SECTION A(1) (35 marks)

1. Simplify $\dfrac{\left(4m^{-3}n^{3}\right)^{-2}}{2m^{-2}n^{5}}$ and express your answer with positive indices.

(3 marks)

$$\frac{\left(4m^{-3}n^{3}\right)^{-2}}{2m^{-2}n^{5}}$$

$= \dfrac{4^{-2}m^{6}n^{-6}}{2m^{-2}n^{5}}$ 1M can be absorbed for $(ab)^n = a^n b^n$ or $(a^m)^n = a^{mn}$

$= \dfrac{m^{6+2}n^{-6-5}}{32}$ 1M can be absorbed for $a^m \div a^n = a^{m-n}$ or $a^m \times a^n = a^{m+n}$

$= \underline{\underline{\dfrac{m^{8}}{32n^{11}}}}$ 1A

2. Let a, b and c be non-zero numbers such that $\dfrac{a}{b}=\dfrac{3}{4}$ and $5a = 8c$. Find $\dfrac{b-c}{a+3c}$.

(3 marks)

$\because a:b = 3:4$ and $a:c = 8:5$

$\therefore a:b:c = 24:32:15$ 1A

$\dfrac{b-c}{a+3c} = \dfrac{32-15}{24+3(15)}$ 1M

$= \underline{\underline{\dfrac{17}{69}}}$ 1A

Alternative Solution

$a = 3k, b = 4k, c = \dfrac{15}{8}k$ for k is a non-zero constant. 1A or equivalent

$\dfrac{b-c}{a+3c} = \dfrac{4-\dfrac{15}{8}}{3+3\left(\dfrac{15}{8}\right)}$ 1M

$= \underline{\underline{\dfrac{17}{69}}}$ 1A

3. Factorize

(a) $x^2 - 25y^2$,

(b) $x^2 - 4xy - 5y^2$,

(c) $x^2 - 4xy - 5y^2 - x^2 + 25y^2$.

(4 marks)

(a) $x^2 - 25y^2 = (x+5y)(x-5y)$ 1A

(b) $x^2 - 4xy - 5y^2 = (x-5y)(x+y)$ 1A

(c)

$x^2 - 4xy - 5y^2 - x^2 + 25y^2$

$= (x-5y)(x+y) - (x+5y)(x-5y)$ 1M using (a) and (b)

$= (x-5y)(x+y-x-5y)$

$= \underline{\underline{-4y(x-5y)}}$ 1A

Alternative Solution

$x^2 - 4xy - 5y^2 - x^2 + 25y^2$

$= -4xy + 20y^2$ 1M for simplification

$= \underline{\underline{-4y(x-5y)}}$ 1A

4. (a) Solve the inequality $x - \dfrac{2x+1}{3} \geq -2$.

(b) How many negative integers satisfy the following compound inequality?

$$x - \frac{2x+1}{3} \geq -2 \text{ or } 3(x-7) < 5x$$

(4 marks)

(a) $3x - (2x+1) \geq -6$ **1M**

$x \geq -5$ **1A**

(b) $3(x-7) < 5x$

$3x - 21 < 5x$

$-21 < 2x$

$x > -\dfrac{21}{2}$ **1A**

$\because x \geq -5$ or $x > -\dfrac{21}{2}$

$\therefore x > -\dfrac{21}{2}$

Required number is 10. **1A**

5. In class A, $\frac{1}{3}$ of students take a mathematics tutorial, In class B, $\frac{2}{7}$ of students take the same tutorial. If the numbers of students taking the tutorial in the 2 classes are the same, and the number of students in class B is 5 more than that of class A, find the number of students in class A.

(4 marks)

Let x be the number of students in class A.	1M
$(x+5)$ is the number of students in class B.	1A can be absorbed
$\frac{1}{3}x = \frac{2}{7}(x+5)$	1M
$7x = 6x + 30$	
$x = 30$	
The number of students in class A is 30.	1A

Alternative Solution	
Let x and y be the numbers of students in classes A and B respectively.	1M
$\frac{1}{3}x = \frac{2}{7}y$	1M
$y = x - 5$	1A
$x = 30$ and $y = 35$	
The number of students in class A is 30.	1A

6. In a polar coordinate system, P is the pole. The polar coordinates of A are $\left(\sqrt{2},\ 310^\circ\right)$. A is rotated clockwise about P through 90° to form B. Denote the axis of reflectional symmetry of ΔPAB by L.
   (a) Write down the polar coordinates of B.
   (b) Describe the geometric relationship between L and AB.
   (c) Find the polar coordinates of the point of intersection of L and AB.

(4 marks)

(a) $B = \left(\sqrt{2},\ 220^\circ\right)$ **1A**

(b) L is perpendicular bisector of AB. **1A** (No mark for only perpendicular)

(c) Required coordinates $= \left(\sqrt{2}\cos 45^\circ, 310^\circ - 45^\circ\right)$

$= \left(1,\ 265^\circ\right)$ **1A + 1A**

7. If a handbag is sold at a 15 % discount on its marked price, then the selling price will be \$2550. If the handbag is sold at a 23% discount on its marked price, then the profit percent is 50%.

(a) Find the marked price of the handbag.

(b) Find the cost of the handbag.

(4 marks)

(a) Marked price = \$2550 ÷ (1 – 15%) = \$3000 1M+1A

(b) Cost = \$3000 × (1 – 23%) ÷ (1 + 50%) = \$1540 1M+1A

8. In **Figure 1**, PQR is a triangle. Points R and T lie on SU such that $QT = RT$. It is given that $PQ = 26$, $PR = 10$ and $QR = 24$. If $\angle QTU = x$, express $\angle PRS$ in terms of x.

(5 marks)

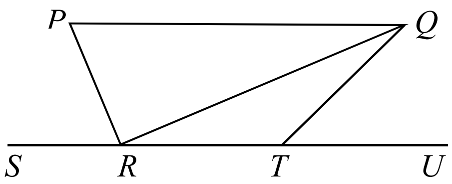


Figure 1

$PR^2 + QR^2 = 100 + 576$ **1M**

$= 676$

$= PQ^2$

$\angle PRQ = 90°$ (Converse of Pyth. Theorem) **1A**

$\angle QRT + \angle RQT = x$ (ext. $\angle$ of Δ) **1M**

$\angle QRT = \angle RQT$ (base $\angle$s, isos. Δ)

$= \frac{x}{2}$ **1A**

$\angle PRS = 180° - 90° - \frac{x}{2}$ (adj. $\angle$s on st. line)

$= 90° - \frac{x}{2}$ **1A**

9. In **Figure 2**, the histogram shows the weights of a group of participants in a fitness class.

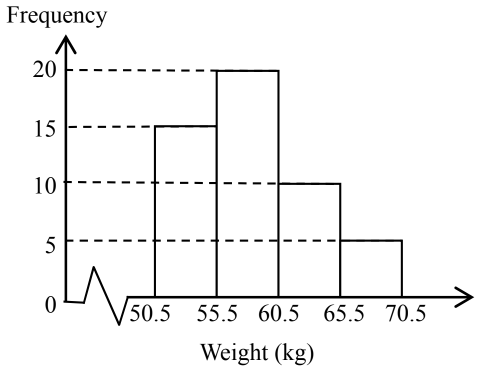


Figure 2

(a) Find the mean weight of the group of participants.

(b) Four new participants join the class and their weights belong to different class intervals. Find the change in the mean weight of all the participants.

(4 marks)

(a) Mean weight

$= \dfrac{53 \times 15 + 58 \times 20 + 63 \times 10 + 68 \times 5}{50}$ kg — **1M** correct class marks

$= \dfrac{2925}{50}$ kg

$= 58.5$ kg — **1A**

(b) New mean weight

$= \dfrac{53 \times 16 + 58 \times 21 + 63 \times 11 + 68 \times 6}{54}$ kg — **1M** for 54

$= \dfrac{3167}{54}$ kg

≈ 58.64814815 kg

Required change

$= +\left(\dfrac{3167}{54} - 58.5 \right)$ kg

$= \dfrac{4}{27}$ kg — **1A** (r.t. 0.148 kg)

SECTION A(2) (35 marks)

10. It is given that $f(x)$ is partly constant and partly varies as $2x^2$. Suppose that $f(-2) = 132$ and $f(3) = 172$.

(a) Find $f(x)$. (3 marks)

(b) Solve the equation $f(x) = 60x$. (2 marks)

(a)

Let $f(x) = k_1 + 2k_2x^2$, where k_1 and k_2 are non-zero constants.

$f(-2) = 132$

$k_1 + 2k_2(-2)^2 = 132$ **1M**

$k_1 + 8k_2 = 132$ .... (1)

$f(3) = 172$

$k_1 + 2k_2(3)^2 = 172$

$k_1 + 18k_2 = 172$ .... (2)

(2) − (1): $10k_2 = 40$

$k_2 = 4$

Substitute $k_2 = 4$ into (1). **1M for substitution or elimination**

$k_1 + (8)(4) = 132$

$k_1 = 100$

$\therefore$ $f(x) = 100 + (4)2x^2$

$f(x) = 100 + 8x^2$ **1A**

(b) $f(x) = 60x$

$100 + 8x^2 = 60x$ **1M**

$8x^2 - 60x + 100 = 0$

$2x^2 - 15x + 25 = 0$

$x = 5$ or $x = \dfrac{5}{2}$ **1A**

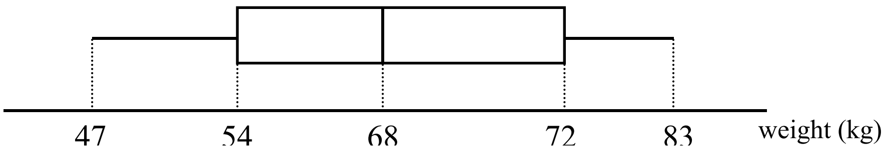
11. 20 people join a fitness training course. The stem-and-leaf diagram below shows the distribution of the weights (in kg) of these 20 people before joining the course.

Stem (tens)	Leaf (units)
5	5 7
6	1 4 7 9 9
7	2 2 6 7 7 8
8	1 3 3 3 5 6
9	0

(a) Find the median, the range and the inter-quartile range of the above distribution.

(3 marks)

(b) After completing the course, the weights of these 20 people are measured again. The box-and-whisker diagram below shows the distribution of the weights of these 20 people in the second measurement.



(i) Is the distribution of the weights of these 20 people in the second measurement less dispersed than that in the first measurement? Explain your answer.

(ii) Peter claims that at least 25% of these 20 people reduce weight after completing the course. Do you agree? Explain your answer.

(4 marks)

(a) Median = 76.5 kg **1A**

Range = 90 – 55

= 35kg **1A**

$$\text{Inter-quartile range} = \frac{83+83}{2} - \frac{67+69}{2}$$

= 15kg **1A**

(b) (i) For the second measurement:

Inter-quartile range= 72 – 54

= 18 kg

> 15 kg

Range = 83 – 47

= 36 kg

> 35 kg **1M**

∴ No, the distribution of the weights of these 20 people in the second measurement is more dispersed than that in the first measurement. **1A f.t.**

(ii) The first (lower) quartile of the distribution of the weights in the second measurement is 54 kg, which is smaller than the weight of the lightest person in the first measurement.

1M

∴ At least 25% of the people reduce weight after the course.

∴ The claim is agreed. **1A f.t.**

12. A right circular cone is divided into two parts by a plane which is parallel to its base. The upper part is a small circular cone and the lower part is a frustum of height 3 cm. The height of the original right circular cone is 12 cm.

(a) Find the ratio of the volume of the small circular cone to that of the original circular cone.

(2 marks)

(b) The base radius of the small circular cone is 18 cm.

(i) Find the volume of the frustum in terms of π.

(ii) A hemisphere and the original right circular cone with same radius are attached together to form a round bottom container. If 10000π cm^3 water is poured into the empty container, find the depth of the water in the container.

(6 marks)

(a) The required ratio

$= \left(\dfrac{12-3}{12}\right)^3$ **1M**

$= \dfrac{27}{64}$ **1A**

(b) (i) Let x cm be the radius of the original cone.

$\dfrac{18}{x} = \dfrac{12-3}{12}$ **1M**

$x = 24$

Volume of the frustum

$= \left[\dfrac{1}{3}\times\pi\times 24^2\times 12 - \dfrac{1}{3}\times\pi\times 18^2\times(12-3)\right]$ cm^3 **1M**

$= 1332\pi$ cm^3 **1A**

Alternative

Volume of the frustum

$= \dfrac{1}{3}\times\pi\times 24^2\times 12\times\left(1-\dfrac{27}{64}\right)$ cm^3 **1M or equivalent**

$= 1332\,\pi$ cm^3 **1A**

(b)(ii) Volume of the hemisphere

$= \frac{2}{3} \times \pi \times 24^3 \text{ cm}^3$ **1M**

$= 9216\,\pi \text{ cm}^3$

Volume of the remaining water in the circular cone

$= 10\,000\,\pi - 9216\,\pi$

$= 784\,\pi \text{ cm}^3$

Volume of the original circular cone

$= \frac{1}{3} \times \pi \times 24^2 \times 12$

$= 2304\,\pi \text{ cm}^3$

Let h cm be the depth of water in the circular cone

$$\frac{(12-h)^3}{12^3} = \frac{2304\pi - 784\pi}{2304\pi}$$

$h = 1.553560732$ **1A**

$\therefore$ The depth of the water $\approx (24 + 1.5535607632)$ cm

≈ 25.55356073 cm

$=$ 25.6 cm (cor. to 3 sig.fig.) **1A**

13. In **Figure 3**, $PQRS$ is a circle. PR and QS intersect at T, and $\angle PTQ = 64°$. It is given that $PR = PS$ and $\overset{\frown}{PQ} : \overset{\frown}{RS} = 3 : 5$.

(a) Find $\angle QPS$.

(4 marks)

(b) If $\overset{\frown}{RS} = 10\pi$ cm, find the area of the shaded region.

(3 marks)

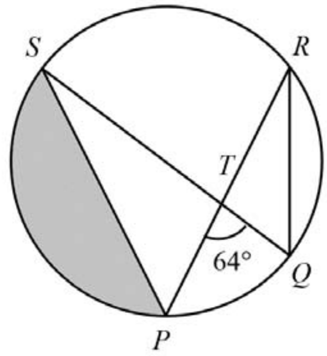


Figure 3

(a) $\angle RPS : \angle PSQ = \overset{\frown}{RS} : \overset{\frown}{PQ}$

$\angle RPS = \frac{5}{3}\angle PSQ$ **1M for** *arcs prop. to ∠s at ⊙^{ce}*

In $\triangle PST$,

$\angle PSQ + \frac{5}{3}\angle PSQ = 64°$

$\angle PSQ = 24°$ **1A**

$\angle RPS = \frac{5}{3} \times 24° = 40°$ for either correct

In $\triangle PRS$,

$\angle PRS = \angle PSR = \frac{180° - 40°}{2} = 70°$

$\angle QSR = 70° - 24° = 46°$

$\angle QPR = \angle QSR = 46°$ **1M for** *∠s in the same segment*

$\angle QPS = \angle QPR + \angle RPS = 46° + 40° = \underline{\underline{86°}}$ **1A**

(b) Let O be the centre and r cm be the radius of the circle.

$\angle SOR = 2\angle SPR = 80°$

$\frac{80°}{360°} \times 2 \times \pi \times r = 10\pi$ **1M for either using** ∠at centre twice ∠ *at* ⊙^{ce}

$r = 22.5$

$\angle POS = 2\angle PRS = 140°$

Area $= \pi \times (22.5)^2 \times \frac{140°}{360°} - \frac{1}{2} \times (22.5)^2 \times \sin 140°$ **1M**

≈ 455.79544

$= 456$ cm^2 (cor to 3 sig. fig.) **1A**

14. Let $f(x) = 2x(x+3)^2 + hx + k$, where h and k are constants. When $f(x)$ is divided by $x^2 - 4$, the remainder is $mx + 20$, where m is a constant. It is given that $f(x)$ is divisible by $x - 2$.

(a) Find h, k and m.

(4 marks)

(b) Someone claims that all the roots of the equation $f(x) = 0$ are integers. Do you agree? Explain your answer.

(4 marks)

(a) Let $f(x) = (x^2 - 4)(ax + b) + mx + 20$, where a and b are constants. **1M**

$\because$ $x - 2$ is a factor of $f(x)$.

$\therefore$ $f(2) = 0$ **1M**

$(2^2 - 4)[a(2) + b] + m(2) + 20 = 0$

$m = \underline{\underline{-10}}$

$f(x) = (x^2 - 4)(ax + b) - 10x + 20 = ax^3 + bx^2 + (-4a - 10)x + 20 - 4b$

From the question,

$f(x) = 2x(x+3)^2 + hx + k = 2x^3 + 12x^2 + (18 + h)x + k$

Compare the coefficients: **1M**

$a = 2$

$b = 12$

$-4a - 10 = 18 + h$

$-4(2) - 10 = 18 + h$

$h = \underline{\underline{-36}}$

$20 - 4b = k$

$k = 20 - 4(12) = \underline{\underline{-28}}$ **1A for all *h*,*k*, *m* correct**

Alternative:

(a) Let $f(x) = (x^2 - 4) \times Q(x) + mx + 20$. **1M**

$\because$ $x - 2$ is a factor of $f(x)$.

$\therefore$ $f(2) = 0$ **1M**

$(2^2 - 4) \times Q(2) + m(2) + 20 = 0$

$m = \underline{\underline{-10}}$

$f(-2) = [(-2)^2 - 4] \times Q(-2) - 10(-2) + 20 = 40$ **1M**

Consider $f(x) = 2x(x+3)^2 + hx + k$

$\because$ $f(2) = 0$

$\therefore$ $2(2)[(2) + 3]^2 + h(2) + k = 0$

$2h + k = -100$ ……………..........(1)

$\because$ $f(-2) = 40$

$\therefore$ $2(-2)[(-2) + 3]^2 + h(-2) + k = 40$

$-2h + k = 44$ ………………(2)

(1) – (2): $4h = -144$

$h = \underline{\underline{-36}}$

(1) + (2): $2k = -56$

$k = \underline{\underline{-28}}$ **1A for all *h*,*k*, *m* correct**

(b) $f(x) = 2x(x+3)^2 - 36x - 28$
$= 2x^3 + 12x^2 - 18x - 28$ **1M**

Using long division,

$$
\begin{array}{r@{}l}
 & \;\; 2x^2 + 16x + 14 \\
x-2 \,\big) & \overline{\;2x^3 + 12x^2 - 18x - 28} \\
 & \underline{\;2x^3 - 4x^2} \\
 & \qquad\quad 16x^2 - 18x - 28 \\
 & \qquad\quad \underline{16x^2 - 32x} \\
 & \qquad\qquad\qquad 14x - 28 \\
 & \qquad\qquad\qquad \underline{14x - 28}
\end{array}
$$

$\therefore \quad f(x) = (x-2)(2x^2 + 16x + 14)$ **1M**
$= 2(x-2)(x+1)(x+7)$

$f(x) = 0$
$2(x-2)(x+1)(x+7) = 0$
$x = 2$ or -1 or -7 **1A**

$\therefore$ Yes, the claim is agreed. **1 f.t.**

SECTION B (35 marks)

15. Suppose 30% of all new drug formulae are active. Out of the active formulae, 60% show side effects. Out of the formulae which are proved to be inactive, 30% can be recomposed to become active, and among these recomposed formulae, 75% show side effect.

(a) (i) What is the probability that a new drug formula will show side effects?

(ii) Given an active drug formula, what is the probability that it will not show side effects?

(5 marks)

(b) Given that a drug formula is active and shows no side effects, the probability that it will be approved by a government is 0.8. Of drug formulae that are active but show side effects, 3% will be approved.

(i) What is the probability that a new drug formula will be approved?

(ii) If 10 independent drug formulae are seeking for approval from the government, what is the probability that exactly half of them will be approved? (3 marks)

(a) (i) Required probability

$= 0.3\times 0.6 + (1-0.3)\times 0.3\times 0.75$ 1M

$= \underline{\underline{0.337\,5}}$ 1A Accept $\frac{27}{80}$ or 0.338 (cor to 3 sig. fig.)

(ii)

Required Probability

$= \frac{0.3\times(1-0.6)+(1-0.3)\times 0.3\times(1-0.75)}{0.3+(1-0.3)\times 0.3}$ 1M for denominator + 1M for numerator

$= \frac{0.1725}{0.51}$

$= \underline{\underline{\frac{23}{68}}}$ or $\underline{\underline{0.338}}$ (cor to 3 sig. fig.) 1A

(b) (i)

Required probability $= 0.337\,5\times 0.03 + 0.1725\times 0.8$

$= 0.148125$

$= \underline{\underline{0.148}}$ (cor. to 3 sig. fig.) 1A (accept 0.148125 or $\frac{237}{1600}$)

(ii)

P(half of them approved) $= C_5^{10}\left(0.148125\right)^5\left(1-0.148125\right)^5$ 1M

≈ 0.008061608

$= \underline{\underline{0.00806}}$ (cor. to 3 sig. fig.) 1A

16. Mr. Wong opens a savings account in a bank. At the beginning of the first year, he deposits $\$x$ in his bank account. In subsequent years, he deposits 8% more than that in the previous year. The interest rate offered by the bank is 2% p.a., compounded yearly. It is given that the balance in his saving account is \$3 374 568 at the end of the 3rd year.

(a) (i) Express, in terms of x, the amount in his savings account at the end of the 2nd year.

(ii) Find x.

(3 marks)

(b) Express, in terms of n, the amount in his savings account at the end of nth year.

(3 marks)

(a) (i) $\$\left[x(1+2\%)+x(1+8\%)\right](1+2\%)$ **1A**

$= \$\left[1.02^2+(1.08)(1.02)\right]x$

$= \$2.142x$

(ii) $\left[1.02^3+(1.08)(1.02)^2+(1.08)^2(1.02)\right]x = 3374568$ **1M** for $(1.08^m)(1.02^n)$

$x = 1\ 000\ 000$ **1A**

(b) $\$\left[(1.02)^n+(1.08)(1.02)^{n-1}+(1.08)^2(1.02)^{n-2}+\cdots+(1.08)^{n-1}(1.02)\right]x$ **1A**

$= \$\dfrac{1.02^n\left[(\frac{1.08}{1.02})^n-1\right]}{\frac{18}{17}-1}x$ **1M**

$= \$17\left[1.08^n-1.02^n\right](1\times10^6)$

$= \$1.7\times10^7\times\left[1.08^n-1.02^n\right]$ **1A**

17. In **Figure 4(a)**, *ABCDEF* is a regular hexagonal paper card. It is given that $AB = 20$ cm.

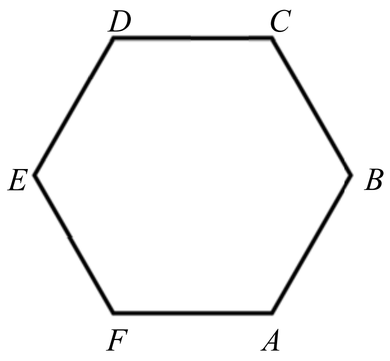


Figure 4(a)

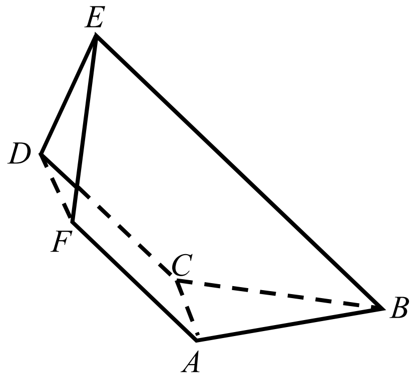


Figure 4(b)

Refer to **Figure 4(b)**, the paper card in **Figure 4(a)** is folded along *EB* such that $AC = DF = 15$ cm.

(a) Find $\angle ABC$. (2 marks)

(b) Find the angle between the planes *ABEF* and *CBED*. (3 marks)

(c) A craftsman built a metal solid statue which is exactly the same as the paper model outlined in **Figure 4(b)**. He claims that the volume of metal used is less than 3100 cm^3. Do you agree? Explain your answer. (4 marks)

(a)

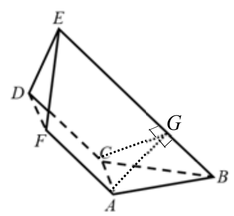
$$\cos\angle ABC = \frac{20^2 + 20^2 - 15^2}{2(20)(20)}$$

$$\angle ABC \approx 44.04862567°$$

$$= \underline{\underline{44.0°}} \text{ (corr to 3 sig. fig.)} \qquad 1\text{A}$$

1M for either

(b) Mark *G* on *BE* such that $CG \perp EB$ and $AG \perp EB$.
The required angle is $\angle AGC$. 1A



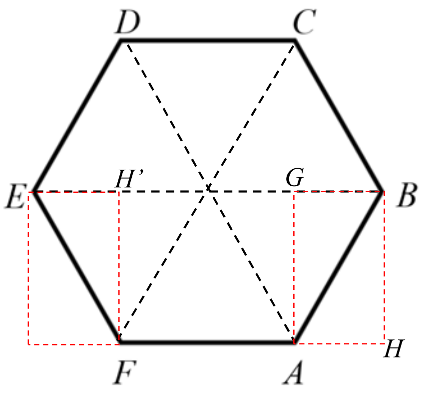
$$AG = CG = \frac{1}{2}\sqrt{20^2 + 20^2 - 2 \times 20^2 \cos 120°} = 10\sqrt{3} \text{ cm} \qquad 1\text{M}$$

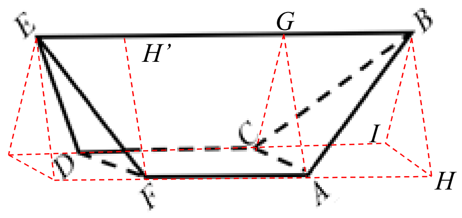
Alternatively, $AG = CG = 20 \times \sin 60° = 10\sqrt{3}$ cm 1M

$$\cos\angle AGC = \frac{\left(10\sqrt{3}\right)^2 + \left(10\sqrt{3}\right)^2 - 15^2}{2\left(10\sqrt{3}\right)\left(10\sqrt{3}\right)}$$

$$\angle AGC \approx 51.31781255°$$

$$= \underline{\underline{51.3°}} \text{ (corr to 3 sig. fig.)} \qquad 1\text{A}$$





(c) Volume of the solid

$= \text{Area of } \Delta AGC \times GH' + 2 \times \text{volume of tetrahedron } ABCG$ 1M

$$= \underbrace{\frac{1}{2}\left(10\sqrt{3}\right)^2}_{\text{1M for area of }\Delta} \times \sin\left(51.31781255°\right) \times 20 + \underbrace{2 \times \frac{1}{3} \times \frac{1}{2}\left(10\sqrt{3}\right)^2 \times \sin\left(51.31781255°\right) \times 10}_{\text{1M for volume of tetrahedron}}$$

≈ 3122.498999

$> 3100 \text{ cm}^3$

Thus the claim is disagreed. 1A f.t.

<u>Alternative Method</u>

Volume of the solid

$= \text{Area of } \Delta AGC \times EB - 2 \times \text{volume of pyramid } AHICB$ 1M

$$= \underbrace{\frac{1}{2}\left(10\sqrt{3}\right)^2}_{\text{1M for area of }\Delta} \times \sin\left(51.31781255°\right) \times 40 - \underbrace{2 \times \frac{1}{3} \times 10 \times 15 \times \left(10\sqrt{3}\right) \times \cos\left(25.65890627°\right)}_{\text{1M for volume of pyramid}}$$

≈ 3122.498999

$> 3100 \text{ cm}^3$

Thus the claim is disagreed. 1A f.t.

18. Let $f(x)=\dfrac{1}{6k-6}\left(x^2-2kx+4k^2+6k-9\right)$, where $k>1$. Denote the vertex of the graph of $y=f(x)$ by V.

(a) Using the method of completing the square, express the coordinates of V, in terms of k.

(3 marks)

(b) Let F and P be a fixed point and a moving point respectively, in the same coordinate system, such that P maintains an equal distance from F and a straight line L: $y=3-k$. The locus of P is the graph of $y=f(x)$, which intersects with a straight line $y=x$ at points B and C. Denote the mid-point of BC by M.

(i) Express the coordinates of F, in terms of k.

(ii) Find the range of values of k.

(iii) Express the coordinates of M, in terms of k.

(iv) Does the perpendicular bisector of FV pass through M? Explain your answer.

(9 marks)

(a) $f(x)=\dfrac{1}{6k-6}\left(x^2-2kx+k^2+3k^2+6k-9\right)$ **1M**

$=\dfrac{1}{6k-6}\left[(x-k)^2+3(k^2+2k-3)\right]$

$=\dfrac{1}{6k-6}(x-k)^2+\dfrac{3(k+3)(k-1)}{6(k-1)}$

$=\dfrac{1}{6k-6}(x-k)^2+\dfrac{k+3}{2}$ **1M** in a form of $a(x-b)^2+c$

Coordinates of V are $\left(k,\ \dfrac{k+3}{2}\right)$. **1A**

(b) (i) Note that x-coordinate of F is k. Let y-coordinate of F be a.

$\dfrac{a+3-k}{2}=\dfrac{k+3}{2}$ **1M**

$a=2k$

Coordinates of F are $(k, 2k)$. **1A**

(ii) Consider $\begin{cases} y=x \\ y=\dfrac{1}{6k-6}\left(x^2-2kx+4k^2+6k-9\right) \end{cases}$.

$(6k-6)x=\left(x^2-2kx+4k^2+6k-9\right)$

$x^2+(6-8k)x+4k^2+6k-9=0$

$\Delta=(6-8k)^2-4(4k^2+6k-9)$

$=48k^2-120k+72$ **1A**

$=24(2k-3)(k-1)$

$\Delta>0$ **1M**

$k<1$ (rej) or $k>\dfrac{3}{2}$ **1A (**withhold if $k<1$ is not rejected)

(iii) x-coordinate of $M = \dfrac{-(6-8k)}{2} = 4k-3$ **1M**

Coordinates of M are $(4k-3,\ 4k-3)$. **1A**

(iv) Equation of the perpendicular bisector of FV:

$$y = \frac{2k + \frac{k+3}{2}}{2} = \frac{5k+3}{4}$$ **1A**

Consider $\dfrac{5k+3}{4} = 4k-3$.

$5k+3 = 16k-12$

$k = \dfrac{15}{11}$ which contradicts with $k > \dfrac{3}{2}$.

$\therefore$ No, perpendicular bisector of FV does not pass through M. **1**