= 9^{n-1}$

Therefore, the sequence is $\frac{3}{4}$, 9, 81, 729, …

Hence, all terms are rational and I is true.

The sequence is not a geometric sequence and II is false.

The 10^{th} term $= 9^{10-1} < 4 \times 10^8$ and III is true.

37. $\begin{cases} \log_4 y = 3x - 1 \\ \log_8 y^3 = 3x^2 + 1 \end{cases}$

$\begin{cases} y = 4^{3x-1} \\ y^3 = 8^{3x^2+1} \end{cases}$

$\begin{cases} y = 2^{6x-2} \\ y = 2^{3x^2+1} \end{cases}$

Therefore, $3x^2 + 1 = 6x - 2$

$3x^2 - 6x + 3 = 0$

$3(x - 1)^2 = 0$

$x = 1$

Therefore, $y = 4^{3(1) - 1} = 16$.

38. Let the radius of the in-circle be r , then

Area of $\Delta BCP = \dfrac{BC \times r}{2} = 5r$

Area of $\Delta ABP = \dfrac{AB \times r}{2} = 4.5r$

Area of $\Delta ACP = \dfrac{CA \times r}{2} = 8.5r$

Area of $\Delta ABC = \sqrt{18 \times (18-9) \times (18-10) \times (18-17)}$

$= 36$.

Therefore area of $\Delta BCP = 36 \times \dfrac{5}{5 + 4.5 + 8.5}$

$= 10$.

39. Let the centre of the circle be O . Join OA and OC .

Since AT and CT are tangents, we have $\angle OCT = \angle OAT = 90°$.

$\angle OCT + \angle AOC + \angle OAT + \angle ATC = 360°$ ($\angle$ sum of polygon)

$90° + \angle AOC + 90° + 42° = 360°$

$\angle AOC = 138°$

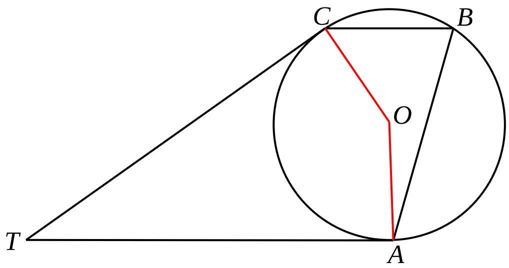
$2\angle ABC = 138°$ ($\angle$ at centre, twice $\angle$ at circumference)

$\angle ABC = 69°$

$\angle BCT + \angle ABC + \angle BAT + \angle ATC = 360°$ ($\angle$ sum of polygon)

$\angle BCT + 69° + 100° + 42° = 360°$

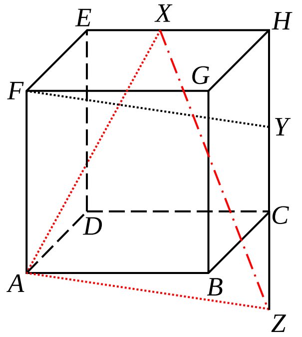
$\angle BCT = 149°$



40. Suppose the length of the cube is $2k$.

Translate FY down $2k$ units. F is translated to A and let Y be translated to Z .

Then the required angle is $\angle XAZ$.



$$XA = \sqrt{(2k)^2 + (2k)^2 + k^2} = 3k$$

$$ZA = \sqrt{(2k)^2 + (2k)^2 + k^2} = 3k$$

$$XZ = \sqrt{k^2 + (3k)^2} = \sqrt{10}k$$

$$\angle XAZ = \cos^{-1}\left(\frac{(3k)^2 + (3k)^2 - (\sqrt{10}k)^2}{2(3k)(3k)}\right) = \cos^{-1}\left(\frac{4}{9}\right) \approx 64° .$$

41. $\begin{cases} x^2 + y^2 - 2x - 6y + 8 = 0 \\ y = kx \end{cases}$

$x^2 + (kx)^2 - 2x - 6(kx) + 8 = 0$

$(k^2 + 1)x^2 - (2 + 6k)x + 8 = 0$

If the circle and the line intersect, we have

$[-(2 + 6k)]^2 - 4(k^2 + 1)(8) \geq 0$

$36k^2 + 24k + 4 - 32k^2 - 32 \geq 0$

$k^2 + 6k - 7 \geq 0$

$(k + 7)(k - 1) \geq 0$

$k \leq -7$ or $k \geq 1$

42. Number of possible unit digits = 4 .

Answer = $4 \times 8 \times 7 \times 6 = 1344$.

43. Required probability $= P(T = 2 \text{ and } M = 1) + P(T = 2 \text{ and } M = 0) + P(T = 1 \text{ and } M = 0)$

$= (0.7)^2(2)(0.6)(0.4) + (0.7)^2(0.4)^2 + 2(0.7)(0.3)(0.4)^2$

$= 0.3808$

44. Let the mean and the standard deviation be μ and σ respectively.

$\begin{cases} \mu - 1.5\sigma = 56 \\ \mu - 0.6\sigma = 74 \end{cases}$

Solving, we have

$\begin{cases} \sigma = 20 \\ \mu = 86 \end{cases}$.

45. The second set of numbers is produced by multiply the first set of number by –2 and then add 1 to the numbers.

Hence, the mean, median and mode would be multiply by –2 and add 1 to the original.

Therefore, $m_2 = -2m_1 + 1$. So I is true.

The standard deviation is multiplied by 2, i.e. $s_2 = 2s_1$. Thus, II is false.

The variance is multiplied by 2^2, i.e. $v_2 = 4v_1$. Thus, III is true.