SECTION A(1) (35 marks)

1. Simplify $\frac{\left(a^3b^2\right)^4}{a^{-2}b^{12}}$ and express your answer with positive indices.

(3 marks)

Solution

$\frac{\left(a^3b^2\right)^4}{a^{-2}b^{12}} = \frac{a^{12}b^8}{a^{-2}b^{12}}$ **1M** $(xy)^n = x^n y^n$

$= a^{14}b^{-4}$ **1M** for $\frac{a^x}{a^y} = a^{x-y}$

$= \frac{a^{14}}{b^4}$ **1A**

2. If $h + \frac{3}{x-4} = 2h - 3$, express x in terms of h.

(3 marks)

Solution

$h + \frac{3}{x-4} = 2h - 3$

$\frac{3}{x-4} = h - 3$ **1M** for putting x on one side

$x - 4 = \frac{3}{h-3}$

$x = \frac{3}{h-3} + 4$

$= \frac{3 + 4(h-3)}{h-3}$ **1M** for fraction addition

$= \frac{4h-9}{h-3}$ **1A**

3. Factorize
(a) $9m^2 - 16$,
(b) $3m^2n - 11mn - 20n$,
(c) $9m^2 - 16 - 3m^2n + 11mn + 20n$.

(4 marks)

Solution

(a) $9m^2 - 16$
$= (3m)^2 - 4^2$
$= (3m + 4)(3m - 4)$ **1A**

(b) $3m^2n - 11mn - 20n$
$= n(3m^2 - 11m - 20)$
$= n(3m + 4)(m - 5)$ **1A**

(c) $9m^2 - 16 - 3m^2n + 11mn + 20n$
$= 9m^2 - 16 - (3m^2n - 11mn - 20n)$
$= (3m + 4)(3m - 4) - n(3m + 4)(m - 5)$ **1M** for (a) – (b)
$= (3m + 4)(3m - 4 - mn + 5n)$ **1A**

4. (a) Find the range of values of x which satisfy both $\frac{4x-3}{5} \geq 2x-3$ and $3-2x<1$.
(b) How many integers satisfy both inequalities in (a)?

(4 marks)

Solution

(a) $\frac{4x-3}{5} \geq 2x-3$ and $3-2x<1$
$4x-3 \geq 10x-15$ and $2<2x$
$12 \geq 6x$ and $x>1$
i.e. $x \leq 2$ and $x>1$ **1A + 1A**
$\therefore\ 1<x\leq 2$ **1M**

(b) Only 1 integer, namely 2, satisfies the result in (a).
Number of integers satisfying both inequalities = 1 **1A**

5. A table is sold at a discount of 20% on its marked price. The selling price of the table is \$2780.
(a) Find the marked price of the table.
(b) After selling the table, the percentage profit is 25%. Find the cost of the table.

(4 marks)

Solution

(a) Let \$$m$ be the marked price of the table.

$(1 - 20\%)m = 2780$ — **1M** 2780÷(1-20%)

$m = 3475$ — **1A**

Thus, the marked price of the table is \$3475.

(b) Let \$$c$ be the cost of the table

$(1 + 25\%)c = 2780$ — **1M** 2780÷(1+25%)

$c = 2224$ — **1A**

Thus, the cost price of the table is \$2224.

6. In a playgroup, the ratio of the number of baby boys to the number of baby girls is 11 : 7. If 8 baby boys and 6 baby girls join the playgroup, then the ratio of the number of baby boys to the number of baby girls is 3 : 2. Find the original number of baby boys in the playgroup.

(4 marks)

Solution

Let $11k$ and $7k$ be the original numbers of baby boys and girls in the playgroup respectively, where k is a positive constant. — **1A** (can be absorbed)

$$\frac{11k+8}{7k+6} = \frac{3}{2}$$ — **1M + 1A**

$22k - 21k = 18 - 16$

$k = 2$

Thus, the original number of baby boys is 22. — **1A**

Let x and y be the original numbers of baby boys and girls respectively.

$$\begin{cases} \dfrac{x}{y} = \dfrac{11}{7} \\ \dfrac{x+8}{y+6} = \dfrac{3}{2} \end{cases}$$ — **1A + 1A**

$$\begin{cases} 7x = 11y \\ 2x - 3y = 2 \end{cases}$$

So, we have $2x - 3\left(\dfrac{7x}{11}\right) = 2$. — **1M**

Solving, we have $x = 22$. — **1A**

Thus, the original number of baby boys in the playground is 22.

7. ABC is a triangle with $\angle B = 90°$. It is given that $AB = (3k - 2)$ cm, $AC = (7k - 2)$ cm and $BC = (5k + 4)$ cm, where k is a constant. Find k.

(3 marks)

Solution

$(3k-2)^2+(5k+4)^2=(7k-2)^2$ **1M**

$9k^2-12k+4+25k^2+40k+16=49k^2-28k+4$

$15k^2-56k-16=0$ **1A** can be absorbed

$k=4$ or $-\frac{4}{15}$ (rej.) **1A**

8. L is a straight line which is perpendicular to a straight line L_1: $3x+4y+8=0$. It is given that the x-intercept of L is -3.

(a) Find the equation of L.

(b) L_1 and L intersect at P. L_1 and L cut the y-axis at A and B respectively. Find the ratio of the area of ΔOPA to that of ΔOPB.

(5 marks)

Solution

(a) Slope of $L_1=-\frac{3}{4}$

Slope of $L=\frac{4}{3}$ **1A**

Equation of L:

$y=\frac{4}{3}(x+3)$ **1M**

$y=\frac{4}{3}x+4$ **1A** or $4x-3y+12=0$

(b) Required ratio

$= OA : OB$ **1M**

$= 2 : 4$

$= 1 : 2$ **1A**

9. In Figure 1, the pie chart shows the distribution of the number of pairs of shoes owned by the students in a group.

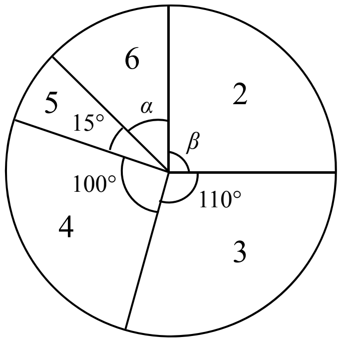

Figure 1

If a student is randomly selected from the group, the probability that the selected student has more than 5 pairs of shoes is $\frac{1}{8}$.

(a) Find the values of α and β.

(b) Find the mean of the distribution.

(5 marks)

Solution

(a) $\frac{\alpha}{360°} = \frac{1}{8}$ — **1M** either

$\alpha = 45°$ — **1A**

$\beta = 360° - 110° - 100° - 15° - \alpha$

$= 90°$ — **1A**

(b) Required mean

$= \frac{2(90) + 3(110) + 4(100) + 5(15) + 6(45)}{360}$ — **1M**

$= \frac{251}{72}$ — **1A** r.t. 3.49

SECTION A(2) (35 marks)

10. The stem-and-leaf diagram below shows the distribution of the weights (in kg) of the players in a volleyball team.

Stem (tens)	Leaf (units)					
4	0	1	3	6	8	
5	2	2	2	2	7	9
6	0	2	4	9	9	
7	1	1	3	k		

It is given that the range of the above distribution is twice of its inter-quartile range.

(a) Find k. (4 marks)

(b) If a player is randomly selected from the team, find the probability that the weight of the selected player is higher than the mode of the distribution. (1 mark)

Solution

(a) Range $= (70 + k - 40)$ kg **1M**

$= (30 + k)$ kg

Inter-quartile range $= \left(69 - \dfrac{48+52}{2}\right)$ kg **1M**

$= 19$ kg

$30 + k = 2(19)$ **1M**

$k = 8$ **1A**

(b) The required probability $= \dfrac{11}{20}$. **1A**

11. Figure 2 shows the graphs for car A and car B travelling on the same straight road between town P and town Q during the period 7:30 to 10:00 in a morning. Car B travels at a constant speed during the period. It is given that town P and town Q are 240 km apart.

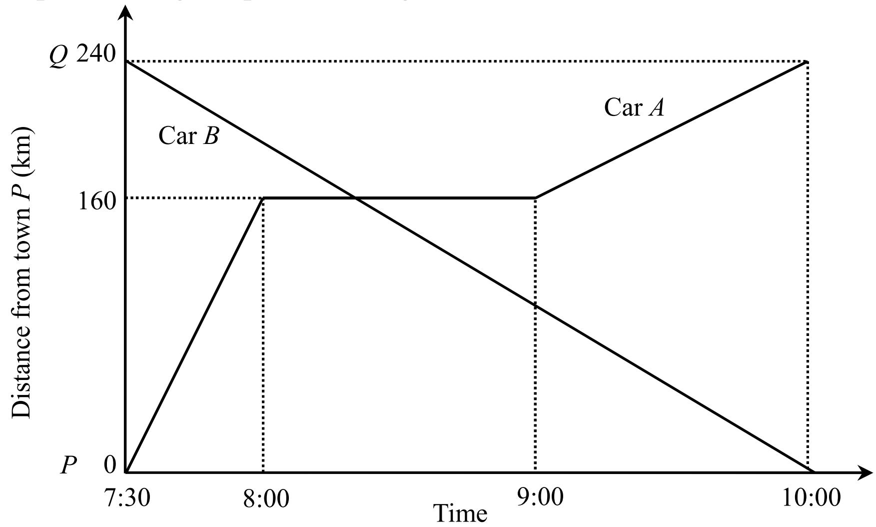

Figure 2

(a) Find the distance of car B from town Q at 8:00 in the morning. (2 marks)

(b) At what time do car A and car B meet? (2 marks)

(c) The driver of car A claims that, after 2 cars meet each other, the average speed of car A is less than that of car B until 10:00. Do you agree? Explain your answer. (2 marks)

Solution

(a) The distance of car B from town Q $= \dfrac{240}{2.5\times 60}\times 30$ **1M** for $\dfrac{240}{2.5}$

$= 48$ km **1A**

(b) Number of minutes needed for Car B to travel 80 km $= 80 \div \dfrac{240}{2.5\times 60}$ **1M**

$= 50$ mins

Cars A and B meet at 8:20 in the morning. **1A**

(c) During the period 8:20 to 10:00 in the morning, Car B travels 160km while Car A travels 80km which is less than 160km. **1M**

The average speed of car A is lower than that of car B.

The claim is agreed. **1A** f.t.

The average speed of car A during the period $= \dfrac{80}{100}$ ← **1M** either $= 0.8$ km/min or 48 km/h The average speed of car B during the period $= \dfrac{160}{100}$ W ← $= 1.6$ km/min or 96 km/h Note that 0.8<1.6. The average speed of car A is lower than that of car B. The claim is agreed. **1A** f.t

12. In Figure 3, the solid consists of a right circular cone and a hemisphere with a common base. The base radius and the height of the circular cone are r cm and 16 cm respectively. It is given that the ratio of the curved surface area of the circular cone to that of the hemisphere is 5 : 6.

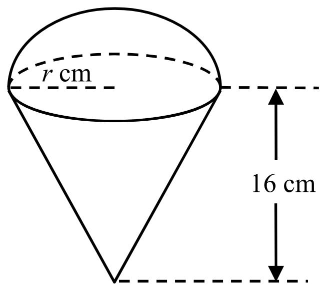

Figure 3

(a) Find the value of r. (2 marks)

(b) Express the volume of the solid in terms of π. (2 marks)

Solution

(a) $\dfrac{\pi r\sqrt{r^2+16^2}}{2\pi r^2} = \dfrac{5}{6}$ **1M** for either $2\pi r^2$ or πrl

$\sqrt{r^2+256} = \dfrac{5r}{3}$

$r^2 + 256 = \dfrac{25}{9}r^2$

$r^2 = 144$

$r = 12$ or $r = -12$ (*rejected*) **1A**

(b) The required volume

$= \dfrac{1}{3}\pi(12)^2(16) + \dfrac{1}{2}\left[\dfrac{4}{3}\pi(12)^3\right]$ **1M** for $\dfrac{1}{3}\pi r^2 h$ or $\dfrac{2}{3}\pi r^3$

$= 1920\pi$ cm^3 **1A**

13. The cost of a wardrobe of weight w kg is \$$C$. C is partly constant and partly varies as $\sqrt{w}$. When $w=16$, $C=1520$ and when $w=25$, $C=1650$.

(a) Find the cost of a wardrobe of weight 36 kg. (4 marks)

(b) Someone claims that the cost of a wardrobe of weight 121 kg is higher than the total cost of two wardrobes of weight 36 kg. Is the claim correct? Explain your answer (2 marks)

Solution

(a) Let $C=k_1+k_2\sqrt{w}$, where k_1 and k_2 are non-zero constants. **1M**

$1520=k_1+\sqrt{16}k_2$ **1M** either one

$k_1+4k_2=1520$ ......(1)

$1650=k_1+\sqrt{25}k_2$

$k_1+5k_2=1650$ ......(2)

(2) − (1): $k_2=130$

By substituting $k_2=130$ into (1), we have

$k_1+4(130)=1520$

$k_1=1000$

$\therefore\ C=1000+130\sqrt{w}$ **1A** (can be absorbed)

The required cost $=\$(1000+130\sqrt{36})=\$1780$ **1A**

(b) When w = 121,

$C=1000+130\sqrt{121}$ **1M**

$=2430$

$<2\times1780$

$\therefore$ The claim is incorrect. **1A** f.t.

14. The cubic polynomial f (x) is divisible by $x - 2$. When f (x) is divided by $x^2 - 4$, the remainder is $kx - 8$, where k is a constant.

(a) Find k. (3 marks)

(b) It is given that when f (x) is divided by x, the remainder is 16. When f (x) is divided by $x + 3$, the remainder is –65. Someone claims that all the roots of the equation f $(x) = 0$ are integers. Is the claim correct? Explain your answer. (5 marks)

Solution

(a) Let $f(x) = (x^2 - 4)q(x) + (kx - 8)$, where $q(x)$ is a polynomial. **1M**

By the factor theorem,

$$f(2) = 0 \quad \textbf{1M}$$

$$(2^2 - 4)\,q(2) + (2k - 8) = 0$$

$$2k - 8 = 0$$

$$k = 4 \quad \textbf{1A}$$

(b) Let $f(x) = (x^2 - 4)(ax + b) + (4x - 8)$, where a and b are constants. **1M**

By the remainder theorem,

$$f(0) = 16 \quad \textbf{1M} \text{ either}$$

$$(0^2 - 4)(0 + b) + (0 - 8) = 16$$

$$b = -6 \quad \textbf{1A}$$

$$f(-3) = -65$$

$$[(-3)^2 - 4](-3a - 6) + [4(-3) - 8] = -65$$

$$a = 1 \quad \textbf{1A}$$

$$f(x) = 0$$

$$(x^2 - 4)(x - 6) + (4x - 8) = 0$$

$$(x - 2)(x + 2)(x - 6) + 4(x - 2) = 0$$

$$(x - 2)[(x + 2)(x - 6) + 4] = 0$$

$$(x - 2)(x^2 - 4x - 8) = 0$$

$$x = 2 \text{ or } x^2 - 4x - 8 = 0$$

$$x = \frac{4 \pm \sqrt{16 - 4(1)(-8)}}{2}$$

$$= 2 \pm 2\sqrt{3}$$

$\therefore$ The roots of the equation $x^2 - 4x - 8 = 0$ are not integers.

$\therefore$ The claim is incorrect. **1A** f.t.

Let $f(x) = (x-2)(ax^2 + bx + c)$ **1M**

$$\left.\begin{array}{l} f(-2) = -16 \ldots(1) \\ f(0) = 16 \ldots(2) \\ f(-3) = -65 \ldots(3) \end{array}\right\}$$ **1M** either one

By (2),

$(0-2)(c) = 16$

$c = -8$

Solve (1) and (3)

$$\begin{cases} -16 = -4(4a - 2b - 8) \\ -65 = -5(9a - 3b - 8) \end{cases}$$

We have

$a = 1,$ **1A**

$b = -4$ **1A**

$f(x) = 0$

$(x-2)(x^2 - 4x - 8) = 0$

$x = 2$ or $x^2 - 4x - 8 = 0$

$$x = \frac{4 \pm \sqrt{16 - 4(1)(-8)}}{2}$$

$$= 2 \pm 2\sqrt{3}$$

$\therefore$ The roots of the equation $x^2 - 4x - 8 = 0$ are not integers.

$\therefore$ The claim is incorrect. **1A** f.t.

15. The coordinates of the points A and B are $(0, 5)$ and $(3, 1)$ respectively. Let P be a moving point in the rectangular coordinate plane such that $AP \perp PB$. The locus of P lies on a circle Γ.

(a) Find the equation of Γ. (2 marks)

(b) Q is a moving point on Γ. C (0,1) is a point in the same plane.

(i) Does Γ pass through C? Explain your answer.

(ii) When the slope of CQ is $\sqrt{3}$, someone claims that $\angle CAQ$ is greater than 100°. Do you agree? Explain your answer. (4 marks)

Solution

(a) Let (x, y) be the coordinates of P.

$\therefore \quad (y-5)(y-1) = -x(x-3)$ **1M**

$x^2 + y^2 - 3x - 6y + 5 = 0$

$\therefore$ The equation of Γ is $x^2 + y^2 - 3x - 6y + 5 = 0$. **1A**

Centre: $(\frac{0+3}{2}, \frac{5+1}{2})$	**1M** either
Radius $= \sqrt{(0-3)^2 + (5-1)^2} \div 2$ $= \frac{5}{2}$	
$\therefore$ The equation of Γ is $(x - \frac{3}{2})^2 + (y-3)^2 = \frac{25}{4}$.	**1A**

(b) (i) Substituting $(0, 1)$ into Γ, we have

L.H.S. $= 0^2 + 1^2 - 3(0) - 6(1) + 5 = 0$

R.H.S. $= 0$

$\therefore$ Γ passes through C. **1**

(ii)

$\because$ The slope of CQ is $\sqrt{3}$.

$\therefore$ $\tan \angle BCQ = \sqrt{3}$ **1M**

$\angle BCQ = 60°$

$\angle ACQ = \angle ACB - \angle BCQ$

$= 90° - 60°$

$= 30°$

$\tan \angle ABC = \frac{4}{3}$ **1M**

$\angle ABC \approx 53.13010235°$

$\angle AQC = \angle ABC$ ($\angle$s in the same segment)

$\angle CAQ = 180° - \angle AQC - \angle ACQ$ ($\angle$ sum of $\triangle$)

$\approx 180° - 53.13010235° - 30°$

$\approx 96.86989765°$

$< 100°$

$\therefore$ The claim is disagreed. **1A** f.t.

SECTION B (35 marks)

16. A queue is randomly formed by 6 boys and 3 girls.

(a) How many different queues can be formed? (1 mark)

(b) Find the probability that all the boys are next to each other in the queue. (3 marks)

Solution

(a) $P_9^9 = 362880$ **1A**

(b) $\dfrac{4!6!}{362880}$ **1M** for denominator **+ 1M** for 4!

$= \dfrac{1}{21}$ **1A** r.t. 0.0476

17. In an examination, the mean of the scores of a class of students is 70. The range of the scores of these students is at most 70. Johnny scored the lowest in the examination. His score is 25 and his standard score is –3. Can the standard score of any student exceed 2? Explain your answer. (4 marks)

Solution

The standard deviation of the scores

$= \dfrac{25-70}{-3}$ **1M**

$= 15$

Highest possible score of any student

$= 25 + 70$

$= 95$ **1A**

Greatest possible standard score of any student

$= \dfrac{95-70}{15}$ **1M**

$= \dfrac{5}{3}$

< 2

No, the standard score of any student cannot exceed 2. **1** f.t.

The standard deviation of the scores

$= \dfrac{25-70}{-3}$ **1M**

$= 15$

Suppose the standard score of a student exceeds 2.

Then the student's score $> 70 + 2(15)$ **1M**

$= 100.$

However, the range $> 100 - 25 = 75$ **1A**

> 70, contradiction.

No, the standard score of any student cannot exceed 2. **1** f.t.

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18. Let $G(n)$ be the nth term of a geometric sequence. It is given that $G(3)=256$ and $G(6)=\frac{1}{16}$.

(a) Find $G(1)$.

(2 marks)

(b) Suppose $G(n)=2^{A(n)}$, for all positive integers n.

(i) Express $A(n)$ in terms of n.

(ii) Hence, or otherwise, find the greatest value of k such that $G(1)G(2)G(3)\cdots G(k)>2022$.

(5 marks)

Solution

(a) Let r be and the common ratio of the sequence.

$$\begin{cases} G(1)\cdot r^2 = 256 \\ G(1)\cdot r^5 = \frac{1}{16} \end{cases}$$ **1M** either one

$$r^3 = \frac{1}{4096}$$

$$r = \frac{1}{16}$$

$G(1)=65536$ **1A**

(b) (i) $G(n)=65536\cdot\left(\frac{1}{16}\right)^{n-1}$

$=2^{16}\cdot 2^{-4(n-1)}$

$=2^{20-4n}$ **1M** expressing in powers of 2

$A(n)=20-4n$ **1A**

(ii) $G(1)G(2)G(3)\cdots G(k)>2022$

$2^{A(1)}\cdot 2^{A(2)}\cdot 2^{A(3)}\cdots 2^{A(k)}>2022$

$2^{A(1)+A(2)+A(3)+\cdots+A(k)}>2022$

$A(1)+A(2)+A(3)+\cdots+A(k)>\frac{\log 2022}{\log 2}$ **1M**

$\frac{k}{2}[16+(20-4k)]>\frac{\log 2022}{\log 2}$ **1M**

$2k^2-18k+\frac{\log 2022}{\log 2}<0$

$0.658227445<k<8.341772554$

Greatest value of k is 8. **1A**

19. (a) In Figure 4(a), $ABCD$ is a paper card in the form of a quadrilateral. It is given that $\angle ABC = 120°$, $AB = AD = 9$ cm and $BC = CD = 15$ cm. Find AC and $\angle CAB$.

(4 marks)

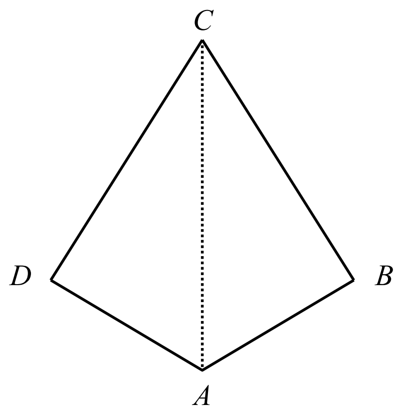

Figure 4(a)

(b) The paper card in Figure 4(a) is folded along AC such that AB and AD lie on the horizontal ground as shown in Figure 4(b). It is given that the angle between the plane ABC and the horizontal ground is 40°. Someone claims that the shortest distance from C to the horizontal ground is less than 8.5 cm. Do you agree? Explain your answer.

(4 marks)

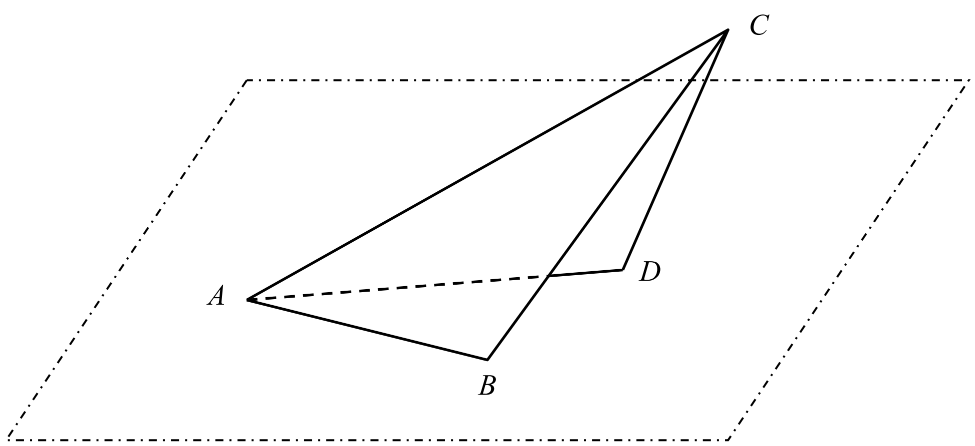

Figure 4(b)

(a) In ΔABC, by the cosine formula,

$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos\angle ABC$ **1M**

$AC = \sqrt{9^2 + 15^2 - 2(9)(15)\cos 120°}$ cm

$= 21$ cm **1A**

By the sine formula,

$\dfrac{AC}{\sin\angle ABC} = \dfrac{BC}{\sin\angle CAB}$ or $\cos\angle CAB = \dfrac{CA^2 + AB^2 - BC^2}{2(CA)(AB)}$ **1M**

$\sin\angle CAB = \dfrac{15\sin 120°}{21}$

$\angle CAB \approx 38.2132107°$ or $141.786793°$ (rej.)

$\approx 38.2°$ **1A**

(b) Let E be the foot of the perpendicular from C to AB.

$CE = AC \sin\angle CAB$ **1M**

$\approx 21 \sin 38.2132107°$ cm

≈ 12.99038106 cm

Denote the projection of C on the horizontal ground by C'.

The angle between the plane ABC and the horizontal ground is $\angle CEC'$. **1M**

$CC' = CE \sin\angle CEC'$ **1M**

$\approx 12.99038106 \sin 40°$ cm

≈ 8.350055988 cm

< 8.5 cm

$\therefore$ The claim is not agreed. **1** f.t.

20. Let $\mathrm{f}(x)=x^2-(4k+2)x+4k^2+7k-4$, where k is a constant and $k\neq -1$. Let U be the vertex of the graph of $y=\mathrm{f}(x)$.

(a) By the method of completing the square, find the coordinates of U in terms of k.
(3 marks)

(b) Let $\mathrm{g}(x)$ be a function. It is known that the graph of $y=\mathrm{g}(x)$ can be obtained by translating the graph of $y=\mathrm{f}(x)$ 5 units downwards and $(k+1)$ units leftwards. Let V be the vertex of the graph of $y=\mathrm{g}(x)$.

(i) Find $\mathrm{g}(x)$ in terms of k.
Hence, or otherwise, write down the coordinates of V in terms of k.

(ii) Let W be the point $(2k+6,\ 2k-6)$.
(1) Find the equation of the circle passing through U, V and W in terms of k.
(2) Someone claims that as k varies, the locus of the circumcentre of ΔUVW is a straight line. Do you agree? Explain your answer.
(9 marks)

<u>Solution</u>

(a) $\mathrm{f}(x) = x^2-(4k+2)x+4k^2+7k-4$

$= x^2-(4k+2)x+(2k+1)^2-(2k+1)^2+4k^2+7k-4$ **1M**

$= (x-2k-1)^2-4k^2-4k-1+4k^2+7k-4$

$= (x-2k-1)^2+3k-5$ **1A**

$\therefore\ U=(2k+1,\ 3k-5)$ **1A**

(b) (i) $\mathrm{g}(x) = \mathrm{f}(x+k+1)-5$ **1A**

$= \left(\left[(x+k+1)-2k-1\right]^2+3k-5\right)-5$ **1M**

$= (x-k)^2+3k-10$

$V=(k,\ 3k-10)$ **1A**

(ii) (1) $m_{UV} = \dfrac{(3k-5)-(3k-10)}{2k+1-k} = \dfrac{5}{k+1}$

$m_{UW} = \dfrac{(2k-6)-(3k-5)}{(2k+6)-(2k+1)} = -\dfrac{k+1}{5}$

$m_{UV} \cdot m_{UW} = \left(\dfrac{5}{k+1}\right)\left(-\dfrac{k+1}{5}\right)$

$= -1$

$\therefore\ UV \perp UW$

$\therefore\ VW$ is a diameter of the circle UVW (converse, $\angle$ in semi-circle)

$\therefore$ The equation of the circle:

$(x-k)[x-(2k+6)]+[y-(3k-10)][y-(2k-6)]=0$ **1M+1A**

$x^2+y^2-[k+(2k+6)]x-[(2k-6)+(3k-10)]y+k(2k+6)+(3k-10)(2k-6)=0$

i.e. $x^2+y^2-(3k+6)x-(5k-16)y+8k^2-32k+60=0$ **1A**

<u>Alternative Solution 1</u>

$m_{UV} = \dfrac{(3k-5)-(3k-10)}{2k+1-k} = \dfrac{5}{k+1}$

$m_{UW} = \dfrac{(2k-6)-(3k-5)}{(2k+6)-(2k+1)} = -\dfrac{k+1}{5}$

$m_{UV} \cdot m_{UW} = \left(\dfrac{5}{k+1}\right)\left(-\dfrac{k+1}{5}\right) = -1$

$\because UV \perp UW$

$\therefore VW$ is a diameter of the circle UVW (converse, $\angle$ in semi-circle)

Centre of the circle $= \left(\dfrac{k+(2k+6)}{2}, \dfrac{(3k-10)+(2k-6)}{2}\right)$ ← **1M** (either)

$= \left(\dfrac{3k+6}{2}, \dfrac{5k-16}{2}\right)$

Radius $= \dfrac{1}{2}VW$

$= \dfrac{1}{2}\sqrt{[(2k+6)-k]^2+[(2k-6)-(3k-10)]^2}$ ←

$= \dfrac{1}{2}\sqrt{2k^2+4k+52}$

$\therefore$ The equation of the circle:

$\left(x-\dfrac{3k+6}{2}\right)^2+\left(y-\dfrac{5k-16}{2}\right)^2=\left(\dfrac{1}{2}\sqrt{2k^2+4k+52}\right)^2$ **1A**

$x^2-(3k+6)x+y^2-(5k-16)y+\dfrac{9k^2+36k+36+25k^2-160k+256-2k^2-4k-52}{4}=0$

i.e. $x^2+y^2-(3k+6)x-(5k-16)y+8k^2-32k+60=0$ **1A**

Alternative Solution 2

Let the equation be $x^2+y^2+Dx+Ey+F=0$.

$$\begin{cases}(2k+1)^2+(3k-5)^2+(2k+1)D+(3k-5)E+F=0 & ...(1)\\ (k)^2+(3k-10)^2+(k)D+(3k-10)E+F=0 & ...(2)\\ (2k+6)^2+(2k-6)^2+(2k+6)D+(2k-6)E+F=0 & ...(3)\end{cases}$$ **1M** (either one)

$(1)-(2): 3k^2+34k-74+(k+1)D+5E=0...(4)$

$(3)-(2): -2k^2+60k-28+(k+6)D+(4-k)E=0...(4)$

From (4),

$5E=-(k+1)D-3k^2-34k+74 \ldots(6)$

Sub. (6) into $5\times(-4)$,

$-10k^2+300k-140+5(k+6)D+(4-k)[-(k+1)D-3k^2-34k+74]=0$

$5(k+6)D-(4-k)(k+1)D=(4-k)(3k^2+34k-74)+10k^2-300k+140$

$(k^2+2k+26)D=-3k^3-12k^2-90k-156$

$D=\dfrac{-3(k+2)(k^2+2k+26)}{k^2+2k+26}$

$D=-3(k+2)$ **1A** (either one)

Sub $D=-3(k+2)$ into (6),

$5E=-(k+1)[-3(k+2)]-3k^2-34k+74$

$5E=3(k^2+3k+2)-3k^2-34k+74$

$5E=-25k+80$

$E=-5k+16$

Sub. $D=-3(k+2)$ and $E=-5k+16$ into (2),

$(k)^2+(3k-10)^2+(k)[-3(k+2)]+(3k-10)(-5k+16)+F=0$

$-F=k^2+9k^2-60k+100-3k^2-6k-15k^2+98k-160$

$-F=-8k^2+32k-60$

$F=8k^2-32k+60$

The equation is $x^2+y^2-3(k+2)x+(16-5y)+8k^2-32k+60=0$. **1A**

(2) Let the circumcentre of ΔUVW be (x, y).

Then $x = \dfrac{3k+6}{2}$ and $y = \dfrac{5k-16}{2}$. **1M**

$$k = \frac{2x-6}{3}$$

$$y = \frac{5\left(\dfrac{2x-6}{3}\right)-16}{2}$$

$$= \frac{5}{3}x - 13$$

$\therefore$ The equation of the locus of the circumcentre of ΔUVW is

$y = \dfrac{5}{3}x - 13$. (or $5x - 3y - 39 = 0$) **1A**

i.e. The locus of the circumcentre of ΔUVW is a straight line as k varies.

$\therefore$ The claim is agreed. **1A** f.t.

Let the circumcentre of ΔUVW be (x, y).

Then $x = \dfrac{3k+6}{2}$ and $y = \dfrac{5k-16}{2}$. **1M**

$$k = \frac{2x-6}{3} \qquad k = \frac{2y-16}{5}$$

$$\frac{2x-6}{3} = \frac{2y+16}{5}$$

$$10x - 30 = 6y + 48$$

$$10x - 6y - 78 = 0$$

$5x - 3y - 39 = 0$ **1A**

which is a straight line

$\therefore$ The claim is agreed. **1A** f.t.