Mock Exam
MATH CP
PAPER 1

YingWa Girls' School

Mock Examination 2018-2019

MATHEMATICS

Compulsory Part

PAPER 1

Marking Scheme

SECTION A(1) (35 marks)

1.

(3 marks)

$$\frac{(2xy^4)^3}{8x^{-2}y^3} = \frac{8x^3y^{12}}{8x^{-2}y^3}$$ 1M

$$= x^5y^9$$ 1M for $x^{3-(-2)}$ or y^{12-3} (can be absorbed)
1A

2.

(3 marks)

$$\frac{Ax}{B} = x + 2C$$

$$Ax = Bx + 2BC$$ 1M for removing denominator (can be absorbed)

$$Ax - Bx = 2BC$$ 1M for putting x on one side

$$x = \frac{2BC}{A-B}$$ 1A or equivalent

3.

(4 marks)

(a) $m^2 - 6mn + 9n^2 = (m-3n)^2$ 1A

(b) $36k^2 - 4m^2 + 24mn - 36n^2 = 36k^2 - 4(m^2 - 6mn + 9n^2)$

$= 36k^2 - 4(m-3n)^2$ 1M for using (a)

$= [6k + 2(m-3n)][6k - 2(m-3n)]$ 1M for difference of two squares

$= 4(3k+m-3n)(3k-m+3n)$ 1A

Alternative Solution

$36k^2 - 4m^2 + 24mn - 36n^2 = 4[9k^2 - (m^2 - 6mn + 9n^2)]$

$= 4[9k^2 - (m-3n)^2]$ 1M for using (a)

$= 4(3k+m-3n)(3k-m+3n)$ 1M for difference of two squares + 1A

4.

(5 marks)

(a) $2x \le x - \dfrac{3+x}{2}$

$4x \le 2x - 3 - x$ — 1M for putting x on one side

$3x \le -3$

$x \le -1$ — 1A

$5x \ge -20$

$x \ge -4$ — 1A

Therefore, we have $x \le -1$ or $x \ge -4$

Thus, the solutions of (*) are all real numbers. — 1A

(b) 1 — 1A

5.

(4 marks)

Let x and y be the number of apples and the number of oranges respectively.

$$\begin{cases} x + y = 148 \ldots\ldots\ldots\ldots\ldots(1) \\ y = (1-15\%)x \ldots\ldots(2) \end{cases}$$ — 1A+1A

So, we have $x + 0.85x = 148$ — 1M for an equation in only 1 unknown

Solving, we have $x = 80$ and $y=68$

$\therefore$ The difference of the number of apples and number of oranges is 12. — 1A

Alternative Solution

Let y be the number of oranges.

$y = (1 - 15\%)(148 - y)$ — 1A for y=(1-15%)x + 1A for x = 148-y

$y = 125.8 - 0.85y$ — 1M for a linear equation

$y = 68$

Note that $68 \div (1 - 15\%) - 68 = 12$.

$\therefore$ The difference of the number of apples and number of oranges is 12. — 1A

Alternative Solution

Difference

$= 148 \div 1.85 \times 0.15$ — 1A for 1.85 + 1A for 0.15 + 1M

$= 12$. — 1A

6.

(4 marks)

(a) $120 \times (1 - 25\%)$ — 1M

$= \$90$

$\therefore$ The selling price of the book is \$90. — 1A

(b) The cost of the book

$= 120 \div (1 + 50\%)$ — 1M

$= \$80$

$< \$90$

$\therefore$ There is a gain after selling the book. — 1 f.t.

7.

(3 marks)

(a) $B = (5, 150^{\circ})$ 1A

(b) Req. distance $= 5\cos 60^{\circ}$ 1M

$= \frac{5}{2}$ 1A

8.

(5 marks)

(a) ΔOBE, ΔCBE, ΔCDE 1A for any, 2A for all

(b) $OB = OD = CD = CB$ (corr. sides, $\cong \Delta$s)

Therefore, $OBCD$ is a rhombus

Alternatively,

$OE = CE$ and $DE = BE$ (corr. sides, $\cong \Delta$s) and $OC \perp BD$ (given)

Therefore, $OBCD$ is a rhombus

2 marks for correct proof with reasons
1 mark for correct proof with some reasons

(c) 60° 1A

9.

(4 marks)

Median $= \frac{9}{5}(21.5) + 32 = \underline{\underline{70.7°F}}$ 1A for finding 21.5 °C from graph + 1A

Interquartile range $= \frac{9}{5}(22.5 - 20) = \underline{\underline{4.5°F}}$ 1M for New IQR $= \frac{9}{5} \times$ Original IQR + 1A

SECTION A(2) (35 marks)

10. (6 marks)

(a)

Since the mean of the 17 student marks is 74, 1M

$k = 74$ 1A

(b) (i)

The standard deviation of the 18 student marks is 9.32737905.

The corresponding interval is (74 – 2×9.32737905, 74 + 2×9.32737905) 1M

≈ (55.34524189, 92.65475811)

Thus, 55 is an *outlier*. 1A

(ii)

There is no change in the median, which is 74 in both cases. 1A

The standard deviation decreases as the extreme datum (the outlier) is removed. 1A

11. (6 marks)

(a)

Let V cm^3 be the volume of water in the vessel.

$$\frac{V-245}{V}=\left(\frac{20}{25}\right)^3$$

$$V=502.0491803$$

The final volume of water in the vessel is 502 cm^3 (*cor. to 3 sig. fig.*) — 1M + 1A, 1A

(b)

Let r cm be the radius of water surface.

$$\frac{1}{3}\times\pi\times r^2\times 25=502.05$$

$$r=4.379139881$$ — 1M

The final area of wet curved surface

$=\pi\times r\times\sqrt{r^2+25^2}$ — 1M

≈ 349.1734811 cm^2

<350 cm^2

The claim is correct. — 1A f.t.

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12. (6 marks)

(a)

Let $C=k_1+k_2n$, where k_1 and k_2 are non-zero constants. — 1A

$20750=k_1+35k_2$ .... (1) — 1M

$23000=k_1+40k_2$ .... (2)

Solving, $k_2=450$

$k_1=5000$ — 1A for both k_1 and k_2

$C=5000+450n$

When $n=50$,

$C=5000+450(50)$

$=27500$

The cost of the camp is \$27500. — 1A

(b)

$34025=5000+450n$ — 1M

$n=64.5$

It is not possible since 64.5 is not an integer. — 1A f.t.

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13 (6 marks)

(a) The distance of car X from town P at 2:30 pm

$= \dfrac{100(120-30)}{120}$ 1M

$= 75$ km 1A

(b) Suppose that car X and car Y meet at the time t minutes after 2:30 pm.

$\dfrac{t+30}{120} = \dfrac{30}{100}$ 1M

$t = 6$

$\therefore$ Car X and car Y meet at 2:36 pm. 1A

(c)

During the period 2:30 pm to 4:00 pm, car Y travels 30 km while car X travels more than 30 km.

$\therefore$ The average speed of car Y is lower than that of car X. 1M

$\therefore$ The claim is disagreed. 1A f.t.

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14. (a) (6 marks)

$f(2) = -50$ 1M

$7(2)^3 + h(2)^2 + k(2) - 4 = -50$

$2k + 52 + 4h = -50$ ……(1)

$f(4) = 0$

$7(4)^3 + h(4)^2 + k(4) - 4 = 0$

$4k + 444 + 16h = 0$ ……(2)

Solving,

$h = -30$ 1A

$k = 9$ 1A

(b)

$f(x) = 0$

$(x-4)(7x^2 - 2x + 1) = 0$ 1A

$\Delta = (-2)^2 - 4(7)$ 1M
$= -24$
< 0

The equation $f(x) = 0$ has one real root and two unreal roots.

$\therefore$ The claim is disagreed. 1A f.t.

15. Let the inclination of L be θ. (5 marks)

$\tan\theta = -\sqrt{3}$ 1M

$\theta = 120°$

The required locus is a pair of angle bisectors, L_1 and L_2. 1A

L_1 and L_2 pass through the origin.

The equation of L_1 is

$y = x\tan 60^\circ$ 1M

$\sqrt{3}x - y = 0$ 1A

(accept $y = \sqrt{3}x$)

The equation of L_2 is

$y = x\tan 150^\circ$

$y = -\dfrac{x}{\sqrt{3}}$

$x + \sqrt{3}y = 0$ 1A

(accept $y = -\dfrac{\sqrt{3}}{3}x$ or $y = -\dfrac{1}{\sqrt{3}}x$)

SECTION B (35 marks)

16.

(2+2 = 4 marks)

(a) No. of teams can be formed

$= C_3^8 + C_2^8C_1^4 + C_1^8C_2^4$ 1M for 3 cases

$= 216$ 1A

Alternative:

$C_3^{12} - C_3^4$ 1M

$= 216$ 1A

(b) Required Probability

$= \dfrac{\dfrac{C_3^8 + C_2^8C_1^4}{C_3^{12}}}{\dfrac{216}{C_3^{12}}}$ 1M for denominator

$= \dfrac{7}{9}$ 1A r.t. 0.778

Alternative:

$\left(1 - \dfrac{C_3^4}{C_3^{12}} - \dfrac{C_2^4C_1^8}{C_3^{12}}\right) \div \left(\dfrac{216}{C_3^{12}}\right)$ 1M $= \dfrac{7}{9}$ 1A r.t. 0.778	$\dfrac{C_3^8 + C_2^8C_1^4}{216}$ 1M for denominator $= \dfrac{7}{9}$ 1A r.t. 0.778	$\dfrac{C_3^{12} - C_3^4 - C_2^4C_1^8}{216}$ 1M $= \dfrac{7}{9}$ 1A r.t. 0.778

17.

(2+3 = 5 marks)

(a) Let a be the first term and r be the common ratio

$a \times ar = 12$ … (1)

$ar^2 \times ar^3 = 972$ … (2) 1M for either one

(2)÷ (1):

$r^4 = 81$

$r = 3$ or $r = -3$ (rej)

By (1), when $r = 3$, $a = 2$ or $a = -2$ 1A

(b) $2 \times 3^{n-1} + 2 \times 3^{n+1} < 3 \times 10^{16}$ 1A

$3^{n-1}(2+18) < 3 \times 10^{16}$

$3^{n-1} < \frac{3}{2} \times 10^{15}$

$(n - 1) \log 3 < \log 1.5 + 15$

$\therefore n < 32.80761936$ 1A

$\therefore$ The greatest value of n is 32. 1A

18.

(3+2+2 = 7 marks)

(a)(i) $m + n = 5 - r$ 1A

$mn + r(m+n) = 3$

$mn + r(5-r) = 3$

$mn = r^2 - 5r + 3$ 1A

(ii) The required equation: $x^2 - (5-r)x + r^2 - 5r + 3 = 0$. 1A

(b) $x^2 - (5-r)x + r^2 - 5r + 3 = 0$ has real roots m and n.

$\therefore \Delta \geq 0$

$[-(5-r)]^2 - 4(1)(r^2 - 5r + 3) \geq 0$ 1M

$r^2 - 10r + 25 - 4r^2 + 20r - 12 \geq 0$

$3r^2 - 10r - 13 \leq 0$

$-1 \leq r \leq \frac{13}{3}$ 1A

(c) By symmetry of m, n and r and (b), $m = -1$

By (a), n and r satisfy $x^2 - [5-(-1)]x + (-1)^2 - 5(-1) + 3 = 0$ 1M for using (a) or (b) (withhold if state sub $r = -1$)

i.e. $x^2 - 6x + 9 = 0$

$\therefore n = r = 3$

$\therefore$ The claim is agreed. 1 f.t.

19.

(4+6 = 10 marks)

(a)(i) In $\triangle ACD$,

$AD^2 = AC^2 + CD^2 - 2 \times AC \times CD \times \cos \angle ACD$ [1M]

$AD = \sqrt{15^2 + 12^2 - 2 \times 15 \times 12 \times \cos 70°}$ cm

≈ 15.68032998 cm

$= 15.7$ cm, *cor. to 3 sig. fig.* [1A]

$BC = AD = 15.7$ cm [1A]

(ii) $AE = AB = CD = 12$ cm

In $\triangle AED$,

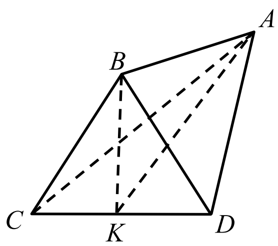
$$\cos \angle AED = \frac{AE^2 + ED^2 - AD^2}{2 \times AE \times ED}$$

$$\approx \frac{12^2 + 10^2 - 15.7^2}{2 \times 12 \times 10}$$

$\angle AED \approx 90.44709029°$

$= 90.4°$, *cor. to 3 sig. fig.* [1A]

(b)(i) Join BK.



The required angle is $\angle BAK$. [1A]

In $\triangle ACK$,

$35° + 70° + \angle AKC = 180°$

$\angle AKC = 75°$ [1A]

$$\frac{AK}{\sin \angle ACD} = \frac{AC}{\sin \angle AKC}$$

$$\frac{AK}{\sin 70°} = \frac{15 \text{ cm}}{\sin 75°}$$

$AK = 14.59262081$ cm

By the sine formula,

$$\frac{CK}{\sin \angle CAK} = \frac{AC}{\sin \angle AKC}$$

$$\frac{CK}{\sin 35°} = \frac{15 \text{ cm}}{\sin 75°}$$

$CK = 8.907150333$ cm

[1M] for finding AK or CK

[1A] for either AK or CK

In $\triangle BCD$,

$$\cos \angle BCD = \frac{BC^2 + CD^2 - BD^2}{2 \times BC \times CD}$$

$$= \frac{15.7^2 + 12^2 - 10^2}{2 \times 15.7 \times 12}$$

$\angle BCD = 39.6°$, *cor. to 3 sig. fig.*

In $\triangle BCK$,

$BK^2 = CK^2 + BC^2 - 2 \times CK \times BC \times \cos \angle BCD$

$BK = \sqrt{8.91^2 + 15.7^2 - 2 \times 8.91 \times 15.7 \times \cos 39.6°}$ cm

$= 10.49039544$ cm

[1M] for finding $\angle BCD$ or BK

In $\triangle ABK$,

$$\cos \angle BAK = \frac{AB^2 + AK^2 - BK^2}{2 \times AB \times AK}$$

$$= \frac{12^2 + 14.6^2 - 10.5^2}{2 \times 12 \times 14.6}$$

$\angle BAK = 45.2°$, *cor. to 3 sig. fig.* [1A]

20.

(3+6 = 9 marks)

(a) $\begin{cases} x^2+y^2=4 \\ y=mx+c \end{cases}$

$x^2+(mx+c)^2=4$

$(m^2+1)x^2+2mcx+c^2-4=0$ [1A]

As L is tangent to C, we have $\Delta = 0$

$(2mc)^2-4(m^2+1)(c^2-4)=0$ [1M for $\Delta = 0$]

$m^2c^2-m^2c^2+4m^2-c^2+4=0$

$\therefore\ c^2=4m^2+4$

i.e. $c^2=4(m^2+1)$ [1 f.t.]

<u>Alternative Solution (Out of Syllabus)</u>

Centre of C = (0, 0) and radius = 2

As L is a tangent to C, $\left|\dfrac{m(0)-0+c}{\sqrt{m^2+1}}\right|=2$ [1M + 1A]

i.e. $|c|=2\sqrt{m^2+1}$

$\therefore\ c^2=4(m^2+1)$ [1 f.t.]

(b) (i) Suppose L: $y=mx+c$ is the tangent from P.

Then $b=am+c$

i.e. $c=b-am$ [1A]

By (a), $c^2=4(m^2+1)$

$(b-am)^2=4(m^2+1)$ [1M for using (a)]

$a^2m^2-2abm+b^2=4m^2+4$

$(a^2-4)m^2-2abm+b^2-4=0$ [1 f.t.]

(ii) By (b)(i), m_1 and m_2 satisfy $(a^2-4)m^2-2abm+b^2-4=0$.

$\therefore\ m_1m_2=\dfrac{b^2-4}{a^2-4}$ [1A]

The tangents are perpendicular $\Rightarrow$ $m_1m_2=\dfrac{b^2-4}{a^2-4}=-1$ [1M]

$\therefore\ b^2-4=-a^2+4$

i.e. $a^2+b^2-8=0$

$\therefore$ The equation of locus of P is $x^2+y^2-8=0$. [1A]

<u>FOR REFERENCE</u>

Question

(c) Consider a circle D with equation $(x-h)^2+(y-k)^2=r^2$ on a rectangular coordinate plane, where h, k and r are constants and $r \geq 0$. If Q is a moving point on the same coordinate plane, and the two tangents from Q to D are perpendicular to each other, write down the equation of locus of Q in terms of h, k and r.

Answer

(c) Equation of locus of Q is $(x-h)^2+(y-k)^2=2r^2$.

[1A for centre (h, k)
1A for R.H.S. $2r^2$
Accept equivalent]