Ying Wa Girls' School
Mock Examination 2018-2019

MATHEMATICS

Compulsory Part

PAPER 1

Question-Answer Book

Time allowed: 2 hours 15 minutes
This paper must be answered in English.

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1.
2. This paper consists of THREE sections: A(1), A(2) and B. Each section carries 35 marks.
3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, and fasten them with string INSIDE this book.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
7. The diagrams in this paper are not necessarily drawn to scale.

Date: 10th January 2019
Period: 1

Name: _____

Class Number: _____

Class: S6 _____

Candidate Number				

	Marker's Use Only	**Examiner's Use Only**
	Marker No.	Examiner No.
Question No.	Marks	Marks
1– 2		
3 – 4		
5 – 6		
7 – 8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		
Total		

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SECTION A(1) (35 marks)

1. Simplify $\dfrac{\left(2xy^4\right)^3}{8x^{-2}y^3}$ and express your answer with positive indices.

(3 marks)

2. Make x the subject of the formula $\dfrac{Ax}{B} = x + 2C$.

(3 marks)

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3. (a) Factorize $m^2 - 6mn + 9n^2$.

(b) Hence, or otherwise, factorize $36k^2 - 4m^2 + 24mn - 36n^2$.

(4 marks)

4. Consider the compound inequality

$$2x \leq x - \frac{3+x}{2} \quad \text{or} \quad 5x \geq -20 \quad \text{.....(*)}$$

(a) Solve (*).

(b) Write down the smallest positive integer satisfying (*).

(5 marks)

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5. In a fruit shop, there are 148 apples and oranges in total. If the number of oranges is 15% less than the number of apples, find the difference of the number of apples and the number of oranges.

(4 marks)

6. The marked price of a book is $120. The book is sold at a discount of 25 % on its marked price.
   (a) Find the selling price of the book.
   (b) If the marked price of the book is 50 % above its cost, determine whether there is a gain or a loss after selling the book. Explain your answer.

(4 marks)

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7. The polar coordinates of a point A are $(5, 30^\circ)$. Point B is obtained by rotating point A anticlockwise about the origin through 120°.
   (a) Write down the polar coordinates of B.
   (b) If O is the origin of the coordinate plane, find the shortest distance from O to AB.

(3 marks)

8. In Figure 1, O is the centre of the semi-circle $ABCD$. It is known that BC // OD and OC intersect BD at E at right angles.

   (a) Name all the triangles congruent to ΔODE.
   (b) Anthony claims that $OBCD$ is a rhombus. Do you agree? Explain your answer.
   (c) Write down the value of $\angle OBC$.

(5 marks)

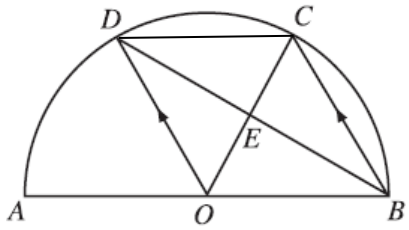

Figure 1

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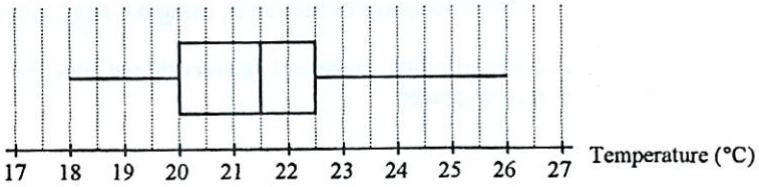
9. In an experiment, the temperatures of a certain liquid under various experiment settings are measured. The box-and-whisker diagram for these temperatures (in °C) is constructed below:



The temperature C (in °C) can be converted to the temperature F (in °F) according to the formula $F = \frac{9}{5}C + 32$. Find the median and interquartile range of the temperature in °F.

(4 marks)

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SECTION A(2) (35 marks)

10. A test is taken by a class of 18 students. The marks are as follows:

55	82	74	70	91	75	79	89	68
79	59	72	79	73	60	71	82	k

where k is Jane's mark.

It is known that the mean mark of the class is the same irrespective of including or excluding Jane's.

(a) Find the value of k.

(2 marks)

(b) A student mark x is classified as an *outlier* if $x < \mu - 2\sigma$ or $x > \mu - 2\sigma$, where μ is the mean and σ is the standard deviation of the set of marks.

(i) Find all the *outlier*(*s*) of the set of the 18 student marks.

(ii) In order to assess the student' performance in the set, all *outliers* are removed from the set. Describe the change in the median and the standard deviation of the student marks due to such removal.

(4 marks)

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11. There is a right circular conical vessel with some water. The original depth of water in the vessel is 20 cm. If 245 cm^3 of water is poured into the vessel without overflowing, the depth of water in the vessel is increased by 5 cm.

(a) Find the final volume of water in the vessel.

(3 marks)

(b) Someone claims that the final area of wet curved surface of the vessel is less than 350 cm^2. Do you agree? Explain your answer.

(3 marks)

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12. The cost (\$$C$) of a camp is partly constant and partly varies directly as the number of participants (n). When $n = 35$, $C = 20750$. When $n = 40$, $C = 23000$.

(a) If there are 50 participants, find the cost of the camp.

(4 marks)

(b) Is it possible that the cost of the camp is \$34025? Explain your answer.

(2 marks)

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13. Figure 2 shows the graphs for car X and car Y travelling on the same straight road between town P and town Q during the period 2:00 pm to 4:00 pm. Car X travels at a constant speed during the period. Car Y comes to rest at 2:30 pm.

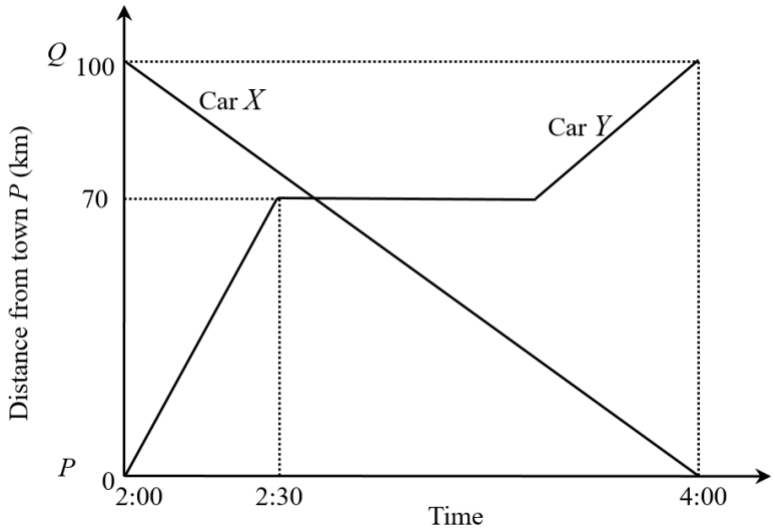

Figure 2

(a) Find the distance of car X from town P at 2:30 pm.

(2 marks)

(b) At what time after 2:00 pm do the two cars first meet?

(2 marks)

(c) The driver of car X claims that the average speed of car X is lower than that of car Y during the period 2:30 pm to 4:00 pm. Do you agree? Explain your answer.

(2 marks)

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14. Let $f(x) = 7x^3 + hx^2 + kx - 4$. When $f(x)$ is divided by $x - 2$, the remainder is -50. It is given that $f(x)$ is divisible by $x - 4$.

(a) Find the values of h and k.

(3 marks)

(b) Tom claims that all the roots of $f(x) = 0$ are real numbers. Do you agree? Explain your answer.

(3 marks)

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15. The equation of the straight line L is $\sqrt{3}x + y = 0$. P is a moving point such that P is equidistant from L and the x-axis. Describe the locus of P and find its equation(s).

(5 marks)

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SECTION B (35 marks)

16. A drama club is formed by 8 girls and 4 boys. A team of 3 students is selected from the club to participate in a competition.

(a) If the team consists of at least one girl, find the number of different teams that can be formed.

(2 marks)

(b) Given that the team consists of at least one girl, find the probability that the number of girls is more than the number of boys in the team.

(2 marks)

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17. The product of the 1st term and the 2nd term of a geometric sequence is 12 while the product of the 3rd term and the 4th term of the sequence is 972.

(a) Find the first term of the sequence.

(2 marks)

(b) If the first term is a positive real number, find the greatest value of n such that the sum of the n th term and the $(n + 2)$ th term is less than 3×10^{16}.

(3 marks)

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18. Let m, n and r be real numbers satisfying

$$\begin{cases} m+n+r=5 \\ mn+mr+nr=3 \end{cases}.$$

(a) (i) Find $m+n$ and mn in terms of r.

(ii) Find a quadratic equation in x, in terms of r, with roots m and n.

(3 marks)

(b) Hence, or otherwise, find the range of possible values of r.

(2 marks)

(c) Someone claims that if m takes the smallest possible value, then $n = r$. Do you agree? Explain your answer.

(2 marks)

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19. Figure 3(a) shows a piece of pentagonal paper card $ABCDE$, where $ABCD$ is a parallelogram. It is given that $AC = 15$ cm, $CD = 12$ cm, $AE = AB$ and $DE = 10$ cm. Suppose that $\angle ACD = 70°$.

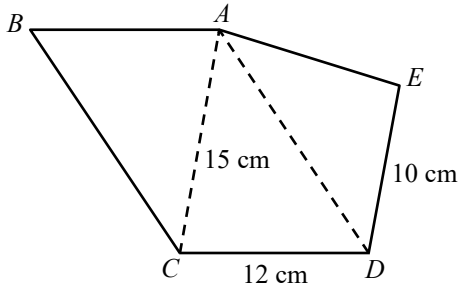

Figure 3(a)

(a) (i) Find the length of AD, and hence the length of BC.
(ii) Find $\angle AED$.

(4 marks)

(b) The paper card in Figure 3(a) is folded along AC and AD such that AB and AE join together as shown in Figure 3(b). K is a point on CD such that $\angle CAK = 35°$. A, C and D are points on the ground. Find the angle between the lines AB and AK.

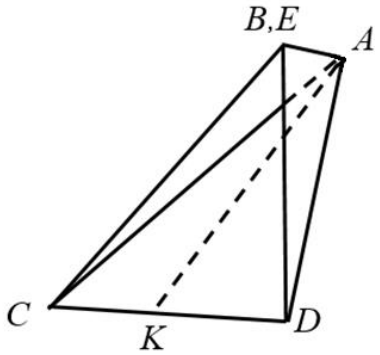

Figure 3(b)

(6 marks)

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20. Let C: $x^2 + y^2 = 4$ be a circle on a rectangular coordinate plane.

(a) Let L: $y = mx + c$ be a tangent to C, where m and c are real constants. Show that $c^2 = 4\left(m^2 + 1\right)$.

(3 marks)

(b) Let $P(a, b)$ be a moving point on the same coordinate plane as C.

(i) A tangent to C with slope m is drawn from P. Prove that $\left(a^2 - 4\right)m^2 - 2abm + b^2 - 4 = 0$.

(ii) Two tangents with slopes m_1 and m_2 respectively are drawn from P to C such that they are perpendicular to each other. Find the equation of locus of P.

(6 marks)

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